Homework 6

Starting from the formula for the Lindhard dielectric function,

$$\epsilon(q,\omega) = 1 - \frac{8\pi e^2}{Vq^2} \sum_{\vec{k}} \frac{f_{\vec{k}} - f_{\vec{q}+\vec{k}}}{\epsilon_{\vec{k}} - \epsilon_{\vec{q}+\vec{k}} - \hbar\omega + i\eta}, \tag{1}$$

show that its imaginary part ϵ_2 is given by

$$\epsilon_2(q,\omega) \approx \frac{\pi}{2} \frac{\lambda^2}{q^2} \frac{\omega}{q v_{\mathrm{F}}} \tag{2}$$

in the limit $q \ll k_{\mathrm{F}}$ and $\omega \ll q v_{\mathrm{F}}$. Here λ is the Thomas-Fermi save vector and v_{F} is the Fermi velocity. Use that to find the imaginary part of the phonon frequency and hence the phonon lifetime for a simple metal as limited by electron-hole pair excitation for the Bohm-Staver approximation as discussed in class.