Homework IV - version of Feb. 20

1. Do problem 5(a) on page 113 of Sander.

2. (a) Write the expression for the Debye-Waller e^{-2W} for a system of dimensionality d. Evaluate, using the Debye model, for $d = 1$ and $d = 2$. Discuss the meaning of your results, in the context of the definition of the exponent W. Are they dependent on the use of the Debye model?

   (b) Consider problem 2 on page 112 of Sander. Would the Debye-Waller factor for this (unphysical) model have the same anomaly as what you found in 2 dimensions above? Why? See if you can answer this without actually doing this Sander problem.

3. Read Section 6.1.1 starting on page 115 of Sander. There are errors in the presentation. Try to find them, and write down the appropriate corrections.

4. Consider a system with N particles (fermions). To avoid notational complication, assume that all of them have spin up. Introduce a complete set of single particle spatial basis functions $\phi_i(\vec{x})$, and an operator $a_i^\dagger$ that creates one of these spin up particles in the spatial state enumerated by i. Suppose these particles are subject to a single particle potential

$$\hat{V} = \sum_{\alpha=1}^{N} V(\vec{x}_\alpha) = \sum_{i,j} \langle i|V|j\rangle a_i^\dagger a_j,$$

where

$$\langle i|V|j\rangle = \int d^3x\, \phi_i^*(\vec{x}) V(\vec{x}) \phi_j(\vec{x}).$$

Use the occupation number representation $\Psi(n_1, n_2, ...)$ to calculate the matrix element of $\hat{V}$ between the state

$$\Psi(n_1, n_2, ..., n_k, ..., n_l, ...),$$

and the state

$$\Psi(n_1, n_2, ..., n_k - 1, ..., n_l + 1, ...),$$

using the identities derived in class for $a_i\Psi$ and $a_i^\dagger\Psi$. Show that your result is equivalent to that derived in class using the Slater determinant form for the wave functions.