Electron gas identities

$k_F^3 = 3\pi^2 n \qquad n = \frac{N}{V}$ ← # electrons; ← volume

ground state energy

$\frac{E}{N} = \frac{3}{5}\epsilon_F$ ← Fermi energy

$\epsilon_F = \frac{\hbar^2 k_F^2}{2m}$

states of one spin in d^3k

is $\frac{V}{(2\pi)^3}$

$$\sum_k F(k) \longrightarrow \frac{V}{(2\pi)^3}\int d^3k\ F(k)$$

Let $\rho(\epsilon) =$ # states of one spin in $d\epsilon$

where $d\epsilon$ is an energy interval

Then $\sum_k g(\epsilon_k) \longrightarrow \int d\epsilon\, \rho(\epsilon)\, g(\epsilon)$

$$\rho(\epsilon) = \sum_k \delta(\epsilon - \epsilon_k)$$

$$\epsilon_k = \frac{\hbar^2 k^2}{2m}$$

Expressions for $\rho(\epsilon)$

$$\frac{\rho(\epsilon)}{V} = \frac{mk}{2\pi^2\hbar^2}, \qquad k = \frac{\sqrt{2m\epsilon}}{\hbar}$$

$$\frac{\rho(\epsilon)}{N} = \frac{1}{\frac{4}{3}\epsilon_F}\left(\frac{\epsilon}{\epsilon_F}\right)^{\frac{1}{2}} \qquad \epsilon > 0$$

and trivially

$$\frac{\rho(\epsilon_F)}{V} = \frac{m k_F}{2\pi^2\hbar^2}$$

$$\frac{\rho(\epsilon_F)}{N} = \frac{1}{\frac{4}{3}\epsilon_F}$$

These are for a single spin