Fourier transform conventions

$$f(\vec{x}) = \sum_{k} e^{i\vec{k}\cdot\vec{x}} f_k \longrightarrow \frac{V}{(2\pi)^3}\int d^3k\, e^{i\vec{k}\cdot\vec{x}} f_k$$

$$f_k = \int \frac{d^3x}{V} e^{-i\vec{k}\cdot\vec{x}} f(x)$$

$$\sum_{k} \rightarrow \int \frac{V}{(2\pi)^3} d^3k$$

$$\frac{1}{V}\sum_{k} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} = \delta(\vec{x}-\vec{x}')$$

or $$\int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} = \delta(\vec{x}-\vec{x}')$$

$$\int \frac{d^3x}{V} e^{i(\vec{k}-\vec{k}')\cdot\vec{x}} = \delta_{\vec{k},\vec{k}'} = \frac{(2\pi)^3}{V}\delta(\vec{k}-\vec{k}')$$

But $$f(t) = \int \frac{d\omega}{2\pi} f(\omega) e^{-i\omega t}$$

$$f(\omega) = \int dt\, f(t) e^{i\omega t}$$