MID-TERM EXAMINATION - Spring 2020

Problem 1.

Calculate the first and second-orders corrections to the energy eigenvalues of a linear harmonic oscillator with the cubic term $-\lambda\mu x^3$ added to the potential. Discuss the condition for the validity of the approximation.

In detail !

Hint: For the validity criterion calculate the ratio: $\dfrac{E_n^{(2)}}{E_n^{(0)}}$

Problem 3.

A one-dimensional linear harmonic oscillator is acted upon by the force $F(t) = \dfrac{F_0\tau/\omega}{\tau^2+t^2}$, $-\infty < t < \infty$. At $t = -\infty$, the oscillator is in the ground state. Using the time-dependent perturbation theory to first-order, calculate the probability that the oscillator is found to be in the excited state at $t = \infty$.

Hint:

$$-\frac{\mathrm{i}}{\hbar}\int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\omega t'} \left\langle 1 \left| H^{(1)} \right| 0 \right\rangle \mathrm{d}t'$$

The integral in the above equation can be evaluated using contour integration. Its value is $(\pi/\tau)\mathrm{e}^{-\omega\tau}$.

Problem 4.

A particle of mass m is acted on by the three-dimensional potential $V(r) = -V_0\mathrm{e}^{-r/a}$ where $\hbar^2/(V_0 a^2 m) = 3/4$. Use the trial function $\mathrm{e}^{-r/\beta}$ to obtain a bound on the energy.

Problem 5.

Calculate the differential cross-section for a central Gaussian potential $V(r) = (V_0/\sqrt{4\pi})\mathrm{e}^{-r^2/4a^2}$ under Born approximation.

Problem 6.

Estimate the ground state of the infinite-well (one-dimensional box) problem defined by

$$V = \begin{cases} 0, & \text{for } |x| < L \\ \infty, & \text{for } |x| > L, \end{cases}$$

using the trial eigenfunction $\phi = |L|^\alpha - |x|^\alpha$ with α the trial parameter and compare it with the exact energy value.

Note. Problem 2 is removed as it is based on writing a code for the variational method and is optional.

Let me know if you are interested for extra points.

Good luck !