MID-TERM EXAMINATION - Spring 2020

Problem 1.

Calculate the first and second-orders corrections to the energy eigenvalues of a linear harmonic oscillator with the cubic term $-\lambda\mu x^3$ added to the potential. Discuss the condition for the validity of the approximation. In detail !

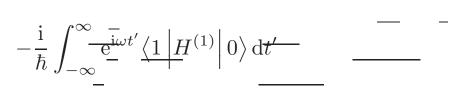
0: $\frac{E_n^{(2)}}{E_n^{(0)}}$

Hint: For the validity criterion calculate the ratio:

Problem 3.

A one-dimensional linear harmonic oscillator is acted upon by the force $\frac{F(t)}{T} = \frac{F_0 \tau / \omega}{\tau^2 + t^2}, \quad -\infty < t < \infty. \text{ At } t = -\infty, \text{ the oscillator is in the ground state. Using the time-dependent perturbation theory to first-order, calculate the probability that the oscillator is found to be in the excited state at <math>t = \infty$.

Hint:



The integral in the above equation can be evaluated using contour integration. Its value is $(\pi/\tau)e^{-\omega\tau}$.

Problem 4.

A particle of mass m is acted on by the three-dimensional potential $V(r) = -V_0 e^{-r/a}$ where $\hbar^2/(V_0 a^2 m) = 3/4$. Use the trial function $e^{-r/\beta}$ to obtain a bound on the energy.

Page 1 of 2

Problem 5.

Calculate the differential cross-section for a central Gaussian potential $V(r) = (V_0/\sqrt{4\pi})e^{-r^2/4a^2}$ under Born approximation.

Problem 6.

Estimate the ground state of the infinite-well (one-dimensional box) problem defined by

$$V = \begin{cases} 0, & \text{for } |x| < L\\ \infty, & \text{for } |x| > L, \end{cases}$$

using the trial eigenfunction $\phi = |L|^{\alpha} - |x|^{\alpha}$ with α the trial parameter and compare it with the exact energy value.

Note. Problem 2 is removed as it is based on writing a code for the variational method and is optional.

Let me know if you are interested for extra points.

Good luck !