LECTURE 15

Start with those 3 pages

Phonons as excitations

THE PROBLEM OF QUANTUM MELTING

Second quantization:

Recall from quantum mechanics:

$$[x,p] = xp - px = i\hbar$$

based on this lets introduce 2 operators, which create and annihilate an excitation when applied to the ground state or vacuum.

$$\begin{cases} b = \dfrac{1}{\sqrt{2\hbar M\omega}}\left(M\omega x + ip\right) \\ b^+ = \dfrac{1}{\sqrt{2\hbar M\omega}}\left(M\omega x - ip\right) \end{cases}$$

lets ~~verify~~ verify that anticommutator is

$$[b, b^+] = 1$$

$$[b b^+] = \frac{1}{2\hbar M\omega}\left\{ (M\omega^2)\underbrace{[xx]}_{=0} + iM\omega[px] - iM\omega\underbrace{[xp]}_{=i\hbar} + \underbrace{[pp]}_{=0}\right\} = \frac{1}{2\hbar} iM\omega [px]$$

$$= \frac{1}{2\hbar}\left(-2i[xp]\right) = \frac{-\cancel{2}i\hbar i}{2\hbar} = 1$$

inversely we can express x and p as

$$x = \sqrt{\frac{\hbar}{2M\omega}}\,(b^+ + b) \qquad p = i\frac{\sqrt{2\hbar M\omega}}{2}\,(b^+ - b)$$

Recall ~~those~~ x, p are operators here! Just a different representation.

For the HARMONIC OSCILLATOR:

Recall $\boxed{\omega = \sqrt{\frac{B}{M}}}$

$$\mathcal{H} = \frac{p^2}{2M} + B\frac{x^2}{2} = \frac{-\cancel{4}\,2\hbar M\omega}{2\cancel{4}\,2M}(b^+ - b)^2$$

$$+ \frac{B\hbar}{2M\omega}(b^+ + b)^2 = -\frac{\hbar\omega}{4}(b^+ - b)^2 + \frac{B\hbar}{2M\sqrt{B/M}}(b^+ + b)^2$$

$\hookrightarrow B = \omega^2 M$

$=$

$$H = \frac{1}{2}\hbar\omega\,(b b^+ + b^+ b) \overset{LHS}{=}$$

$$= \hbar\omega\, b^+ b + \frac{1}{2}\hbar\omega = \hbar\omega\,(n + \tfrac{1}{2})$$

$[b b^+] = 1$

where $n = b^+ b$ is the occupation number (the number of ~~excitations~~ particles in the ground state)

for the states with excitations present:

$$b|n\rangle = \sqrt{n}\,|n-1\rangle$$
$$b^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

the philosophy of bosons the more the marier!

the amplitude grows as $\sim\sqrt{n+1}$!

b - anihilator
b^+ - creation operators.

they increase (decrease) the # of bosons by $+$ (-1).

for $n \equiv b^+ b \Rightarrow n|n\rangle = b^+ b|n\rangle = b^+\sqrt{n}\,|n-1\rangle$

$$= \sqrt{n}\cdot\sqrt{n}\,|n\rangle = n|n\rangle$$

for the groundstate: $b|0\rangle = 0$

$$|n\rangle = \frac{1}{\sqrt{n!}}\,(b^+)^n\,|0\rangle.$$

Now we are ready for something interesting.

Quantum Melting.