Today's Plan

- Recap
- Superconductivity and the Higgs Boson
- Notes on the Final
- Current Research Topics

- Recap.

- Type I vs. Type II Superconductivity

$\xi > \lambda$
$\lambda \geq \xi$

- Experimental Overview

$C_V \rightarrow \Delta \rightarrow \Delta$ (also determined by optical methods)

$g(E)$
$(T=0)$
Two fluids = normal + superconducting \& c. s.

Empirically

\[ \eta_s = \eta \left[ 1 - \left( \frac{T}{T_c} \right)^4 \right] \]

Microwave/IR properties

- \( \kappa_w < \Delta \) \& superconducting (\( \rho = 0 \))
- \( \kappa_w > \Delta \) \& normal (\( \rho \neq 0 \)).

Isotope effect

Empirically \( m^2 T_c = \) constant

Importance of electron-photon interaction!

Thermodynamics

\[ \Delta f = \frac{1}{2} \mu_0 H_c^2 \quad \text{condensation energy} \]

\[ H_c(0) = \left( \frac{2\pi k^2}{\mu_0 \varepsilon_F} \right)^{1/2} T_c. \]
\[ \Delta C_p \bigg|_{T=T_c} = \mu_0 T V \left( \frac{dH_c}{dT} \right)_p^2 \]

Ohm's law:
\[ I = \frac{V}{R} \]

Tunnelling

NIS

\[ I \]

\[ \Delta \left( \frac{2e}{eV} \right) \]

Tunnelling

\[ eV > \frac{\Delta}{2e} \]

SIS

\[ I \]

\[ I_c \]

Josephson current

\[ \frac{\Delta}{2e} \]
Type II superconductors (No one of only elemental II SCs).

Materials w/ high $T_c$'s $\Rightarrow$ Type II

\[ \xi \sim \frac{1}{T_c} \]

\[ \xi \geq \lambda \]

Estimates for $H_{c1}, H_{c2}$.

$H_{c1}$ field associated w/ nucleation of a fluxoid.

\[ H_{c1} \sim \frac{\Phi_0}{\pi \lambda^2} \]

achieve 1 flux

$\sim \pi \xi^2 H_{c2}$. Each one $\leftrightarrow$ 1 flux (density of ones $\leftrightarrow$ $\xi$).

1 penetrates sample almost uniformly

$H_{c2}$ fluxoids packed together tightly!

How are chained? $H_{c1}, H_{c2} \leftrightarrow H$ (stability of superfluid state)

may particle

\[ f = f_{\text{line}} + f_{\text{mag. penetration of } B} \]

neutralize the state $f < 0$.

\[ (H_{c2}^2 \xi^2 - B^2 d^2) \text{ in SC material} \]
Threshold field for stability \( B = H_{c_1} \) \( f = 0 \)

\[
\frac{H_{c_1}}{H_C} \approx \frac{\tilde{\xi}}{\lambda}
\]

\[
(H_{c_1}, H_{c_2})^{1/2} \sim H_C
\]

\[\prod \xi \Lambda H_C \approx \Phi_0\]

- Upper limit on critical field (see 5A).

\[H_{c_2} < H_p = \frac{H_C}{\sqrt{\chi_p}}\]

Pauli limit

- Notes on the Final.

Topics covered this semester

Early Solid State Physics (Einstein, Debye)

I. Free Electron Theory

- Geometry of Solids and Lattice Vibrations
- Electronic Structure: Metals and (Band) Insulators

... 

II. Semiconducting Devices

- Graphene/2d materials
- Experimental Determination of Band Structure
- Introduction to Superconductivity
and

$$\left( H_{c1}, H_{c2} \right)^{1/2} \sim H_c.$$ 

N.B.

We've determined $H_{c2}$.

$$\xi \sim \frac{V_F}{\Delta} \sim \frac{k_F}{m \Delta}.$$

$$H_{c2} \xi^2 \sim \Phi_0 \Rightarrow H_{c2} \sim \frac{1}{\xi} \sim \frac{(m^*)^2}{\xi} \frac{\Delta^2}{k_F^2}.$$ 

Pauli-limited term (magnetic field breaks spin singlet pair) 

$$g \mu_B H \sim \Delta \quad \text{superconductor}.$$ 

$$H_p \sim \frac{\Delta}{g \mu_B}.$$ 

If $m^* > m$, $H_p < H_0$. 

↓

superconductors are Pauli-limited!
Contents of Final Exam

- Dec. 18, 3-12 pm, SEC 212
- Bring 2 equation sheets + calculator
- 10 Problems/Questions (P/Qs) @ 10 points each

- 5 P/Qs from Homework/Midterm
- 3 P/Qs Part II
- 1 P/Q Part I
- 1 P/Q ??

Overall Review
- Go over all HW, Midterm, readings, class notes

Pre-Midterm (quick review)

- Specific Heat of Solids (Einstein, Debye)
- Free Electron Theory (Drude, Sommerfeld)
- Bravais Lattices + Miller Indices
- Scattering
  - Elastic (Debye-Waller factors)
  - Inelastic scattering
  - Neutrons vs. X-rays

Electronic Structure
- Bloch's Theorem
- Tight-Binding Model
- Nearly Free Electron Model
Handouts

- Fermi Gas of Atoms
- Quasicrystals
- Packing of Ellipsoids
- X-Ray Diffraction: The First Study
- Anti-Fragg Scattering

Post-Midterm Review (again please go over all hw, readings, class notes)

Semiconductors

- Band structure $\Delta \lesssim 2 \text{eV}$

- Intrinsic

\[
n = \rho \quad \rho = \frac{1}{V} \int_{E_{\text{F}}}^{\infty} f(E) g(E) \, dE = \n \approx 2 \left( \frac{m_e k_B T}{2\pi \hbar^2} \right) e^{-\frac{E_f}{k_B T}}
\]

(analogue for $p$)

- Extrinsic

Acceptors and donors

Intrinsic + Extrinsic Regimes $n(T)$

\[
\downarrow \quad \downarrow \quad \downarrow \quad \downarrow
\]

carriers determined thermally by donors/acceptors

activated
Law of Mars-Achim

\[ np = 4 \left( \frac{k_B T}{2\pi \hbar^2} \right)^3 \left( \frac{m_e m_h}{m} \right)^{3/2} e \]

\[ \Delta / kT \]

- Mobility + Conductivity (why Drude theory OK)
- Hall Effect
- Direct vs. Indirect gaps / Optical Properties

Semi-conductor Devices.
- pn junctions
  - basic operation
  - application: solar cells, LEDs of negative resistance
  - junction transistor
  - MOSFET
- Tunnel diodes
  - importance

Graphene: Basic Properties, What's All The Excitement...

Experimental Determination of Band Structure
- Optical Absorption
- Tunneling (g(E,F)) and Scanning Tunneling Microscopy (local g(E,F))
- ARPES

Superconductivity
- Key experimental signatures
- \( \rho \), Meissner effect
- Perfect conductor vs. perfect diamagnet? / Law
- London Equation \( j \propto A \Rightarrow \lambda \) London penetration
- Length Scales

\( \lambda = \frac{\hbar}{m} \) (known derivatives!)

- Flux Quantization (Dirac monopole)

\[ \text{electron pairs} \]

- Josephson junction

\[ S \mid I \mid S \]

DC Josephson

\[ J = J_0 \sin \delta \quad \delta = \theta_2 - \theta_1 \]

- AC Josephson

\[ J = J_0 \sin \left[ \delta(0) - \frac{2eVt}{h} \right] \]

SQUID

\[ J_{\text{tot}} = \frac{J_0}{\phi_0} \cos \frac{n \Phi}{\Phi_0} \]

Two-junction interferometer
Experimental Overview

Thermodynamics of Superconductors

Tunnelling

Type I vs. Type II Superconductors

Readings

- Photonics: Semiconductors of Light
- Carbon Nanotube Electronics / Nanomaterials + Transistors
- High Tc Superconductivity in the Iron Prticiples
- Birth of Topological Insulators / 2-D Nanocrystals
- Superconductivity and the Higgs Boson

- Magnets (Nb$_3$Sn)

Supercconductors can carry much larger currents than ordinary wires $\Rightarrow$ higher fields
Produces higher fields than all but strongest electromagnets - much cheaper to run
since no energy dissipated as heat (at present must be cooled to cryogenic temperature)

LHC - Nb$_3$Sn magnets operate at 1.9 K $\Rightarrow$ 8.3 T
- Each magnet stores 7 MJ
- All magnets 10.4 GJ

Central Solenoid with 46 kA $\Rightarrow$ 13.5 T
Protons accelerated from 450 GeV $\Rightarrow$ 7 TeV

Bending magnets
Inconceptual Underpinnings

Broken symmetry \Rightarrow Underlying H invariant

PM \rightarrow FM \quad \text{Ground state breaks symmetry}

Liquid \rightarrow Solid

Broken rotational, translational, Galilean symmetries

\Rightarrow \text{Phasons}

\Rightarrow \text{Goldstone bosons.}

\Rightarrow \text{Gapless (massless!)}

Ball at top - rotational symmetry results from broken symmetry reduces gravitational energy \Rightarrow rolls downhill

No cost in angular motion \Rightarrow massless Goldstone boson
Is all spontaneous symmetry-breaking accompanied by massless Goldstone bosons?

(Gauge invariance/particle conservation vs. massless Goldstone bosons?)

Schwinger: possible when quantum mechanics + massless and strong coupling mode.

Anderson: 1. Example of plasma collective long excitation of electron gas.

\[ e(\omega) = 1 - \frac{\omega^2}{\omega_p^2} \]

\[ \omega_p = \left( \frac{4\pi ne^2}{m} \right)^{1/2} \]

\( \omega < \omega_p \) reflected (radar)

\( \omega > \omega_p \) transmitted (starlight)

Photon acquires mass.

Completely classical! Gauge invariance + particle mass!
Supersymmetry

Meissner effect $\Rightarrow \lambda \neq 0 \Rightarrow$ photon has mass.

QFT

Range of force $\propto \frac{1}{\text{mass of particle carrying it}}$

$\downarrow$

Massless photon $\Leftrightarrow$ 0-range E-M forces.

Limited penetration $\Leftrightarrow$ photon has mass $\lambda \neq 0$.

Massless Andronic: Massless photon + Goldstone boson

(CED) (symmetry-breaking)

Importance of tiny-range ($\propto 1/R$) forces! "cancel each other out"

$\downarrow$

Exactly what happens: disappearance of Goldstone boson in superconductors! Massive photon