Homework Assignment #1
(due 9/18/15) Physics 601

Reading: Simon chapters 1 – 3

Problems/Questions:

1. In the Einstein model, atoms are treated as independent oscillators. The Debye model, on the other hand, treats atoms as coupled oscillators vibrating collectively. However the collective modes are regarded here as independent. Explain the meaning of this independence and contrast it with that in the Einstein model.

2. In the spirit of the Einstein model, using reasonable parameters, what is the amplitude of zero-point-motion of atoms in a solid?

3. What is the specific heat (per unit volume) of vacuum at room temperature? Here the vacuum can be considered to contain black-body radiation (photons) in thermal equilibrium at room temperature. Please recall the dispersion relation for photons (Hint: It is similar to a certain model we use for phonons) and note that there is no upper limit on the number of photon "modes".

4. In the Debye approximation we assume the dispersion $\omega = \nu q$. Please consider instead a three-dimensional solid with one atom/unit cell where the dispersion is given by $\omega = \nu q^2$

   a) Calculate the density of modes $g(\omega)$.

   b) What is the cutoff frequency $\omega_{\text{max}}$, equivalent to the Debye frequency?

   c) What is the temperature-dependence of the specific heat at low temperatures?

5. Use the equation

$$m \left( \frac{d^2 r}{dt^2} + \frac{r}{r^3} \right) = -eE$$
for the electron drift velocity \( V \) to show that the conductivity at frequency \( \omega \) is

\[
\sigma(\omega) = \sigma(0) \left\{ \frac{1 + i\omega \tau}{1 + (i\omega \tau)^2} \right\}
\]

where \( \sigma(0) = ne^2 r/m \)

6. Show that if the random velocity of the electrons were due to the thermal motion of a classical electron gas, the electrical resistivity would increase with the temperature as \( T^{-3/2} \).

7. In the Drude model, the probability of an electron having a collision in any infinitesimal time \( dt \) is \( dt/\tau \).

   a) Show that an electron picked at random at a given moment had no collision during the preceding \( t \) seconds with probability \( e^{-t/\tau} \). Show that it will have no collision during the next \( t \) seconds with the same probability.

   b) Show that the probability that the time interval between two successive collisions of an electron falls in the range between \( t \) and \( t+dt \) is \( (dt/\tau) e^{-t/\tau} \).

   c) Show as a consequence of (a) that at any moment the mean time back to the last collision (or up to the next collision) averaged over all electrons is \( \tau \).

   d) Show as a consequence of (b) that the mean time between successive collisions of an electron is \( \tau \).

   e) Part (c) implies that at any moment the time \( T \) between the last and next collision averaged over all electrons is \( 2\tau \). Explain why this is not inconsistent with the result in (d) (A thorough explanation should include a derivation of the probability distribution for \( T \)). A failure to appreciate this subtlety led Drude to a conductivity only half of what we derived in class. He did not make the same mistake in the thermal conductivity, and then got a number of the Lorenz number that was in very good agreement with experiment.