HOMEWORK 3 - 2 problems

Phonons

P1. Consider a 1D crystall with next-nearest-neighbor interactions. The crystal contains atoms of mass m. Only the interactions up to the n-n-n are taken into account. and are modeled by springs with the force constant K and that for the n-n-n given by K'.

Hint: $F = \frac{1}{2} K \sum_j (u_j - u_{j+1})^2 + \frac{1}{2} K' \sum_j (u_j - u_{j+2})^2$

① Compute the dispersion relation of the normal modes.

② Find the condition on K' so that the dispersion curve peaks inside the 1st BZ.

③ Find the expression for group and phase velocities. Evaluate them under the condition found in ②.

P2. An experiment produced the following values of the specific heat of potassium at low T's given in the table below.

T	C_v	T	C_v	T	C_v	T	C_v
0.2604	0.5852	0.2885	0.6578	0.3644	0.8858	0.4515	1.177
0.2781	0.6306	0.2894	0.6657	0.3734	0.9180	0.4578	1.208
0.2953	0.6786	0.3067	0.7104	0.3935	0.9733	0.4835	1.302
0.2501	0.5592	0.3270	0.7687	0.3994	1.003	0.4969	1.353
0.2650	0.5969	0.3379	0.7962	0.4231	1.021	0.5435	1.551
0.2698	0.6066	0.3478	0.8362	0.4274	1.102	0.5944	1.786

(1) Plot C_v vs. T and $\frac{C_v}{T}$ vs. T^2

(2) Perform a linear square fit to the data for $\frac{C_v}{T}$ to $\gamma + AT^2$ and get γ and A.

(3) Estimate the Debye temperature of potassium at low Ts (give answer in Kelvin)

Hint: Specific heat per mole

$$\lim_{T \to 0} C_v = \frac{12\pi^4 T^3}{5\Theta_D^3} N_A k_B, \quad N_A = 6.022 \cdot 10^{23} \text{ mole}^{-1}$$