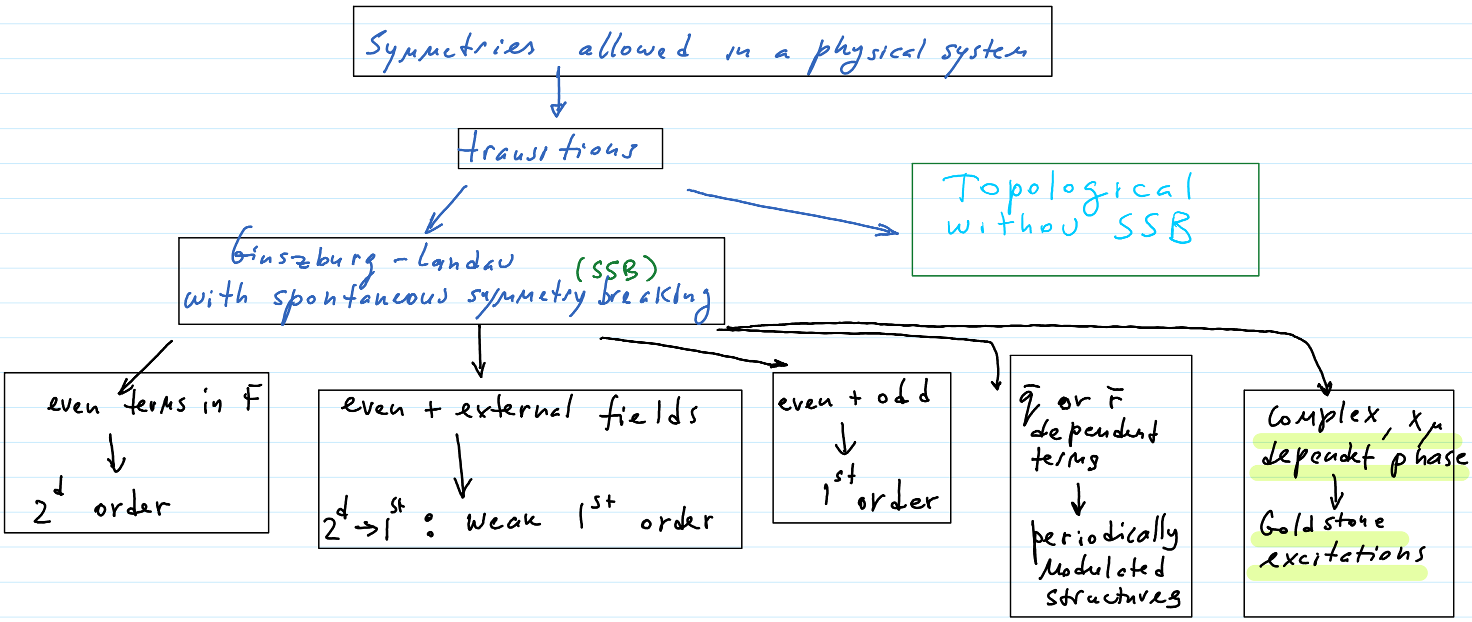
What kinds of OP?

Thursday, September 20, 2018 9:37 AM

OP can be a scalar – periodic charge density in a crystal
vector – magnetisation in a FM or FE
complex scalar – electron condensate wavefunction in SC
tensor – liquid crystals, anisotropic SC or superfluidity in ^3He or high T_c SC.



IN THIS LECTURE WE WILL LEARN ABOUT MODERN IMPLIMENTATION OF G-L. THEORY WITHIN QUANTUM FIELD THEORY.

What happens when nature breaks symmetry ?

Saturday, September 1, 2018 9:10 PM

- **Phase transitions** We saw that in Landau's example, the parameter a in the free energy was temperature dependent. At a temperature T_c, at which a changes sign, a phase transition takes place. The transition separates two distinct states of different symmetry. The low-temperature phase has lost some symmetry, more precisely it is missing a symmetry element.[5]
- **New excitations** Our philosophy has been that every particle is an excitation of the vacuum of a system. When a symmetry is broken we end up with a new vacuum (e.g. a vacuum with $M = -M_0$). The fact that the vacuum is different means that the particle spectrum should be expected to be different to that of the unbroken symmetry state (such as $M = 0$ in our example). We will see that new particles known as Goldstone modes can emerge upon symmetry breaking.[6]
- **Rigidity** Any attempt to deform the field in the broken symmetry state results in new forces emerging. Examples of rigidity include phase stiffness in superconductors, spin stiffness in magnets and the mechanical strength of crystalline solids.
- **Defects** These result from the fact that the symmetry may be broken in different ways in different regions of the system, and are topological in nature. An example is a domain wall in a ferromagnet. These are described in Chapter 29.

Reading for this section:
QFT for GA . Ch. 26

Excitation spectrum

Thursday, September 20, 2018 10:00 AM

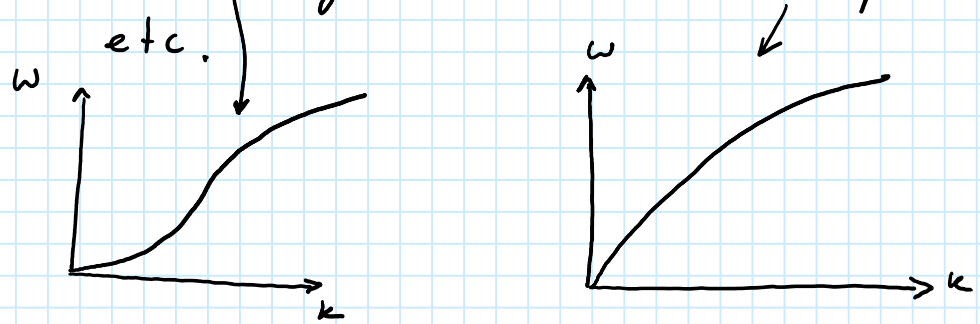
Recall our approach was to describe condensed matter from the excitation spectrum point of view.

Very generally, we can ask why and what kind of excitations we can expect in a symmetry broken state.

Experimentally there are many examples of such excitations:

- isotropic ferromagnet with SPIN WAVES [MAGNONS]
- crystal with acoustic phonons [PHONONS]

etc.



is there any general rule which tells if those excitation really exists?

Meet the Goldstone theorem:

If at the transition we break a continuous symmetry, there must exist in the ordered state of this material a collective mode or collective excitation with gapless energy spectrum.

But what about Superconductivity?

Do we live in superconducting Universe ?

A PECULIAR DEPARTURE.

Thursday, September 20, 2018 3:25 PM

In the electro-weak theory of Weinberg-Salam there is a combined U(1) x SU(2) gauge symmetry. Due to coupling to the Higgs field whose symmetry is spontaneously broken
one gauge field remains massless (the photon) and the other three become massive. These massive particles are the W+, W-, and Z bosons.

One of the key ideas first emphasized by Phil Anderson in 1963 was that a massless gauge field can acquire a mass in the presence of a coupling to a spontaneously broken field. A concrete realization of this occurs in superconductors. In the Meissner effect a superconductor thicker than the penetration depth expels magnetic fields. This is like the photon acquires a mass.

From <http://condensedconcepts.blogspot.com/2012/07/the-higgs-boson-and-condensed-matter.html>

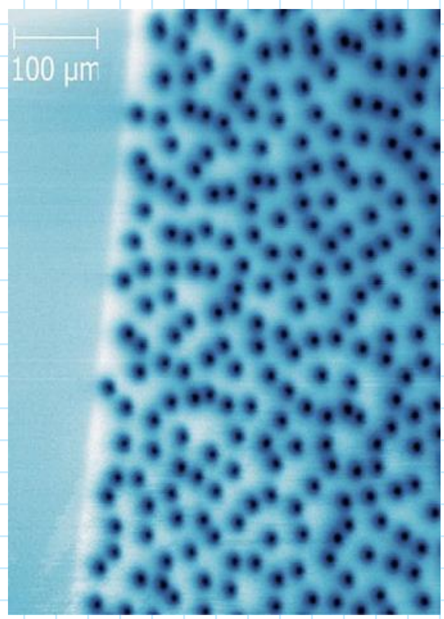
In a type II superconductor, vortices are allowed in the superconducting order parameter field. **Can such vortices occur in the Higgs field?** They may have been important in the early universe.

On fascinating thing is that for the Higgs field the crucial ratio [between the London penetration length and the superconducting coherence length] that determines whether type II behavior is possible is the ratio of Higgs boson mass to W mass. The **LHC results suggest that type II behavior is possible!**

!?

From P. Coleman's book, "Introduction to many-body..." page 246.

Shortly after the importance of this mechanism for relativistic Yang Mills theories was noted by Higgs and Anderson, Weinberg and Salem independently applied the idea to develop the theory of "electro-weak" interactions. According to this picture, the universe we live is a kind of cosmological Meissner phase, formed in the early universe, which excludes the weak force by making the vector bosons which carry it, become massive. It is a remarkable thought that the very same mechanism that causes superconductors to levitate lies at the heart of the weak nuclear force responsible for nuclear fusion inside stars. In trying to discover the Higg's particle, physicists are in effect trying to probe the cosmic superconductor above its gap energy scale.



Vortices in a 200-nm-thick YBCO film imaged by scanning SQUID microscopy

LG theory in QFT

Friday, August 31, 2018 2:47 PM

1

AND NOW WE ENTER THE WORLD OF QUANTUM FIELDS.

(JUST ONCE IN THIS COURSE)

Field theory and L-G theory.

– Breaking symmetry with Lagrangian.

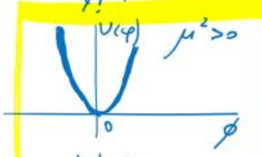
What we do in QFT is searching for a ground state of $\varphi(x)$

Recall:
$H = K + P$
$\mathcal{L} = K - P$

For simplicity lets start with a simple model:

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)^2 - U(\phi) \quad \text{where } U(\varphi) = \frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4!}\varphi^4$$

Now we move to a very interesting case: $\mu^2 < 0$!



notice
$\mathcal{L} \to \mathcal{L}$
if $\varphi \to -\varphi$

in this case

$$\frac{\partial U}{\partial \varphi} = 0 \quad 0 = -\mu^2 \varphi + \frac{\lambda}{3!}\varphi^3$$

$$\left(0, \pm\sqrt{6\mu^2/\lambda}\right)$$

and $\dfrac{\partial^2 U}{\partial \varphi^2} = 0 \Rightarrow -\mu^2 + \dfrac{\lambda \varphi^2}{2}$

$= 0 \Rightarrow \dfrac{\partial^2 U}{\partial \varphi^2} < 0$ for $\varphi = 0$

and $= +2\mu^2$ for $\boxed{\varphi_0 = \pm\sqrt{6\mu^2/\lambda}}$

This is very strange as our system has two new vacua → ground state is <u>brocken</u> in $\varphi_0 \to -\varphi_0$ <u>symmetry</u> and it happens <u>spontaneously</u>

▶ What happens to excitations in the new ground state?

To investigate this lets select a new vacuum. e.g. $+\varphi_0$, and excite the field around the ground state. The Taylor expansion gives:

$$U(\varphi - \varphi_0) = U(\varphi_0) + \left(\frac{\partial U}{\partial \varphi}\right)\Big|_{\varphi_0}(\varphi - \phi_0) + \frac{1}{2!}\frac{\partial^2 U}{\partial \varphi^2}\Big|_{\varphi_0}(\varphi - \varphi_0)^2 + \dots =$$

$$= \underbrace{U(\varphi_0)}_{\text{Const}} + \mu^2 \underbrace{(\varphi - \varphi_0)}_{\equiv \phi'}{}^2 + \dots$$

The final Lagrangian is:

$$\boxed{\mathcal{L} = \tfrac{1}{2}(\partial \varphi')^2 - \mu^2 \varphi'^2 + O(\varphi'^3)}$$

Lets compare this to the original theory

$$\mathcal{L} = \tfrac{1}{2}(\partial \varphi)^2 - \frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4!}\varphi^4 \qquad \mathcal{L} \xrightarrow[\varphi \to -\varphi]{} \mathcal{L}$$

$$\boxed{\mu \to \sqrt{2}\,\mu}$$

Notice, the Lagrangian don't break the symmetry, it's still $\varphi \to -\varphi$ invariant. But the symmetry is broken in the <u>ground state</u>. As the result, the vacuum gets a non-zero amplitude $\varphi_0 = \left(\frac{6\mu^2}{\lambda}\right)^{1/2}$ and becomes heavier. } !!

2

Goldstone Modes

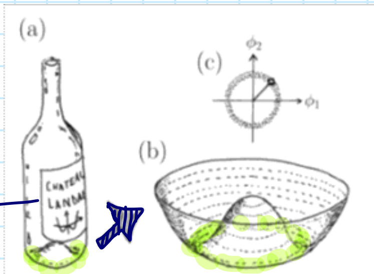
Goldstone modes easer to explain a 2D model.

Consider a 2-component (or 2D) QFT:

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \varphi_1)^2 + (\partial_\mu \varphi_2)^2\right] + \frac{\mu^2}{2}(\varphi_1^2 + \varphi_2^2) - \frac{\lambda}{4!}(\varphi_1^2 + \varphi_2^2)^2$$

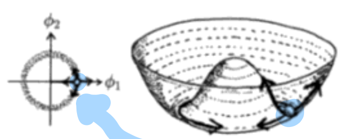
← it has SO(2) rotational symmetry around internal $\varphi_1(x) - \varphi_2(x)$

There are infinite number of local minima.



the Label says "CHATEAU LANDAU"

Fig. 26.6 (a) The potential for the $SO(2)$ symmetry breaking looks like the bottom of a punted wine bottle. (b) There is a maximum at the point $\phi_1 = \phi_2 = 0$, but surrounding this there is a set of minima which lie on a circle. (c) The circle of minima are shown on a ϕ_1-ϕ_2 plot (this is therefore viewing the surface sketched in (b) from 'above'). The symmetry can then be broken by choosing a particular point in the circle of minima and setting this to be the ground state. We can then examine small deviations away from that point.



$$U(x) = -\mu^2/_2\, x + \frac{\lambda}{4!}x^2$$

$$x \equiv \varphi_1^2 + \varphi_2^2 \Rightarrow \frac{\partial U}{\partial x} = 0 \Rightarrow$$

$$x_{min} \equiv \varphi_1^2 + \varphi_2^2 = 6\mu^2/\lambda$$

Lets imagine we break symmetry e.g. $(\varphi_1, \varphi_2) = \left(+\sqrt{6\mu^2/\lambda}, 0\right)$

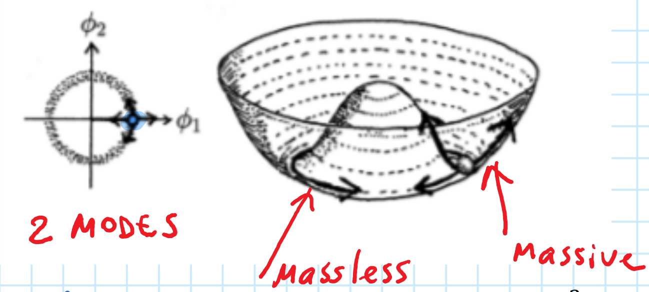
and investigate the excitations around the ground state.

$$\varphi_1' = \varphi_1 - \sqrt{6\mu^2/\lambda}\ ; \quad \varphi_2' = \varphi_2$$

See next figure to get a better idea.

LG theory of QFT

Friday, August 31, 2018 6:57 PM



Again as before we can check what happens if we consider our Lagrangian for the new ground state:

Consider $U(\varphi_1, \varphi_2) = \frac{-\mu^2}{2}(\varphi_1^2 + \varphi_2^2) + \frac{\lambda^2}{4!}(\varphi_1^2 + \varphi_2^2)^2$

we expand around minimum $(\varphi_1, \varphi_2) = \left(\sqrt{\frac{6\mu^2}{\lambda}}, 0\right)$

and get: $\frac{\partial^2 U}{\partial \varphi_1^2} = 2\mu^2 \qquad \frac{\partial^2 U}{\partial \varphi_2} = 0$

$\Rightarrow$

$$\mathcal{L} = \frac{1}{2}\left[(\partial\varphi_1)^2 + (\partial\varphi_2)^2\right] - \underbrace{\mu^2}_{\text{Massive or gapped}}(\varphi_1')^2 + \underbrace{0}_{\text{massless or gapless in solid state}} \times (\varphi_2')^2 + O(\varphi'^3)$$

So the particle with field φ_1' has mass and φ_2' is massless. See Fig. above

, the excitations in φ_2' are gapless!

(in particle physics this would be massless)

The vanishing of the mass is the result of Goldstone theorem: Breaking a continuous symmetry always results in massless excitations known as a Goldstone mode.

Breaking symmetry in a gauge theory

4

The most amazing effect occur when we apply the same ideas to the broken ground state in a gauge theory.

Here we want to discuss the famous Higgs mechanism.

Consider a scalar field theory:

$$\mathcal{L} = (\partial^\mu \psi^\dagger - igA^\mu \psi^\dagger)(\partial_\mu \psi + igA_\mu \psi) + \mu^2 \psi^\dagger \psi - \lambda(\psi^\dagger \psi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

this theory can describe for exemple an electron interacting with photons.

The important point - this theory is gauge invariant, i.e

$$\begin{cases} A_\mu \to A_\mu - \frac{1}{q} \partial_\mu \alpha(x) \\ \psi \to \psi e^{i\alpha(x)} \end{cases}$$

The theory as written above describes

2 massive scalar particle

$E_p = (p^2 + \mu^2)^{1/2}$ and

2 transverse polarized massless photons

$E_p = |p|$

2 transverse polarized massless photons

$E_p = |p|$

Higgs Anderson mechanism

Monday, September 3, 2018 10:54 AM

Now we will break the symmetry
Lets move to the polar coordinates:

$\psi(x) = \rho(x)\, e^{i\theta(x)}$ and select some unique angle θ_0 for all x.

So we seat at this specific state and want to know what excitations can emerge around this state?

$$\partial_\mu \psi + iq A_\mu \psi = \left(\partial_\mu \rho(x)\right) e^{i\theta(x)} +$$

$$+ i\left(\partial_\mu \theta(x)\right) \rho e^{i\theta} + q A_\mu \rho e^{i\theta} =$$

$$= (\partial_\mu \rho) e^{i\theta} + i\rho e^{i\theta}\left(\underline{\partial_\mu \theta + q A_\mu}\right)$$

Compare to $(\partial_\mu \psi + iq A_\mu \psi)$

$(\partial_\mu + iq A_\mu)\psi$

we introduce a new gauge field

$$A_\mu + \frac{1}{q}\partial_\mu \theta(x) \equiv C_\mu$$

So the term

$$(\partial^\mu \psi^+ - iq A^\mu \psi^+)(\partial_\mu \psi - iq A_\mu \psi) =$$

$$= (\partial_\mu \rho)^2 + \rho^2 q^2 C_\mu C^\mu$$ ← SHOW THIS

Higgs Anderson Mechanism

Monday, September 3, 2018 11:03 AM

We can also transform the field energy $F_{\mu\nu} = \partial_\mu A^\nu - \partial_\nu A^\mu$
$= \partial_\mu C^\nu - \partial_\nu C^\mu$

Finally

$$\mathcal{L} = (\partial_\mu \rho)^2 + \rho^2 q^2 C^\mu C_\mu + \mu^2 \rho^2 -$$
$$- \lambda \rho^4 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad \approx C^2$$

Now BREAK THE SYMMETRY:

$$\rho_0 = \sqrt{\mu^2 / 2\lambda} \qquad \theta_0 = 0$$

Lets introduce the excitations around the ground state:

$$\frac{\chi}{\sqrt{2}} = \rho - \rho_0$$

SHOW THIS

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \chi)^2 - \mu^2 \chi^2 - \sqrt{\lambda}\, \mu\, \chi^3 - \frac{\lambda}{4} \chi^4$$
$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{M^2}{2} C_\mu C^\mu +$$
$$+ q^2 \left(\frac{\mu^2}{\lambda}\right)^{1/2} \chi\, C_\mu C^\mu + \frac{1}{2} q^2 \chi^2 C_\mu C^\mu + \ldots$$

$$+ g^2 \left(\frac{\mu^2}{\lambda} \right)^{1/2} \chi C_\mu C^\mu + \frac{1}{2} g^2 \chi^2 C_\mu C^\mu + \dots$$

here $M = g \sqrt{\mu^2/\lambda}$

Higgs Anderson Mechanism

Monday, September 3, 2018 11:10 AM

7

Now we see that we have
χ field excitations with mass $\sqrt{2}\mu$

But C^μ which was massless gauge field analog of A_μ now has mass M.

Also θ which was massless now is NOT in the broken symmetry theory and we instead have a massive term C^μ

The massless photon field A_μ has "eaten" $\theta(x)$ and got mass $C_\mu(x)$

So we have:

$$\chi(x) \quad E_p = \left(p^2 + (\sqrt{2}\mu)^2\right)^{1/2}$$

+

3 Vector fields

$$C_\mu(x) \qquad E_p = \left[p^2 + \left(\frac{g^2\mu^2}{\lambda}\right)\right]^{1/2}$$

Summary: By applying a gauge transformation

Summary: By applying a gauge transformation $A_\mu \to C_\mu$ we remove massless Goldstone modes and gained mass in excitations combining it with the gauge field.

e.g. $\left\{ \begin{matrix} 2 \times \text{massive scalar particles} \\ 2 \times \text{massless photons} \end{matrix} \right\} \to \left\{ \begin{matrix} 1 \times \text{massive scalar particle} \\ 3 \times \text{massive vector particles} \end{matrix} \right\}$

In simpler words: If you have a gauge invariant theory, e.g. when your phase depends on the point in space x_μ, e.g. $\varphi(x_\mu) = \varphi_0(x_\mu)\, e^{i\theta(x_\mu)}$

By breaking this symmetry we destroy UNPHYSICAL MASSLESS Goldstone modes, by giving those excitation mass via the interaction with a gauge field (e.g. photons)

In particle's words it gives mass to otherwise massless bosons, and known as Higgs particles: Z and W bosons

Similar mechanism was proposed by P. Anderson for superconductivity.

IF THIS STILL NOT CLEAR READ

Chapter 26, QFT for gifted

One more surprise (without proof).

Order in reduced dimensions:

All what we have discussed in the past 3 lectures was refering to 3D world. But what about low dimensional materials?

Berezinskii - Mermin - Wagner theorem

Continuous symmetry cannot be broken at FINITE T's in any system with $D \leq 2$ with short-range interactions.

Sometimes, you can hear that there is no Long-range ordering in Low-D materials.

{ Read WIKIPEDIA ARTICLE on: MARMIN – WIGNER theorem }