Introductory discussion.

Phys 502 - 2019
There are some important difficult questions:

1. Why atoms would condense?
2. What happens to the elementary particle when they are inside a digital lattice?
3. What kind of new excitations are possible?
4. Are there any kind of new defects in the digital space?

5. Why metallic Fermi surface stable?

Hint: The modern notion is that physics of electrons in solids is not very different from the low-energy regime in particle physics read G. Volovik, “The universe in the drop,” Oxford U. press.
Symmetries

- Systems described by $H$ or $\mathcal{L}$
- Symmetry is a transformation which leaves $H$ invariant
  
  e.g. $r \to -r$, plane reflections
  
  $t \to -t$ in non-magnetic

Landau idea: a specific new phase must break a certain symmetry.

  e.g. Liquid $\to$ Solid

  PM $\to$ FM or AFM

  FM breaks spin rotation
  time reversal

  AFM in addition breaks

  translational symmetry
  between 2 sublattices

The pattern of symmetry is characterized by the "order parameter"
Order parameter is some physical quantity which transforms differently under some symmetry operation from a Hamiltonian. In simple words it is something which is \( \langle OP \rangle \neq 0 \) in a new phase.

e.g. FM: \( \langle M \rangle \rightarrow -\langle M \rangle \)

if \( t \rightarrow -t \)

but \( H \rightarrow H \)

\( t \rightarrow -t \)

condensed xtal: OP is the Fourier transformed atomic density \( \mathbf{q} \)-space

\[
\frac{2\pi}{d} \text{ between atoms}
\]

but nothing like this for liquids
e.g. super fluid
also to superconductors but charge-neutral
characterized by a complex order parameter
as in Math!
which is a phase of the global wave function
Superfluid breaks this OP which corresponds
to the particle number conservation
more on that in Lecture 2
on Landau-Ginzburg theory.

Is there anything beyond symmetry?
Welcome to the 21st century!
e.g. FQHE no symmetry is broken.

 Band Metal $\leftrightarrow$ Insulator
both have the same symmetry!
or e.g. in strongly correlated materials
metal $\leftrightarrow$ insulator transition

We need a new idea which is
not connected to the OP broken phase.
Here is a very simplified view.
You will learn soon that
the difference
- in insulator all bands are filled and separated by a large gap from the empty bands (states)
- in metals at least one band is partially filled
  \[ \downarrow \]
  at least 1 Fermi surface

So we have a topological invariant
  \[ \Rightarrow \] the number of Fermi surface sheets

This number is unaffected by the smooth deformation of the F.S.
Otherwise if this number changes
the system undergoes a quantum phase transition (Lifshitz transition)
In short, many gaped materials may have non-trivial topological properties characterized by a topological order parameter.

\[ \text{e.g.} \quad \Phi \text{HEs} \]

- Topological insulators
- Weyl semimetals (no gap here!)
- Superconductors

\[ \quad \downarrow \text{in SC no spontaneous symmetry breaking} \]

but the ground state degeneracy depends on the topology.

So the topological OP will have to be a measure of topology and entropy.

The end.