1 [5 pts] Using the Levi-Civita symbol $\epsilon$, show that
\[ \vec{A} \times (\vec{B} \times \vec{C}) = \vec{B} (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{A} \cdot \vec{B}). \]

2 [5 pts] Consider a parallelopiped whose edges at one vertex are given by three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$. Show that it has a volume given by $\vec{A} \cdot (\vec{B} \times \vec{C})$. Note that the volume is the area of one face times the distance (measured perpendicular to that face) between that face and the one opposite.

3 [5 pts] Show that, if $\vec{u}$ and $\vec{v}$ are irrotational fields (i.e. $\vec{\nabla} \times \vec{u} = 0 = \vec{\nabla} \times \vec{v}$) then $\vec{u} \times \vec{v}$ is solenoidal (i.e. $\vec{\nabla} \cdot (\vec{u} \times \vec{v}) = 0$).

4 [5 pts] Work out the Leibnitz rule for the curl of a product of a scalar field and a vector field; that is, express $\vec{\nabla} \times (\rho \vec{v})$ in terms of $\rho$, $\vec{\nabla} \rho$, $\vec{v}$, and $\vec{\nabla} \times \vec{v}$.

5 [5 pts] Show that any solution of the equation
\[ \vec{\nabla} \times \vec{\nabla} \times \vec{A} - k^2 \vec{A} = 0 \]
with $k \neq 0$ automatically satisfies the vector Helmholtz equation
\[ \nabla^2 \vec{A} + k^2 \vec{A} = 0 \]
and the solenoidal condition
\[ \vec{\nabla} \cdot \vec{A} = 0. \]