1. Minimal surface of revolution, another way: This time, rotate the curve $y = y(x)$ around the $x$ axis.
   
a) Show that the area is
   $$2\pi \int_1^2 y \sqrt{1 + y'^2} dx = \int_1^2 f(y, \dot{y}) dx$$  (1)

   b) Using calculus of variations, derive a differential equation for $y(x)$.
   
c) Use the conservation of $y \frac{\partial f}{\partial \dot{y}} - f$ to find solutions to the differential equation.

2. Semiholonomic constraints: Let us say that we have $m$ general semi-holonomic constraints.
   $$f_\alpha(q_1, \ldots, q_n, \dot{q}_1, \ldots, \dot{q}_n, t) = 0, \text{ for } \alpha = 1, \ldots, m.$$  (2)

   a) Work out the first variation of
   $$J = \int_1^2 [L + \sum_\alpha \mu_\alpha(t)f_\alpha] dt.$$  (3)

   and derive the equations of motion.

   b) Find the contribution to generalized force $Q_j$ from the semiholonomic constraints.

3. Time-dependent Lagrangian: Dissipation may be introduced via time dependence.

   a) Find the equation of motion for the Lagrangian
   $$L = e^{\gamma t} \left( \frac{m}{2} \dot{x}^2 - \frac{m\omega^2}{2} x^2 \right).$$  (4)

   b) Provide an interpretation of the parameter $\gamma$ from the equation of motion for $x$. 
c) If we define \( h = \dot{x} \frac{\partial L}{\partial \dot{x}} - L \), we know that

\[
\frac{dh}{dt} = -\frac{\partial L}{\partial t}.
\]  

(5)

Define \( E = \frac{m}{2} \dot{x}^2 - \frac{m\omega^2}{2} x^2 \). Using the consequence of (5), find \( \frac{dE}{dt} \) in terms of \( x, \dot{x} \) and the parameters.