**Course:** Classical Mechanics  
**Problem Set:** 1  
**Due:** 19th Sept 2016

**Read the background material first!** For certain calculations, we will represent vector $\vec{a}$ as $(a_1, a_2, a_3)$, and we have to play with index sums.

**Kronecker Delta:**

\[
\delta_{ij} = \begin{cases} 
1, & \text{if } i = j, \\
0, & \text{otherwise.}
\end{cases}
\]  

(1)

**Summation Convention:** Repeated indices gets summed over (old time physicists love it and mathematicians hate it!):

\[a_i b_i \text{ indicates } \sum_i a_i b_i.\]  

(2)

So $\vec{a} \cdot \vec{b} = a_i b_i = \delta_{ij} a_i b_j$. This last example has two repeated sums.

**The Levi-Civita Symbol:**

\[
\epsilon_{ijk} = \begin{cases} 
1, & \text{if } (i, j, k) \text{ is an even permutation of } (1, 2, 3), \\
-1, & \text{otherwise.}
\end{cases}
\]  

(3)

That means

\[
\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1, \\
\epsilon_{213} = \epsilon_{132} = \epsilon_{321} = -1.
\]  

(4)

Note that the cross-product $\vec{a} = \vec{b} \times \vec{c}$ could be represented in terms of the Levi-Civita symbol.

\[a_i = \epsilon_{ijk} b_j c_k.\]  

(5)

The last equation has two indices to which summation convention is applied. Now, on to the homework problems!
1. Vectors, indices and all that: Remember that the summation convention is on.

a) Show that
\[ \epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}. \]  
(6)

b) Using Eq. (6), show that
\[ \vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}. \]  
(7)

c) The Biot-Savart law says that the magnetic field due to moving particle \( j \), with charge \( q_j \), position \( \vec{r}_j \) and velocity \( \vec{v}_j \) is given by
\[ \vec{B}_j(\vec{r}) = \frac{\mu_0 q_j \vec{v}_j \times (\vec{r} - \vec{r}_j)}{4\pi |\vec{r} - \vec{r}_j|^3} \]  
(8)

The Lorenz force due to particle \( j \)'s magnetic field on particle \( i \), with charge \( q_i \), position \( \vec{r}_i \) and velocity \( \vec{v}_i \), is given by
\[ \vec{F}_{ji} = q_i \vec{v}_i \times \vec{B}_j(\vec{r}_i) \]  
(9)

Compute \( \vec{F}_{ji} \) using Eq. (7). By exchanging the indices, compute \( \vec{F}_{ij} \). Is \( \vec{F}_{ji} = -\vec{F}_{ii} \)?