1. Consider a system of 3 weights suspended by pulleys:

\[ l_1, l_2 = \text{total rope lengths} \]

\[ x \]

Assume that the pulleys and the cords are massless and that there is no friction. Each pulley has a radius \( r \) and the coordinates of masses \( m_1, m_2, m_3 \) are \( x_1, x_2, x_3 \) respectively. Write down the equations of motion for all 3 masses. Solve the resulting equations with \( x_i(t=0) = x_{i,0} \) and \( x_{i,0}(t=0) = 0 \), \( i = 1, 2, 3 \).
Consider a rotating horizontal rod of the total length $L$. Two spheres of mass $m$ can glide along the rod as shown in the Fig. below:

Imagine that the rod is given an initial angular velocity $\omega_0$ and the spheres glide without friction. Assume that the spheres collide with the stops and the collision is completely inelastic. Compute the final angular velocity $\omega_1$, the change in the kinetic energy $\Delta T_{\text{rot}}$, and where did the extra energy go or come from?
Consider a spherical pendulum of mass $m$ and length $b$:

$$\ddot{\theta} + c \dot{\theta} + \frac{g}{b} \sin \theta = 0$$

Find the generalized momenta and write down (but do not solve!) Hamilton's EoM for this system. Comment on momentum conservation and find a cyclic coordinate (if any).
Consider an arbitrary function \( g(q, p, t) \), where \( q = \{q_i\}_{i=1}^n \) are generalized coordinates and \( p = \{p_i\}_{i=1}^n \) are canonical momenta.

(a) Write down an EoM for \( g \) using Poisson brackets

(b) Write down EoMs for \( p_j \) & \( q_j \), \( j = 1, \ldots, n \) using Poisson brackets

(c) Find \( [p_i, p_j] \), \( [q_i, q_j] \) and \( [p_i, q_j] \)

(d) Using results from (a), write down the conditions for \( g \) to be a constant of the motion of the system.
The potential energy between two neutral atoms is given by:

$$v(r) = 4 \varepsilon \left[ \left( \frac{e}{r} \right)^{12} - \left( \frac{e}{r} \right)^{6} \right],$$

where \( r \) is the separation distance.

Atom masses:

\( m_1, m_2 \)

(a) Find the equilibrium separation \( r_0 \).

(b) Find the frequency of small oscillations about the equilibrium position. Qualitatively, how will the average separation \( \langle r \rangle \) vary with the amplitude of the oscillation? [This question does not assume small oscillations]