Solutions to hw 5

2 14.4

(a) The displacement current is

\[ \mathbf{J}_d (\mathbf{r}) = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \frac{d}{dt} \left( \frac{\mathbf{q} \cdot \mathbf{r}^3}{4 \pi r^2} \right) = \frac{\mathbf{I} \cdot \mathbf{r}}{4 \pi r^2} \]

(b) By symmetry & Ampere's law \( \mathbf{B} \) produces no \( B^\perp \) field. The entire \( B^\perp \) field comes from \( \mathbf{J}_d \)

\[ \mathbf{B} = \begin{cases} B(r, \phi) \hat{\phi}, & t > 0 \\ -B(r, \phi) \hat{\phi}, & t < 0 \end{cases} \]

Use Ampere's law in integral form

\[ B(2\pi r \sin \theta) = \mu_0 x (\text{current into disk}) \]

\[ \text{current into cap } = \oint \hat{n} \cdot (\text{solid angle})x^2 \]

\[ \text{solid angle } = 2\pi \int_0^{\theta_1} \sin \theta \, d\theta = 2\pi (1 - \cos \theta) \]
\[ B(2\pi r \sin \theta) = \frac{I}{4\pi r} \frac{1 - \cos \theta}{\sin \theta} \pi^2 \]

\[ B(r_1 \theta) = \frac{I}{4\pi r} \frac{1 - \cos \theta}{\sin \theta} \pi^2 \]

For \( r < 0 \), \( \theta \to \pi - \theta \), i.e.,

\[ B(r_1 \theta) = \frac{I}{4\pi r} \frac{1 + \cos \theta}{\sin \theta} \frac{\pi}{2} < \theta < \pi \]

2. 14.9

Counter-clockwise \( I \& E \) to compensate the increase of \( \vec{B} \) into the page, i.e.,

\[ V_1 > 0 \text{ and } V_2 < 0 \]

Faraday + Ohm

\[ I(R_1 + R_2) = B \pi r^2 \]

\[ V_1 = I R_1 = \frac{R_1}{R_1 + R_2} \frac{B \pi r^2}{r^2} \]

\[ V_2 = -I R_2 = -\frac{R_2}{R_1 + R_2} \frac{B \pi r^2}{r^2} \]
\[ \Delta \Psi = \oint d\mathbf{l} + E = \frac{1}{R} \oint d\mathbf{l} + E = \frac{1}{R} \oint d\mathbf{l} \frac{d\phi}{dt} = \frac{1}{R} \int d\phi \]

Thus, \[ \Delta \Psi = \frac{\Delta \phi}{R} \]

\( \text{flux of monopole} \)

\( \text{change in magnetic} \)

\( \phi = \phi_1 + \phi_2 = \frac{\mu_0 I}{2} \)

By symmetry \( \phi_1 = \phi_2 \), i.e., \( \phi_1 = \phi_2 = \frac{\mu_0 I}{2} \)

When the monopole is just above the loop \( \phi_{\text{loop}} = \phi_1 = \frac{\mu_0 I}{2} \)

Just below the loop \( \phi_{\text{loop}} = -\phi_2 = -\frac{\mu_0 I}{2} \)

The charge that goes thru the loop as the monopole goes thru is \( \Delta \Psi = \frac{\mu_0 I}{R} \)

Note that there is also charge \( \frac{\mu_0 I}{2R} \) that passes as it approaches the loop from above & \( \frac{\mu_0 I}{2R} \) as it flies away from below.