Zangwill 7.13 The Plane-Cone Capacitor A capacitor is formed by the infinite grounded plane \( z = 0 \) and an infinite, solid, conducting cone with interior angle \( \pi/4 \) held at potential \( V \). A tiny insulating spot at the cone vertex (the origin of coordinates) isolates the two conductors.

(a) Explain why \( \varphi(r, \theta, \phi) = \varphi(\theta) \) in the space between the capacitor “plates”.
(b) Integrate Laplace’s equation explicitly to find the potential between the plates.

Zangwill 7.17 An Incomplete Cylinder The figure below shows an infinitely long cylindrical shell from which a finite angular range has been removed. Let the shell be a conductor raised to a potential corresponding to a charge per unit length \( \lambda \). Find the fraction of charge which resides on the inner surface of the shell in terms of \( \lambda \) and the angular parameter \( p \). Hint: Calculate \( Q_{\text{in}}?Q_{\text{out}} \).

Zangwill 7.24 A Complex Potential Give a physical realization of the electrostatic boundary value problem whose solution is provided by the complex potential

\[
f(w) = i \frac{V_1 + V_2}{2} + \frac{V_1 - V_2}{2} \ln \left[ \frac{R + iw}{R - iw} \right].
\]