Physics 502: Problem Set 3 (DUE ON FRIDAY 2/24)

1) A hydrogen atom is in the ground state at \( t = -\infty \). A weak electric field \( E(t) = E_0 \hat{z} e^{-|t/\tau|} \) is applied starting at time \( t = -\infty \).

a) Using first order TDPT, find the probabilities for the atom to end up in each of the \(|21m\rangle\) states (i.e., for each of the three values of \( m \)). (You may use that \( \langle 210\vert z\vert 100 \rangle = \frac{2\pi^3}{3^5}a_0 \).

b) Discuss the limit \( \tau \to \infty \); does the result agree with what you would expect based on the adiabatic approximation?

2) Consider a composite system made of two spin 1/2 particles with Hamiltonian:

\[
H = \begin{cases} 
0 & \text{if } t < 0 \\
\frac{4\Delta}{\hbar^2} \vec{S}_1 \cdot \vec{S}_2 & \text{if } t > 0
\end{cases}
\]

(1)

Suppose the system is in \(|-+\rangle\) for \( t \leq 0 \). Find as a function of time the probability of being found in each of the following states, \(|++\rangle\), \(|+-\rangle\), \(|-+\rangle\) and \(|--\rangle\) by:

a) Solving the problem exactly.

b) Using first order TDPT. Under what conditions does (b) give the correct results?

3) A perturbation consisting of a series of \( N \) oscillatory pulses

\[
V(t) = \sum_{j=1}^{N} A e^{-i\omega_0 t} f(t - t_j)
\]

(2)

is applied to a system in which two discrete levels \( s \) and \( n \) are separated in energy by \( \hbar\omega_0 \). Here \( A \) is an operator, \( f(t) \) is an envelope function describing the shape of each pulse, and \( t_j \) is the time of arrival of the \( j \)th pulse.

a) Show that, in first-order time-dependent perturbation theory, the probability of excitation from \( s \) to \( n \) is proportional to \( N^2 \).

b) This peculiar result is a consequence of the fact that each pulse is applied in phase, i.e., the excitation is coherent. How does the probability scale with \( N \) if the excitation is instead incoherent, i.e., each term in the expression for \( V(t) \) is multiplied by a random phase \( e^{i\phi_j} \)? (If you think of the photons emerging from a light bulb or a laser as being the pulses, you can appreciate the fact that coherent sources such as lasers may have very different effects on matter than incoherent ones.)
4) A hydrogen atom initially in its $1s$ ground state is subject to an electric field $E(t) = E_0 \cos(\omega t)\hat{z}$ whose frequency is large enough that $\hbar \omega$ exceeds the ionization energy of 1 Ry. Assuming a plane-wave final state, what is the rate for transitions to an ionized state, and what is the angular distribution of emitted electrons? (Note: you will need to evaluate the matrix element $\langle \vec{k} | z | 1s \rangle$. To proceed with the rest of the problem, it is sufficient to evaluate this to leading nonzero order in the small $k$ limit. For bonus points, figure out how to evaluate the matrix element exactly.)

5) In class we derived the $\omega = 0$ version of the transition probability as a function of time for transitions of $s \rightarrow n \neq s$:

$$P(s \rightarrow n) = \frac{4}{\hbar^2} |V_{ns}|^2 \sin^2 \frac{\omega_{ns} t}{\omega_{ns}}$$  \hspace{1cm} (3)

We did this starting from the result for $\omega \neq 0$ obtained using TDPT, and then taking $\omega \rightarrow 0$. Rederive this result more directly using time-independent perturbation theory.