1. [8 points] One particle sits in a two-dimensional version of the “half harmonic oscillator,” that is, \( V(x, y) = \frac{1}{2}m\omega^2(x^2 + y^2) \) if \( x > 0 \) and \( = +\infty \) otherwise. (You may use all the results of Problem 1 on HW 6.)
   
   (a) Show that the Hamiltonian is separable, and list the first few energy eigenvalues and the corresponding state labels.
   
   (b) What is the degeneracy of levels at \( E = 6\hbar\omega \)?
   
   (c) Write out the ground state and first excited state wavefunctions in the coordinate representation \( \psi(x, y) \).
   
   (d) For both states in part (c), convert the wavefunction to polar coordinates \( \psi(r, \theta) \).

2. [4 points] Let \( |s\rangle \) and \( |t\rangle \) be arbitrary vectors in \( \mathcal{V}_1 \), and \( |x\rangle \) and \( |y\rangle \) be arbitrary vectors in \( \mathcal{V}_2 \). (Let’s use the notation developed in class that the basis vectors of \( \mathcal{V}_1 \) are \( |i\rangle \) and those of \( \mathcal{V}_2 \) are \( |\alpha\rangle \), so that \( |s\rangle = \sum_i c_i |i\rangle \), etc.) Show that
   
   \[
   (\langle s| \otimes \langle x|) (|t\rangle \otimes |y\rangle) = \langle s|t\rangle \langle x|y\rangle.
   \]

3. [4 points] If the Hamiltonian for a system of two (distinguishable) particles is separable, \( H = H_1 + H_2 \), show that the time-evolution operator for the whole system takes the form \( U(t) = U^{(1)}(t) \otimes U^{(2)}(t) \).
   
   (You might get some hints from pp. 254-5 in Shankar, where this is almost proven.)

4. [8 points] Three similar particles are to be placed into a 1D simple harmonic oscillator potential in such a way that the total energy is \( \frac{9}{2}\hbar\omega \) or less.
   
   (a) Show that the number of states is 20, 7, or 1 if the particles are distinguishable, bosons, or fermions, respectively.
   
   (b) Write out correctly normalized and symmetrized (or antisymmetrized) state vectors for the bosonic and fermionic states having energy exactly \( \frac{9}{2}\hbar\omega \). [You do not need to go to the coordinate representation; when I say “write out” I mean something like \( \frac{1}{\sqrt{3}}(|2|0\rangle|0\rangle + |0|2\rangle|0\rangle + |0|0\rangle|2\rangle \).]

5. [6 points] Shankar Ex. 10.3.4 on p. 278 (two particles in 1D box). [The last sentence of the problem statement means you can ignore spin.]