Due date: Wednesday, Oct. 9

Reading: Finish Shankar Ch. 4. Also, start reading Ch. 5.

1. [10 points] Referring to Ex. 4.2.1 on p. 129:
   (a) Obtain eigenvalues and normalized eigenvectors for all three operators $L_x$, $L_y$, and $L_z$.
   (b) Answer Part (2) of 4.2.1.
   (c) Suppose the system is prepared in the state with $L_x = 0$, and then $L_z$ is measured. What are the possible outcomes and their probabilities?
   (d) Suppose the system is prepared in the state with $L_x = 0$, and then $L_z$ and $L_y$ are measured, in that order. What is the probability that the results of both measurements will be +1?
   (e) Answer Part (5) of 4.2.1.
   (f) Answer Part (6) of 4.2.1.

2. [6 points] Read the discussion on pp. 133-134 of Shankar about the statistical operator (or “density matrix”). Show that the time-dependent statistical operator $\rho(t)$ obeys the evolution equation
   \[
   \frac{d\rho(t)}{dt} = -\frac{i}{\hbar} [H, \rho(t)] .
   \]
   You may start from $\rho(t) = \sum_i p_i |\psi_i(t)\rangle \langle \psi_i(t)|$, where the statistical weights $p_i$ are independent of time, and where each $|\psi_i(t)\rangle$ obeys the time-dependent Schrödinger equation.

3. [6 points] Shankar Ex. 4.2.3, p. 139 (expectation value of momentum operator).

4. [8 points] A particle in 1D has a wavefunction $\psi(x) = \langle x | \psi \rangle = 1/(x^2 + a^2)$ where $a$ is a positive real constant. Obtain a correctly normalized version of this wavefunction and compute $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, and $\langle p^2 \rangle$. Also compute the correctly normalized $\psi(p) = \langle p | \psi \rangle$.
   (Note: In this kind of problem you are allowed to evaluate definite or indefinite integrals using formulas obtained from books or programs like Maple or Mathematica, but please briefly cite your source.)