1) Prove Schwartz inequality:

\[ |\langle v|w \rangle| \leq |v||w| \]  

and triangle inequality

\[ |v + w| \leq |v| + |w| \]  

2) a) Do functions defined on the interval \([0...L]\) and that vanish at the end points \(x = 0\) and \(x = L\) form a vector space?  
   b) How about periodic functions obeying \(f(L) = f(0)\)?  
   c) How about all functions with \(f(0) = 4\)?

3) Consider the vector space \(V\) spanned by real \(2\times2\) matrices.  
   a) What is its dimension?  
   b) What would be a suitable basis?  
   c) Consider three example vectors from this space:

\[
\begin{align*}
|1\rangle &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \\
|2\rangle &= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}; \\
|3\rangle &= \begin{pmatrix} -2 & -1 \\ 0 & -2 \end{pmatrix}
\end{align*}
\]  

Are they linearly independent? Support your answer with details.

4) Consider the two vectors \(\vec{A} = 3\hat{i} + 4\hat{j}\) and \(\vec{B} = 2\hat{i} - 6\hat{j}\) in the 2-dimensional space of the x-y plane. Do they form a suitable set of basis vectors? (Explain.) Do they form an orthonormal basis set? If not, use Gram-Schmidt algorithm to turn them into an orthonormal set.

5) Assume the two operators \(\Omega\) and \(\Lambda\) are Hermitian. What can you say about
   a) \(\Omega\Lambda\)  
   b) \(\Omega\Lambda + \Lambda\Omega\)
6) Consider the matrix
\[
\Omega = \begin{pmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
1 & 0 & 0
\end{pmatrix}
\] (4)

a) Is it Hermitian?
b) Find its eigenvalues and eigenvectors.
c) Verify that \(U^\dagger \Omega U\) is diagonal, \(U\) being the matrix formed by using each normalized eigenvector as one of its columns. (Show that \(U\) is unitary!)
d) Calculate \(\exp(i\Omega)\) and show it is unitary.

7) Consider the "Theta-funcion"
\[
\theta(x - x') = \begin{cases}
1 & x \geq x' \\
0 & \text{otherwise}
\end{cases}
\]
Show that \(\delta(x - x') = \frac{d\theta(x - x')}{dx}\) by multiplaying on the r.h.s with an arbitrary square-integrable function \(f(x)\) and integrating over all \(x\).

8) Consider a ket space spanned by the eigenkets \(\{|a_i\rangle\}\) and eigenvalues \(\{a_i\}\) of a Hermitian operator \(A\) of dimension \(n\). There is no degeneracy.

a) Prove that operator
\[
\prod_{i=1}^{n}(A - a_i)
\]
is a null operator \(|0\rangle\) in this space.
b) What type of projector is this operator
\[
\prod_{j=1, j \neq i}^{n} \frac{1}{a_i - a_j}(A - a_j)
\]?