1) This problem concerns Clebsch-Gordan coefficients \( \langle jm | j_1 m_1, j_2 m_2 \rangle \).

a) What are allowed values of total \( j \) for the addition of angular momenta \( j_1 = 3 \) and \( j_2 = 1 \)?

b) Explain why \( \langle 44 | 33, 11 \rangle = 1 \). [Notation \( \langle jm | j_1 m_1, j_2 m_2 \rangle \)]

c) Find \( \langle 43 | 32, 11 \rangle \) (Hint: Use the spin lowering operator).

2) A system is in a state described by the wavefunction \( \psi(r) = f(r)(x + iy + z) \), where \( f(r) \) is a radial wave function. If \( L_z \) is measured, what are the possible values of the measurement, and their probabilities?

Note that \( Y_{00} = \sqrt{\frac{1}{4\pi}} \), \( Y_{1\pm 1} = \frac{\pm i}{\sqrt{8\pi}} \sin \theta e^{\pm i\phi} \) and \( Y_{10} = \sqrt{\frac{3}{4\pi}} \cos \theta \).

3) Consider a system of two non-identical fermions, each with spin \( 1/2 \). One is in a state with \( S_{1y} = \frac{\hbar}{2} \), while the other is in a state with \( S_{2x} = -\frac{\hbar}{2} \). What is the probability of finding the system in a state with total spin quantum numbers \( s = 0 \)?

4) A particle of reduced mass \( \mu = 470 \text{ MeV}/c^2 \) is moving in a spherical potential well of range \( a \) and depth \( V_0 = -76.73 \text{ MeV} \). \([V(r) = V_0 \text{ for } |r| < a \text{ and } V(r) = 0 \text{ for } |r| > a]\).

The particle is bound in the 1s ground state with binding energy \( E = -2.225 \text{ MeV} \).

(This is supposed to be a very simple model of the deuteron). Note: \( \hbar c = 197.327 \text{ MeV fm} \).

a) Solve the Schroedinger equation for both \( r < a \) and for \( r > a \).

b) Using the boundary conditions at \( r = a \), extract the size of the "potential range" \( a \).

c) Calculate the probability that a measurement of \( r \) will find \( r > a \), i.e. the particle is outside the range of the potential (which is of course forbidden classically).

5) Two elementary particles of spin \( s_1 \) and \( s_2 \) are bound by an attractive spin-dependent potential, as specified by the Hamiltonian

\[
H = \frac{p^2}{2\mu} + U(r) + V(r)S_1 \cdot S_2
\]  

(1)

where \( r \) and \( p \) are relative coordinate and momentum; \( \mu \) is the reduced mass; \( U(r) \) and \( V(r) \) are two different spherically symmetric potentials; and \( S_1 \) and \( S_2 \) are the spin operators for two particles. (We ignore the center-of-mass motion).
a) The Hamiltonian can also be written as

\[ H = \left[ \frac{p^2}{2\mu} + U \right]^{(c)} \otimes I^{(s)} + V^{(c)} \otimes \left[ \frac{1}{2}(S^2 - S_1^2 - S_2^2) \right]^{(s)} \]  \hspace{1cm} (2)

Briefly explain the notation used above, explain why certain terms appear before or after the '⊗' and show how the last terms involving spins was obtained, in which \( S = S_1 + S_2 \)

b) Show that a vector of the form \(|\psi_{nsm}\rangle = |\chi_{ns}\rangle \otimes |sm_1s_2\rangle\) is an eigenvector of \( H \) if \(|\chi_{ns}\rangle\) obeys the effective Schroedinger equation

\[ \left[ -\frac{\hbar^2 \nabla^2}{2\mu} + U + VC_s \right] |\chi_{ns}\rangle = E |\chi_{ns}\rangle \]  \hspace{1cm} (3)

with \( C_s = (\hbar^2/2)[s(s + 1) - s_1(s_1 + 1) - s_2(s_2 + 1)] \). Here the state \(|sm_1s_2\rangle\) is built from states \(|s_1m_1, s_2m_2\rangle\) according to the usual rules for addition of angular momenta.