ANGULAR MOMENTUM

Cartesian angular momentum operators

$$[J_i, J_j] = i\hbar\varepsilon_{ijk}J_k$$
$$[J^2, J_i] = 0$$

$$J^2\,|jm\rangle = \hbar^2 j(j+1)\,|jm\rangle$$
$$J_z\,|jm\rangle = \hbar m\,|jm\rangle$$

$j = 0,\ 1/2,\ 1,\ 3/2,\ 2,\ ...$

For a given j: $\quad m = -j,\ -j+1,\ ...,\ j \quad$ ($2j+1$ values)

Angular momentum raising and lowering operators

$J_\pm = J_x \pm iJ_y$, so that

$$[J^2, J_\pm] = [J^2, J_z] = 0$$
$$[J_z, J_\pm] = \pm\hbar J_\pm$$
$$[J_+, J_-] = 2\hbar J_z$$

$$J^2\,|jm\rangle = \hbar^2 j(j+1)\,|jm\rangle$$
$$J_\pm\,|jm\rangle = \hbar\sqrt{j(j+1) - m(m\pm1)}\ \ |j\ m\pm1\,\rangle$$

Lowering operators for first few j values

$j = 1/2$:

$$J_-|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = \hbar|\tfrac{1}{2}\ \tfrac{-1}{2}\rangle$$

$j = 1$:

$$J_-|11\rangle = 2\,\hbar\,|10\rangle$$
$$J_-|10\rangle = 2\,\hbar\,|1\,\text{-}1\rangle$$

$j = 3/2$:

$$J_-|\tfrac{3}{2}\,\tfrac{3}{2}\rangle = \sqrt{3}\,\hbar\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle$$
$$J_-|\tfrac{3}{2}\,\tfrac{1}{2}\rangle = 2\,\hbar\,|\tfrac{3}{2}\ \tfrac{-1}{2}\rangle$$
$$J_-|\tfrac{3}{2}\ \tfrac{-1}{2}\rangle = \sqrt{3}\,\hbar\,|\tfrac{3}{2}\ \tfrac{-3}{2}\rangle$$