CHAPTER 6

Energy and Its Conservation

When you drive a car you have to stop from time to time to add fuel to the tank. In a house or apartment a bill arrives each month from the electric company. And all of us have to keep breathing and eating. In each case we use *energy* that is never created or destroyed, only transformed from one kind to another.

The gasoline in the car makes it possible for the car to move, as some of its internal ("chemical") energy is changed to energy of motion ("kinetic" energy). Electric currents heat the stove or the toaster, make the lamps light up, and the fans and vacuum cleaner run. In each of these cases energy is transformed at the generating station, usually from some kind of stored internal energy, or from the kinetic energy of wind or water. It becomes "electric" energy that can be transmitted through wires, and in turn transformed to the energy that we use as heat or light or motion. The food we eat gives us the energy stored within it, which is then transformed in our bodies to the energy that we need to exist, to move, and to function.

There is hardly a concept more pervasive than energy in all of science. The food we eat is fuel that we count in energy units. We transform it to kinetic energy when we move and to gravitational potential energy when we climb. Changes of material, whether physical, as from ice to water, or chemical, as in the burning of wood, are accompanied by changes of energy. We would not exist without the energy radiated by the sun, which is liberated there by its nuclear reactions. The state of each atom and molecule is characterized by its internal energy, that is, by the motion of its components and by their distribution in space and the resulting electric potential energy.

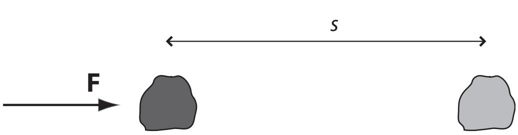
Our experience with the various kinds of energy brought us to the realization that energy cannot be *made* or created, and that it does not disappear. This is the *law of conservation of energy*. No exceptions to it have been found, and it has become a guiding principle that plays an essential role in every part of science.

6.1 Work: not always what you think

In ordinary language "work" can refer to a variety of activities, from shoveling snow and sawing wood to writing and thinking. In physics its meaning is more limited, but also more precise: you are working when you exert a force on an object and the object moves in the direction of the force (or of a component of the force). You may want to call it work just to hold a book or a rock, even when it doesn't move, but the physicist's definition wouldn't include that. If the rock doesn't move, you're not working.

If the rock does move, and there is a force on it in the direction of the motion, we define the work done on it by the force as equal to the force on the rock times the distance that the rock moves, or $W = Fs$.

EXAMPLE 1



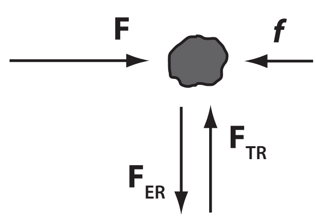
What is the work done by the force $F = 3\,\mathrm{N}$?

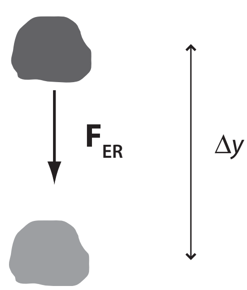
Ans.:
If the force is 3 N, and the rock moves 4 m, the work done on the rock by the force is (3 N) (4 m), or 12 Nm.

The unit Nm is also called the *joule* (J).

The force, F, (3 N) is in the same direction as the distance, s, (4 m) through which the rock moves. The work (W) done by this force is Fs, or 12 J.

This is so even though there are three other forces on the rock. Two of them are vertical, the weight of the rock ($F_{ER} = Mg$) and the upward force on it by the table (F_{TR}). There is no vertical acceleration, and so these two forces add up to zero. Since the rock does not move in the vertical direction, these two forces do not do any work on the rock. There is also the force of friction (f), in the direction opposite to the direction of the motion. However, we are asked about the work done by only one force, the 3 N force, and so we do not need to consider the work done by any of the other forces.





It is not easy to think of a situation where there is only a single force acting on an object unless we make some assumptions that simplify the situation. A freely falling rock is a good example, but only if we assume that we can neglect the air resistance. The weight, $Mg = F_{\mathrm{ER}}$, is then the only force that we need to consider.

As the rock falls a distance Δy, the work done on it by the weight is $Mg\Delta y$.

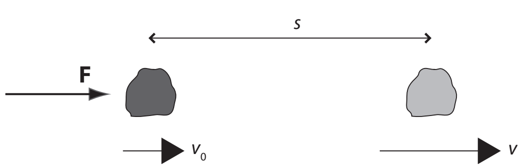
That the work is equal to the force times the distance is the basic definition when the force is constant and in the same direction as the motion. We can expand this statement so that it also works for other cases. If there are several forces on the rock, all in the direction of the motion of the rock, each one does work on it, equal to the magnitude of the force times the distance. If the directions are not the same, we can take care of that too, and will do so later.

6.2 Energy of motion: kinetic energy

The most basic kind of energy is energy of motion, or *kinetic energy*. Let's see how it is related to the quantities that we know, to force and acceleration, and to mass and speed.

Go back to our rock. Suppose that just a single force acts on it. We already know that the force will cause the rock to accelerate. Perhaps it starts from rest, or it may already be moving. The force will cause it to move faster, and after some time it will have a new and higher speed.

If we know the force and the distance, can we find out just how much faster the rock will be moving? Yes, we can. We know Newton's second law of motion, and we can find the acceleration. Once we know the acceleration, we can find how much faster the rock will be moving:

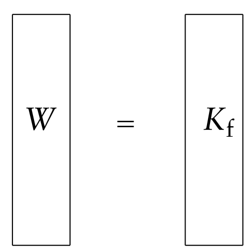


Initial velocity: $v_0 = 0$
Final velocity: v
F is the only force on the rock, and so $F = Ma$
Work done on the rock: $Fs = Mas$

Because the force is constant, the acceleration is constant too, and we can use the third relation for constant acceleration from Chapter 3, $v^2 = v_0^2 + 2as$. Since the block starts from rest, $v_0 = 0$, and $v^2 = 2as$, or $as = \frac{1}{2}v^2$. The work done by the force, Fs, or Mas, is therefore equal to $\frac{1}{2}Mv^2$.

We know what happens, but we are going to say it differently. This will open up a whole new way of looking at the same situation. We say that the work done on the rock *changes its energy of motion*, which we call its *kinetic energy*. We define the kinetic energy by saying that it is the work done on the rock, starting from rest, to the point where its speed is v. We see that this is equal to $\frac{1}{2}Mv^2$.

To visualize the energy changes we can use bar charts, in which the height of each bar represents an amount of energy or work done.

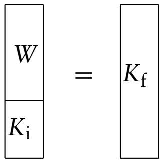


$$W = K_{\mathrm{f}}$$

The two bars have the same height, illustrating the relation that the work done on the rock (W) equals the increase (from zero to K_{f}) in the kinetic energy.

If the rock starts out with some speed v_0, we can again use Chapter 3's third relation for constant acceleration, $v^2 = v_0^2 + 2as$, but this time we have to include the term in v_0 so that the work done by the force is $Fs = Mas = \frac{1}{2}Mv^2 - \frac{1}{2}Mv_0^2$. We see that doing work on the rock is a mechanism for changing its kinetic energy.

We can write this as $\frac{1}{2}Mv_0^2 + Fs = \frac{1}{2}Mv^2$: the initial kinetic energy of the rock ($K_{\mathrm{i}} = \frac{1}{2}Mv_0^2$) plus the work done on it (W) is equal to the final kinetic energy ($K_{\mathrm{f}} = \frac{1}{2}Mv^2$).



That $K_i + W = K_f$ is always true when W is the total work (which we can call W_T) done on an object. W_T is the sum of the amounts of work done by all of the forces on it. It is also the work done by the *net force*, i.e., by the vector sum of all of the forces acting on the object.

The statement that *the total work done on an object is equal to the increase in its kinetic energy* is so universal that it is given a special name. It is called the *work-energy theorem*.

We have to remember that we are not talking about any changes inside the rock or other object. We are using the model in which the object acts as a particle, in other words, as an object without any internal structure. Only then does the work-energy theorem apply.

EXAMPLE 2

A 0.2 kg hockey puck is at rest on horizontal frictionless ice. It is then pushed with a steady horizontal 2.5-N force and moves 1.5 m.

(a) Set up a mathematical description that relates the work to the kinetic energy. Make an energy bar chart.

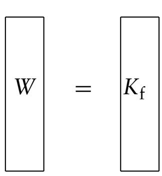
Determine how fast the puck is moving after it has gone the 1.5 m.

(b) Repeat part (a) for the same situation, but with the puck initially moving with a speed of 3 m/s in the direction of the force.

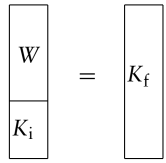
Ans.:

(a) The work, W, done by the force, F, when the puck moves a distance, s, is Fs, or (2.5 N)(1.5 m), which is equal to 3.75 Nm or 3.75 J. Since $W = K_f$, this is also equal to the final kinetic energy, K_f.

You are, however, asked to find the velocity, v_f, which is related to the kinetic energy by $K_f = \frac{1}{2}MV_f^2$, or $v_f = \sqrt{\frac{2K_f}{M}}$, which is $\sqrt{\frac{(2)(3.75)}{0.4}}$, or 4.33 m/s.



(b) This time we first have to find the initial kinetic energy, K_i, which is $(\frac{1}{2})(0.4)(9) = 1.8$ J. W is still 3.75 J, so that $K_i + W$ is $1.8\text{ J} + 3.75\text{ J} = 5.55$ J. This is K_f, so that $v_f = \sqrt{\frac{2K_f}{M}} = \sqrt{\frac{(2)(5.55)}{0.4}} = 5.27$ m/s.



We could have kept each unit in the expression for v_f before evaluating it. However, because each of the quantities (energy and mass) was expressed in the appropriate SI units (J and kg), the result for v_f is automatically in the correct SI unit (m/s). If energy and mass were not in SI units, they would first have to be converted. (Some other system of consistent units would work also, but we will stick to the SI system.)

Note also that the mathematical expressions are in terms of work and energy. These are the quantities related by the relevant physical law, namely that the kinetic energy of the puck is increased by the amount of work that is done on it. It is only after we calculate the final kinetic energy that we can find the final velocity. (We cannot add the velocities!)

6.3 Energy of position: potential energy

Gravitational potential energy

When we lift a rock from the floor to the table, we give it energy. This energy depends on where the rock is, so that we can call it *energy of position*. The energy is stored. It has the "potential" to be changed to kinetic energy if the rock falls back down to the floor. That's why it is called *potential energy*.

We are talking about the system containing both the rock and the earth. Take the rock in your hand and lift it. Do it at constant speed so that we don't have to think about a change in its kinetic energy. Your hand exerts a force and does work on the system. The force exerted by your hand, F_{HR} (hand on rock), as the rock moves with constant speed, is equal in magnitude and opposite in direction to the gravitational force of the earth, the weight, F_{ER} (earth on rock), of the rock. The work done by your hand creates *potential energy* (P). It is work done by a force that is in the direction opposite to the direction of the gravitational force, or *against* the gravitational force. We call this potential energy *gravitational potential energy*. We define *the increase in the gravitational potential energy* as the *work done against the gravitational force.*

For the system containing the earth and the rock, the forces exerted by the earth on the rock and by the rock on the earth are internal to the system. The force of your hand is external to this system. It acts to separate the earth and the rock, and increases the gravitational potential energy of the system.

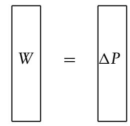
As long as the only other object besides the rock is the earth, there is not much likelihood of confusion, and we may talk about the potential energy "of the rock" while keeping in mind that we are really talking about energy shared by the rock and the earth.

The words *the increase in the gravitational potential energy* and *the work done against the gravitational force* mean the same thing. We can use one or the other. In any one problem we can use either *the increase in the potential energy* or *the work against the gravitational force*, but not both.

Your hand lifts the rock, increases its distance from the earth, and increases the potential energy. Without the presence of the earth, the gravitational potential energy would not change. We see that the potential energy is a property of both the rock and the earth. It is a property of the *system* consisting of both.

How much potential energy did we give the system as the rock moved up? Because the rock moves with constant velocity, the upward force of your hand, F_{HR}, has the same magnitude as the downward gravitational force, F_{ER}. The work (W) done by F_{HR} is the work done against the gravitational force. If the rock goes up a distance y, the force F_{HR} does an amount of work equal to $(F_{HR})(y)$.

The increase in the potential energy (ΔP) of the rock is therefore $F_{HR}y$, and since F_{HR} and $F_{ER} = Mg$ have the same magnitude, it is also equal to Mgy: $W = \Delta P = Mgy$, $\Delta P = P_f - P_i$.



So far we have talked only about a rock so near to the earth that the gravitational force on it can be considered to be constant. Later we will consider the more general situation where objects are so far from each other that the variation of the gravitational force, as given by Newton's law of gravitation, has to be taken into account.

EXAMPLE 3

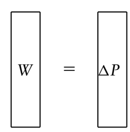
You do 40 J of work, lifting a rock 1.2 m at constant velocity.

(a) Set up a mathematical description that relates work and energy. Make an energy bar chart.
How much energy does the rock gain? What kind of energy does it gain?

(b) Determine the mass of the rock.

Ans.:

(a) There is no change in kinetic energy. We will neglect air resistance. Therefore the only energy that is changed is the gravitational potential energy: $W = \Delta P$.



The work done on the rock to lift it is 40 J, and so the rock gains 40 J of potential energy.

(b) The work (W) done against the gravitational force (Mg) is Mgy, where $W = 40\,\text{J}$, $y = 1.2\,\text{m}$, and g is, as usual, $9.8\,\text{m/s}^2$. $M = \frac{W}{gy} = \frac{40}{(9.8)(1.2)} = 3.40\,\text{kg}$.

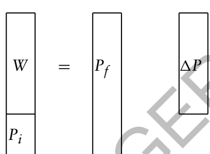
Some problems can be solved by using either the concept of force or the concept of energy. But in general there are two great advantages of using energy rather than force. One is that energy is a scalar quantity, while force is a vector quantity. The whole problem of directions and adding vectors falls away when we use the concept of energy. The second advantage is that with energy we need to consider only the initial and final energies. Except for work done on the system, or other changes in its energy, we don't need to think about what happens in between. When we use forces, on the other hand, we have to be able to follow every motion in detail: we have to know the forces at every point and at every moment.

The reference level

The change in the potential energy depends on the height, but we have not said anything about where the height is measured from. As long as we talk only about differences in potential energy, the question doesn't come up. When we want to talk about the actual amount of potential energy, we need to measure it from some *reference level*.

Because only differences in potential energy matter, it makes no difference where we take the reference level to be. It could be the floor, the ceiling, sea level, or any other convenient height. We are free to put it wherever it seems simplest. Regardless of where we choose it to be, the difference in height, $y_1 - y_2$, and the difference in the potential energy, ΔP, will be the same.

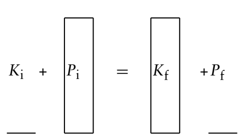
If the initial potential energy is P_i, and work W is then done on the rock to lift it, the final energy is given by $P_\text{i} + W = P_\text{f}$, or $W = P_\text{f} - P_\text{i} = \Delta P$. If we choose a different reference level, P_i will be different, but so will P_f, and the increase $P_\text{f} - P_\text{i} = \Delta P$ will remain the same. The bar chart illustrates both of these relations.



Mechanical energy and its conservation

After lifting the rock let's let it drop back, starting from rest at the top. We can use different ways to describe what happens. One is to consider the system of the rock and the earth. Its total energy remains constant. On the way down its potential energy decreases as the rock speeds up and gains kinetic energy.

$K_\text{i} = 0, P_\text{f} = 0$
$K_\text{i} + P_\text{i} = K_\text{f} + P_\text{f}$
$P_\text{i} = K_\text{f}$



We can also consider the system containing only the rock. The gravitational force (Mg) is now an external force. It does work on the rock so that it speeds up and gains kinetic energy.

How much kinetic energy does it get? Let's go back to the description in terms of the force. The definition of work tells us that on the way down the work done on the rock is equal to the force on the rock times the distance back down. The magnitude of the force is Mg, the distance is again y, and the amount of work is therefore equal to Mgy. This is the amount of kinetic energy that the rock gains on the way down.

Now we have to be careful. Is the force Mg the only force on the rock? Not quite. There is also air resistance, and there may be other forces. For now let's assume that we can neglect them. (We also neglected air resistance when the rock was on the way up.) We will come back to this question later to see the effect of air resistance.

For now, while we neglect air resistance, we see that in the system of the rock and the earth the potential energy that is lost as the rock falls is equal to the kinetic energy that is gained. The sum of the two energies remains constant while the rock falls.

We give this sum a new name, *mechanical energy*, and can then say that the mechanical energy remains constant while the rock falls.

The earth and the rock interact via the gravitational interaction. The gravitational force is *internal* to the system of the earth and the rock. Within this system there is kinetic energy and potential energy. As long as there is no force from *outside* the system, the sum of the two is constant.

As the rock falls, the kinetic energy of the system increases (mostly of the rock, but to a minute extent also of the earth) and the potential energy decreases. When the rock moves away from the earth, the potential energy increases and the kinetic energy decreases. The sum of the two, the mechanical energy, remains constant. Other objects or interactions are *outside* the system. They give rise to forces that *change* the mechanical energy of the system. These *external* forces may be air resistance, or the force of your hand as you separate the earth and the rock, i.e., as you lift the rock. In the absence of any external forces, the mechanical energy of the system remains constant.

This is our first encounter with the law of conservation of energy. It is in a very restricted form, because we are neglecting air resistance and other forces. Only a single force, the gravitational force, the weight, does work.

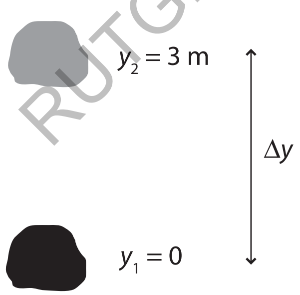
Under these circumstances, however, it always holds: in the system containing only the earth and another object, the weight of the object is the only force that does work. It is an internal force, and the mechanical energy of the system remains constant.

Remember that the choice of what we call *the system* is up to us. We can also look at the system containing only the rock. For this system, with the earth outside it, the gravitational force is an external force. As long as we continue to use the model where the rock is considered to be a particle, i.e., without internal energy, the only energy that it has is kinetic energy.

We will gradually remove all the restrictions and introduce other kinds of energy. We will then see the law of conservation of energy as one of the most fundamental and general laws in all of science.

We will talk about other kinds of potential energy. We distinguish the one that we started with by calling it gravitational potential energy, but if there is not much chance that we will confuse it with others, we will just call it potential energy.

EXAMPLE 4



A rock has a mass, M, of 2 kg, and is lifted a distance, y, of 3 m, at constant velocity.

(a) How much potential energy does it gain on the way up? (From its initial amount, P_1, to its final amount, P_2.)

(b) How much kinetic energy does it gain on the way down, if it drops the 3 m, starting from rest? (From K_2 and P_2 at the top, to K_3 and P_3 when it is back at the bottom.)

(c) What is its speed when it gets back to the starting point?

Ans.:
The system is that of the rock and the earth. As the rock is lifted, work (W) is done on it by your hand. The complete energy relation is $W + P_1 + K_1 = P_2 + K_2$. We can choose the reference level so that $P_1 = 0$. The kinetic energy remains the same ($K_1 = K_2$), so that we are left with $W = P_2$.

$$\boxed{W} = \boxed{P_2}$$

On the way down, $P_2 + K_2 = P_3 + K_3$. It starts from rest ($K_2 = 0$). It goes back to the point where $P_3 = 0$. That leave $P_2 = K_3$.

$$\boxed{P_2} = \boxed{K_3}$$

The rock's weight is Mg, or 19.6 N. On the way up it gains potential energy Mgy, or 58.8 J. On the way down it gains an amount of kinetic energy equal to 58.8 J. Since this is equal to $\frac{1}{2}Mv_3^2$, $v_3 = \sqrt{\frac{2K_3}{M}} = \sqrt{\frac{(2)(58.8)}{2}} = 7.67$ m/s.

EXAMPLE 5

Go to the website phet.colorado.edu and open the simulation *Energy Skate Park*. Check "potential energy reference." Drag the reference level to the lowest point of the skater's path. (The skater's position is indicated by the red dot below the skates.) Slow down the motion as much as possible by using the slider at the bottom. Click on "bar graph."

(a) Observe what happens to the potential energy, (P), the kinetic energy, (K), and the total (= mechanical) energy, (E). How do K and P compare when the skater is half-way down the track? Three quarters of the way down? What is the speed half-way down, compared to its maximum value?

(b) Reset to remove the bar graph. Click on "energy vs. position." Check "potential" and uncheck all others. (All units on the graph are SI units.) Check "measuring tape" and use the tape to measure the maximum height.

Calculate the weight of the skater (in N). Calculate his mass in kg. Are these results reasonable?

(c) Look at K and E and compare to your results in part (a).

Ans.:

(a) E is constant. P and K change form zero to E. Their sum is constant and equal to E. At the top $K = 0$ and P is at its maximum value, which is equal to E. At the bottom $P = 0$ and K is equal to E.

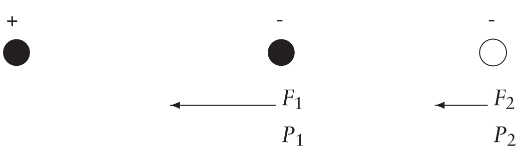
P is proportional to the height, h, above the reference level ($P = Mgh$). Therefore, when the skater is at half the maximum height the potential energy has half of its maximum value, and so has K. When he is one-quarter of the way from the bottom P is at $\frac{1}{4}$ of its maximum and $K = \frac{3}{4}E$.

Half-way down $K = \frac{1}{2}K_{\text{max}}$, or $\frac{1}{2}Mv^2 = \left(\frac{1}{2}\right)\left(\frac{1}{2}Mv^2_{\text{max}}\right)$. $v^2 = \frac{1}{2}v^2_{\text{max}}$, or $v = \frac{1}{\sqrt{2}}v_{\text{max}}$.

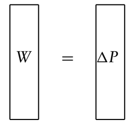
(b) For a maximum height of 4.1 m and a maximum potential energy ($= Mgh$) equal to about 3000 J, the weight is $\frac{P}{h}$ or 732 N, and the mass is $\frac{P}{gh}$ or 75 kg. This is equivalent to 164 pounds, which seems reasonable.

Electric potential energy

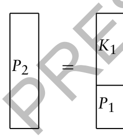
Just as the earth and all other objects attract each other by the gravitational force, positive and negative charges attract each other by the electric force. If we want to separate the charges, we have to pull them apart with a force that acts in a direction opposite to the electric force that attracts them. The force that we exert does work on the charges, and we can talk about electric potential energy in close analogy to the way we talked about gravitational potential energy.



The negative charge is attracted to the positive charge, as described by Coulomb's law. If we push it from point 1 to point 2, doing work, W, it gains potential energy: $P_1 + W = P_2$ or $W = P_2 - P_1 = \Delta P$.



If we let it "fall back" from rest at point 2 to point 1, it will lose potential energy (from P_2 to P_1) and gain kinetic energy (from $K_2 = 0$ to K_1). The sum of the kinetic and potential energies remains constant, just as in the gravitational case, as long as we neglect any forces except for the electric force between the positive and the negative charge. $K_2 + P_2 = K_1 + P_1$ and $K_1 = P_2 - P_1 = \Delta P$.



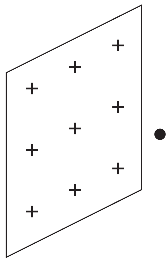
The electric force between two charges is analogous to the gravitational force between two masses. There is the added feature that while all masses are treated equally by the gravitational force, there are two kinds of charge. The electric force can be one of attraction or repulsion, depending on the sign of the charges. Apart from that, the gravitational force and the electric force, as well as the gravitational potential energy and the electric potential energy, are entirely analogous to one another. *The work done against the electric force is equal to the increase in the electric potential energy.*

To find the amount of electric potential energy that the negative charge gains as it is pushed away from the positive charge is not so easy. We can't just say that the work done is the electric force times the distance, because the force becomes smaller as the negative charge moves away. There is no single value for the force! It is still possible to calculate the change in potential energy, and we will do it later.

When we talked about gravitational potential energy we considered only situations at or near the surface of the earth. Because all of the points at the earth's surface are approximately at the same distance from the center of the earth we could take the gravitational force on an object to be constant. If the object moves to an earth satellite, that might no longer be a good approximation. And it certainly won't be if it goes to the moon.

EXAMPLE 6

A single electron ($M_e = 9.11 \times 10^{-31}$ kg) is released from rest near a large positively charged surface. It loses 6×10^{-18} J of electric potential energy after moving 0.2 m.

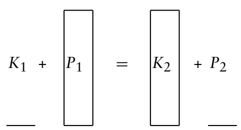


(a) Set up a mathematical description relating the various energies.

(b) Determine the electron's velocity after it has moved the 0.2 m.

Ans.:

(a)



$$P_1 + K_1 = P_2 + K_2$$
$$K_1 = 0, P_2 = 0$$
$$P_1 = K_2$$

(b) The only energies in this problem are the electric potential energy and the kinetic energy. (We will assume that there is no change in the gravitational potential energy, and that there are no forces other than the electric force.)

As the electron loses potential energy, it gains kinetic energy.

$P_1 + K_1 = P_2 + K_2$, where $K_1 = 0$. We can choose our reference level so that $P_2 = 0$. Then $P_1 = 6 \times 10^{-18}$ J, $P_1 = K_2$, and $K_2 = 6 \times 10^{-18}$ J.

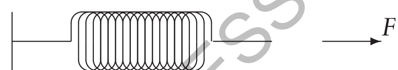
$v_2 = \sqrt{\frac{2K_2}{M}} = \sqrt{\frac{(2)(6\times10^{-18})}{9\times10^{-31}}} = 3.63 \times 10^6$ m/s, and it moves toward the plane.

Springs: elastic potential energy

Here is another potential energy. Suppose a spring is held stationary at one end and you pull on the other. The spring pulls on your hand in the opposite direction. *The work that you do against the elastic force of the spring is equal to the increase in the spring's elastic potential energy.*

If you let go, the potential energy that was stored in the spring is converted into kinetic energy. If we ignore air resistance, and also any friction in the spring, so that we leave only the elastic force that the spring exerts, the sum of the kinetic and potential energies will again be constant. As the potential energy decreases, the kinetic energy increases.

Like the electric force, the elastic force is not constant as the object on which it acts moves. As the spring is stretched, more and more force is required to stretch it further. We can therefore again not simply say that the work done by the elastic force is equal to the force times the distance.



Here is a spring that starts unstretched. We will use the subscript "1" for the energies there, and take the elastic potential energy P_1 to be zero. Since it is not moving, K_1 is also zero.

It is then stretched by the force, F, until it has a potential energy P_2, with K_2 still zero. When it is released, the potential energy is changed to kinetic energy. When it gets to the original starting point, the potential energy is again zero, and all of the potential energy has changed to kinetic energy (K_3).

$$P_1 = 0 \qquad K_2 = 0 \qquad P_3 = 0$$
$$P_1 + W = P_2$$
$$K_2 + P_2 = K_3 + P_3$$
$$P_2 = K_3$$

EXAMPLE 7

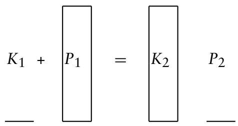
A ball whose mass is 0.02 kg is shot horizontally from a spring gun. Initially the spring is compressed and stores 10 J of energy.

(a) Set up a mathematical description that relates the energies.

(b) Determine how fast the ball is moving just after it has been fired from the gun.

Ans.:

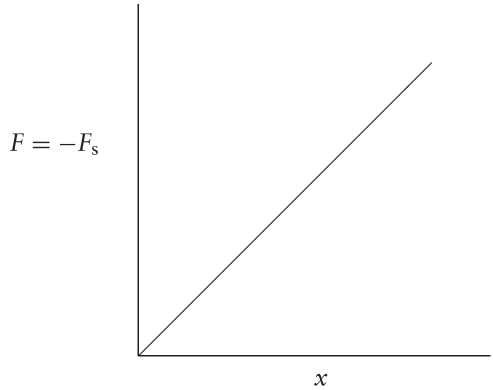
Just after the ball leaves the gun, it has not yet moved vertically. Hence there is no change in the gravitational potential energy. We neglect any frictional losses. The only changes are then in the elastic potential energy (P), which is lost by the spring (10 J), and the kinetic energy, which is gained by the ball.



$K_1 + P_1 = K_2 + P_2,\ K_1 = 0,\ P_2 = 0$, and $P_1 = K_2$

$K_2 = 10\,\text{J} = \frac{1}{2}Mv_2^2$

$v_2 = \sqrt{\frac{(2)(10)}{0.02}} = 31.6\,\text{m/s}$



Hooke's law and the expression for the elastic potential energy

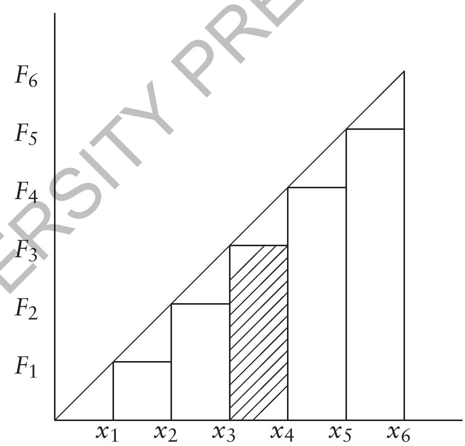
Experiments show that for most springs the force that a spring exerts is, to a good approximation, proportional to the displacement from the equilibrium position. This is so until the spring is stretched so much (beyond the "elastic limit") that it distorts permanently and no longer returns to its original shape.

For a force smaller than that of the elastic limit, the force exerted by the spring is $F_s = -kx$, where k is a proportionality constant that is characteristic for the particular spring, and is called the *spring constant*. The *minus* sign shows that the force that the spring exerts is in a direction opposite to the displacement from the equilibrium configuration. The relation $F_s = -kx$ is called *Hooke's law*.

How much work has to be done on the spring to stretch it from its equilibrium position ($x = 0$) to some new position, where the displacement is x?

First we will shift from the force $F_s(= -kx)$ that the spring exerts to the external force with which the spring is being pulled. We will call this force F. It is equal in magnitude and in a direction opposite to F_s, and so is equal to kx. The graph shows the straight line $F(x)$.

If we want to find the work that is done to stretch the spring, we can't just multiply "the force" by the displacement, as we did for constant forces, because now the force changes. In the next figure we have divided the area below the line into a number of vertical strips, six to start with. For each strip the force does not vary as much, and the work done by F is approximately equal to the area of the strip. For example, the area of the crosshatched strip is $(F_3)(x_4 - x_3)$.



We can repeat this procedure for the other intervals. The work during all six intervals is then the area of all six strips. This is a little less, but close to the triangular area under the line.

If we had used more than six intervals, the area of the strips would be closer to the area under the line, and it becomes equal to it in the limit as the strips become narrower and their number becomes larger. We can see that the work done on the spring is, in fact, equal to the triangular area under the line, which is equal to $\left(\frac{1}{2}\right)(kx)(x)$ or $\frac{1}{2}kx^2$. This is the work done *against* the spring's force, and is therefore equal to the elastic potential energy that the spring gains when it is stretched from $x = 0$ by the amount x.

The same method can be used for other forces that are not constant. On a graph of F against x, the work done by F is again equal to the area under the curve.

For the special case where F and x are related *linearly*, in other words, when the graph of F

against x is a straight line (as in the case of the spring), the average force is equal to half of the final force. For the stretching of the spring from $x = 0$ to x, this is $\frac{1}{2}kx$, and the work done is $(\frac{1}{2}kx)(x)$, or $\frac{1}{2}kx^2$, as before. But when the graph of F against x is not a straight line, the average force is not half of the final force. The work is then still the area under the curve of F against x, but it is no longer $\frac{1}{2}kx^2$.

Go to the PhET website (http://phet.colorado.edu) and open the simulation *Masses and Springs*.

Move the friction slider to the far left so that there is no friction. Hang one of the masses on one of the springs and watch it oscillate up and down. Click on "Show energy..." and watch the changes in the various energies: kinetic, gravitational potential, and elastic potential. Where is each energy at its maximum and at its minimum? Where is the kinetic energy zero? Note that you can change the reference level for the potential energy.

Explore the changes in the motion with the amount of mass and the softness of the spring. (Note that you can slow the motion down.)

As long as the force follows Hooke's law (and this is programmed into the simulation) the mass moves with simple harmonic motion, i.e., proportional to $x = \sin \omega t$. Here $\omega = \frac{2\pi}{T}$, where T, the *period*, is the time for one complete oscillation. (Add T to t to see that x comes back to the same value after the time T.)

Here is how we can see the relation between Hooke's law and simple harmonic motion. Write Hooke's law as $a = -\frac{k}{m}x$. Remember that a is the slope of the slope (the *second derivative*) of x. Is there a function that has the property that the slope of its slope is the negative of the function itself? Yes! It is the sine or the cosine or some combination of the two.

We can make the relation quantitative. Let $x = A\sin \omega t$, which is the same as $x = A\sin\frac{2\pi t}{T}$. The slope of the slope of this function can be shown to be $-\omega^2 x$. Comparing this relation to Hooke's law shows that $\frac{k}{m} = \omega^2$, which leads to $T = 2\pi\sqrt{\frac{m}{k}}$.

6.4 Friction and the loss of mechanical energy

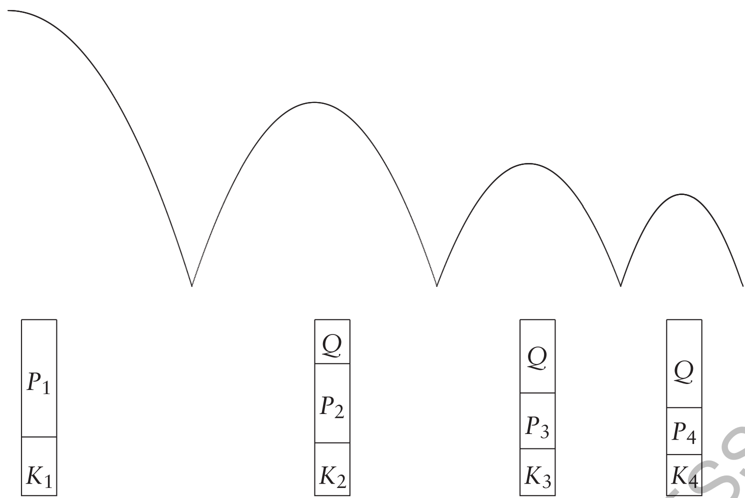
In each of the three cases (gravitational potential energy, electrical potential energy, and elastic potential energy) the term *potential energy* implies that energy is stored, and that it can be retrieved and converted into an equivalent amount of kinetic energy. Each of the kinds of potential energy changes when the position changes. The gravitational potential energy changes when the rock or other object moves closer or farther from the earth. The electric potential energy changes when the charges move closer to each other or farther apart. The spring's elastic potential energy changes when it is stretched or unstretched. As long as there are no forces other than the gravitational force, the electric force, and the elastic force, the sum of the kinetic energy and the various potential energies is constant, and an object that comes back to a particular position will have the same kinetic energy there that it had at that position at an earlier time. A bouncing ball, for instance, will come back to the same height.

But wait a minute. That doesn't really happen! If I drop a ball, it won't bounce back to the same height. Each time it bounces, it will come back, but not to the same height as before. After some time it will stop bouncing and just sit there.

The ball loses potential energy on the way down, but the kinetic energy that it gains is not the same, because the force of air resistance slows it down. Air resistance causes some of the mechanical energy to be lost. Some mechanical energy is also lost when the ball is distorted when it comes in contact with the floor, and when some sound is produced as the ball bounces.

When we said that the bouncing ball will come back to the same height after it bounces, we assumed that we can neglect the effects of all forces on the ball except for the one that the earth exerts on it, i.e., its weight. For a real ball this is clearly not a good assumption. We can not simply talk about the ball as if it were a particle. Unlike a particle, it takes up space and encounters air resistance. It gets squeezed and deformed as it bounces. It has an internal constitution that we have to consider.

When the ball finally stops, some of its potential energy has been lost but there is no kinetic energy to take its place. The sum of the two, the mechanical energy, is no longer the same. Some of it has been transformed to another kind of energy. It has been *dissipated*. We will use the symbol Q for the mechanical energy that has been dissipated.



What spoils the earlier story is that there are forces that we have neglected. One of them is air resistance. It prevents us from getting all the potential energy back and keeping the mechanical energy constant.

Air resistance slows the ball down whether it is on the way up or on the way down. This is very different from what the weight does: the weight speeds the ball up on the way down and slows it down on the way up.

Some mechanical energy has been used up to do work against air resistance. You can't get this part of the mechanical energy back. It is gone. It has been *dissipated*. That's why air resistance is called a *dissipative* force.

EXAMPLE 8

A ball whose mass is 0.5 kg is dropped from rest at a height of 2 m above the floor. It bounces and comes back to a height of 1.5 m before dropping again. How much mechanical energy has been dissipated by the time it gets to 1.5 m after the first bounce?

Ans.:

$$P_1 = P_2 + Q$$

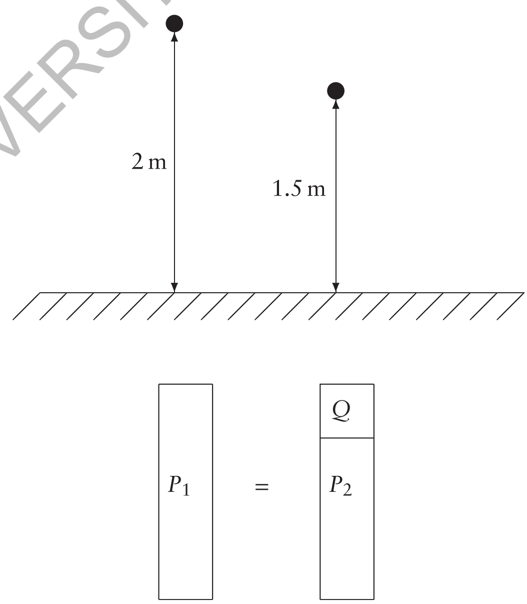
$$Q = P_2 - P_1 = Mg(y_2 - y_1) = Mg\Delta y$$

Use the floor as the reference level and measure P from there. At the starting point (point 1) $P_1 = Mgy_1 = (0.5)(9.8)(2) = 9.8$ J.

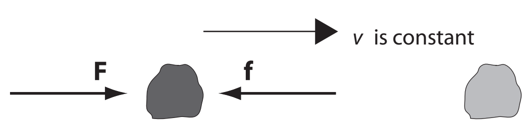
After the bounce, at 1.5 m (point 2), $P_2 = Mgy_2 = (0.5)(9.8)(1.5) = 7.35$ J.

The kinetic energy is zero at both points. Hence the mechanical energy that has been dissipated is $P_2 - P_1$, or 2.45 J.

The amount of mechanical energy $Mg\Delta y = (0.5)(9.8)(0.5)$ J $= 2.45$ J is lost to the system. It is *dissipated*.



Push a rock along a table so that it moves with constant velocity. You apply a force. The rock moves. You do work. There is another force, and it works against you. That is the force



of friction. It works against you regardless of whether you pull or push. If you turn around and push the rock back, the force of friction also turns around, and still opposes the force with which you push the rock.

The work that you do does not increase the rock's energy. It gains neither kinetic energy nor potential energy. Both P and K remain constant.

W	$=$	Q
K_1		K_2

If there were no friction, the block would accelerate. The work done on it would result in increased kinetic energy. But for a large-scale, macroscopic object, one that you can see, there is always some friction. The increase in the rock's kinetic energy is always less than the work that you do as you push the block.

Go to the PhET website and open the simulation *Masses and Springs*.

Let there be some friction, and explore the various energies, this time including the dissipated energy, here called *thermal energy*.

EXAMPLE 9

You push a block 0.8 m along a horizontal table with a force of 2 N. The block moves with constant velocity.

(a) Set up a mathematical description that relates the energies.

(b) Determine how much mechanical energy is dissipated.

Ans.:

(a) $K_1 = K_2, \quad P_1 = P_2, \text{ and } W = Q.$

(b) $W = Fs = (2\,\text{N})(0.8\,\text{m}) = 1.6\,\text{J}.$

This is the amount of mechanical energy that is dissipated.

W	$=$	Q
K_1		K_2

In the real, large-scale ("macroscopic") world there are always friction, air resistance, or other dissipative forces. The ball does not keep bouncing. The mechanical energy does not stay constant.

It was a revolutionary discovery when it was realized that when the mechanical energy disappears, something else appears, namely an equivalent amount of another kind of energy, which we have not yet considered.

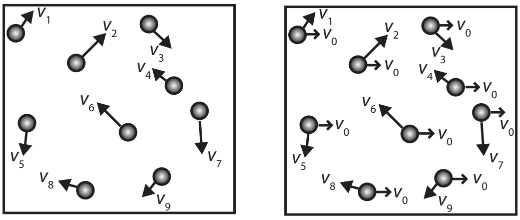
As the block slides along the table, its temperature increases. That's a sign that its *internal energy* increases. Some of the energy is transferred to the table and the air, and their temperature increases, showing that their internal energy also increases. Mechanical energy is lost, but internal energy is gained. The *total energy* remains constant. It is *conserved.*

There are two great insights that led to our understanding of the process that accompanies the decrease of mechanical energy. The first is that it was shown experimentally that a definite amount of internal energy is produced each time a definite amount of mechanical energy is lost.

The second is that on the *microscopic* level, that of atoms and molecules, mechanical energy doesn't disappear at all. What we observe as heating and an increase in internal energy on the macroscopic or large-scale level is, in fact, simple to understand on the microscopic level. The microscopic particles, the atoms and molecules that make up a macroscopic object, are all in motion, randomly, and at all times, each with its own kinetic and potential energy. A change in the total internal energy is actually a change in the kinetic and potential energies of these particles.

In a gas the atoms or molecules move freely, and the internal energy is primarily kinetic energy. In a solid they vibrate about their equilibrium positions, and the internal energy is kinetic and potential in roughly equal parts.

We use the term *internal energy* (U) as the macroscopic quantity that represents the sum of the microscopic kinetic and potential energies of the particles that make up the macroscopic object. One way in which the internal energy can change is when the particles speed up, as they do when the temperature increases. It can also change in other ways, for example, when it is squeezed, or when there is a *phase change* such as that from ice to water. In these two cases the microscopic potential energy changes.



The internal kinetic energy is that of motion of the atoms and molecules in all posssible directions, randomly. This is very different from the motion of an object as a whole, where the atoms move together, all in the same direction. The schematic diagram on the left shows a number of molecules moving randomly. The diagram on the right shows an additional velocity, v_0, for each molecule, representing a movement of the whole piece of material with this velocity. (In reality the velocities of the individual molecules are generally much larger than the velocity of the whole piece.)

We use the word *heat* or *heat energy* only for energy that is transferred from one object or system to another. A material does not *contain* a certain amount of heat, as if heat were a kind of substance. Heating is a *process* by which energy can be transferred from one piece of material to another.

In this sense heat is similar to work. Work is not *contained* in an object. It also represents a process by which energy is transferred from one object to another. There is, however, a profound difference between work and heat. Work can increase mechanical energy, i.e., potential energy and kinetic energy, and it can also change the internal energy.

On the other hand, when an object is heated, its internal energy increases, but there are severe limits on how much of the energy can be changed to mechanical energy. We will come back to this distinction later when we discuss *the second law of thermodynamics*.

EXAMPLE 10

The typical random speed of helium atoms at room temperature is about $v_a = 1300$ m/s.

(a) What is the ratio of the internal Kinetic energy, U, of a mole of helium at room temperature to the kinetic energy, K, of the same amount of gas in a balloon moving up at the rate of $v_b = 1.3$ m/s?

Ans.:
For N atoms $U = (N)(\frac{1}{2}mv_a^2)$ and $K = (N)(\frac{1}{2}mv_b^2)$. The ratio $\frac{U}{K}$ is equal to $(\frac{v_a}{v_b})^2 = 10^6$.

The internal energy of the gas is about a million times as large as the kinetic energy of the balloon!

6.5 Internal energy and the law of conservation of energy

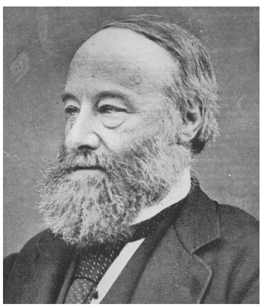
At first it doesn't seem unreasonable to think of heat as some kind of substance, a fluid that can flow between objects. After all, when a hot body and a cold body are brought together, we say colloquially that heat "flows" from one to the other, until they are at the same temperature. It used to be thought that the heat generated by rubbing two bodies together somehow resulted from the squeezing out of this substance.

The crucial experiments that showed that no such substance exists were done by Benjamin Thompson (1753–1814), one of the strangest figures in the history of science. He was born in Massachusetts, but became a spy for the British, and then fled to England. He was knighted by King George III, and later was in the employ of the Elector of Bavaria, who made him a general and a count of the Holy Roman Empire. He became known as Count Rumford, choosing the name of a village near where he was born.

Benjamin Thompson, Count Rumford.
Courtesy of MIT Press.

In 1798 he was in charge of the making of cannons. The drilling process is accompanied by a very large amount of heating. He first tried to measure the weight of the elusive substance, (termed "caloric") that was supposed to represent heat, and did not find any. Even more interestingly, he showed that there seemed to be no limit to the amount of heating that could be produced. He concluded that heat had to be something quite different from a material substance.

Detailed quantitative experiments were later made by James Prescott Joule (1818–1889), who showed that for a given amount of work done, a definite amount of internal energy (often colloquially called heat) is generated. He stirred water in a container with a paddle wheel. The work was done and measured by a weight attached to the paddle wheel. The internal energy that was produced was determined by measuring the amount of water and the rise of its temperature. Look for "Joule apparatus" on the internet.



James Prescott Joule.

The work done is equal to Mgy, where Mg is the weight and y is the vertical distance through which it moves. In our modern SI units, with M in kg, Mg in newtons, and y in meters, the work is measured in units that we now call *joules*. Heat units were developed independently, at a time when their connection to energy was unknown. The *calorie* is the amount of energy that raises the temperature of 1 gram of water by one degree Celsius. Joule's experiments, in 1847 and later, led to the number of joules equivalent to one calorie, close to the modern value of 4.186 J/cal.

A quite different experiment had already been done in 1807 by Joseph Louis Gay-Lussac. He showed that a gas that was allowed to expand into a container while pushing a piston would cool, while no such temperature change occurred if the gas did not do any work during the expansion.

These experiments were crucial for the realization that mechanical energy and internal energy represent different forms of the same kind of quantity and can be transformed from one form to the other. They led to the establishment and acceptance of the *law of conservation of energy*.

In its most succinct form the law says that *in an isolated system the total energy is constant*. In other words, different kinds of energy can be converted from one to the other, but without any change, by creation or disappearance, of the total amount of energy.

Alternatively, we can count the amount of energy input to a system and say that the system's energy is increased by the net work done on it, and, in addition, by the net amount of heating that is done on it.

Initially the law of conservation of energy applied only to heat and mechanical energy. Other forms of energy were introduced later, especially electromagnetic energy, and the law was generalized to include them. In 1905, as the result of the special theory of relativity, it was seen that the mass of an object could also be transformed to energy, and that the law of conservation of energy needed to be generalized to include this fact. With the inclusion of all forms of energy, as well as mass, it continues to be a cornerstone of modern science.

EXAMPLE 11

Go to the website phet.colorado.edu and open the simulation *Energy Skate Park*. Check "potential energy reference." Drag the reference level to the lowest point of the skater's path. Slow down the motion as much as possible by using the slider at the bottom. Click on "bar graph." Click on "track friction." Put the slider marked "coefficient of friction" at a point two spaces from the left end.

"Pause," put the skater at a point near the top of the track, and "Resume."

What happens to K, P, to the internal (= thermal) energy, U, and to the total energy (E)? How is the mechanical energy related to the total energy at the start? What is it when the skater has come to rest?

Ans.:

The total energy, E, is still constant. The mechanical energy is still $P + K$, but it is no longer equal to the total energy because it is gradually being transformed to thermal energy, i.e., to the internal energy of the skater, the track, and the air. At the start the mechanical energy is the total energy. At the end it is zero and all of it has been transformed to thermal energy.

EXAMPLE 12

How much potential energy does a person whose mass is 75 kg gain as he or she climbs the Empire State Building ($h = 443.2$ m)? How many food calories are equivalent to this amount?

Ans.:
$\Delta P = Mgh = (75)(9.8)(443.2) = 3.26 \times 10^5$ J

1 cal = 4.186 J, and so this is equivalent to 7.79×10^4 cal. However, a food calorie, as listed on a cereal box, is equal to 1000 cal or a kilocalorie. We will write it as a Calorie with a capital C, i.e., 1 kcal = 1 Cal. The answer is therefore 77.9 kcal or 77.9 Cal.

This would be the number of food calories used up in climbing to the top of the Empire State Building if the process were 100% efficient. The actual efficiency of the human body ranges approximately from 3% to 20%, and so you would need at least 400 Cal.

The human body is a complicated engine. The food we eat combines with the oxygen we breathe to liberate the energy that we use. This energy maintains the body's temperature and is, in part, converted to mechanical energy when we move. Excess food leads to stored material and we gain weight. Excess exercise draws on material that contains stored energy.

The quiescent body, as it lies motionless or sleeping, uses energy at a rate called the basal metabolic rate (BMR). The BMR can be measured, for example, by measuring the amount of oxygen that is breathed in and the amount of carbon dioxide that is breathed out in a given time.

You can estimate your BMR crudely by multiplying your body weight in pounds by 10 to get the BMR in kcal/day. A better approximation is the Harris–Benedict equation, which comes in two versions: one for males: $66 + 13.7W + 5H - 6.8A$, and one for females: $655 + 9.6W + 1.85H - 4.7A$, where W is the mass in kg, H is the height in cm, and A is the age in years.

Estimates of the energy used in a variety of activities can be found in many publications. (For example, *Exercise and Weight Control*, The President's Council on Physical Fitness and Sports, http://www.fitness.gov, *Essentials of Exercise Physiology*, W. D. McArdle, F. I. Katch, and V. L. Katch, Lea and Febiger, Philadelphia, 1994). The numbers are often given to three significant figures, but they depend on many factors, both external and internal to the body, and are necessarily quite approximate.

To a reasonable approximation, away from rest and from extremes of stress, the energy used in level walking, running, swimming, and bicycling is constant for a given distance. (At twice the speed you then use the same energy in half the time.) The values are proportional to the weight. (If you weigh 10% less, you use 10% less energy as you move.)

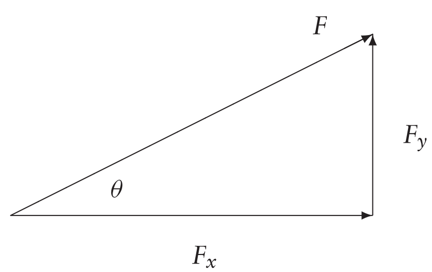
Here are numbers for the energy in kcal used per mile, averaged from several sources: (Remember that these are rough guides, good to perhaps ±15%.)

walking: 85
running: 110
swimming: 400
bicycling: 40

You can see that it takes quite a while to use up the number of Calories represented by a candy bar (150 to 250 and more), or by a glass of orange juice (100).

6.6 Work and energy revisited

Let's look at our definition of work again to see how it can be used when the force and the displacement are not in the same direction. In that case we use only the part of the force that is in the direction of the displacement. This is the only part of the force that does work. We call that part the *component* of the force in the direction of the displacement. If the angle between the force (F) and the displacement (x) is θ, that component

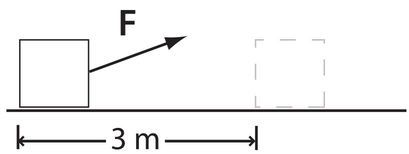


(the x component) is $F \cos \theta$, and the work done by the force is $Fx \cos \theta$.

The figure shows the force, **F**, and its x component, F_x. It also shows the y component, F_y. Since $\cos\theta = F_x/F$ and $\sin\theta = F_y/F$, the x component is $F\cos\theta$ and the y component is $F\sin\theta$.

If F and x are at right angles, so that $\theta = 90^o$ and $\cos\theta = 0$, the component of the force in the direction of x is zero, and the force does no work.

EXAMPLE 13



You move a block across a horizontal frictionless surface by pulling with a 4-N force on a rope that is attached to it at an angle of 30° with the horizontal. How much work have you done when the block has moved 3 m?

Ans.:
Work is done only by the component of the force that is in the direction of the motion. This component, F_x, is $(4)(\cos 30°)$ N, or 3.46 N. $W = F_x s = (3.46)(3) = 10.39\,\text{Nm} = 10.39\,\text{J}$.

EXAMPLE 14

The moon travels around the earth at a steady speed in an orbit that is close to being circular. Use the approximation that the path is exactly circular and neglect the influence of the sun and of any other astronomical objects.

What is the nature of the forces that act on the moon? Give an expression for its acceleration. Is any work done on it? How do its kinetic and potential energies change?

Ans.:
The only force on the moon is the gravitational attraction by the earth, F_{em}, and it is at right angles to the path. No work is done, the kinetic and potential energies each remain constant, and the moon's motion continues. The dissipative forces are so minute that the assumption of constant mechanical energy is extremely close.

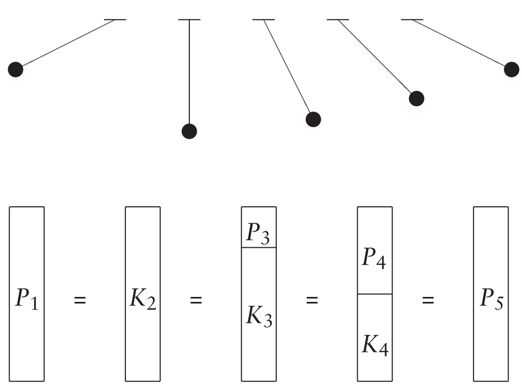
If the moon moves in a circle, there must be an acceleration toward the center (the centripetal acceleration) and it has to be equal to $\frac{v^2}{r}$. If it were bigger or smaller, the moon would not continue to move in a circular path. The net force that gives rise to the acceleration is toward the center, and its magnitude must be equal to $\frac{Mv^2}{r}$. This is not an additional force. The gravitational force is the only force.

EXAMPLE 15

A pendulum consists of a string suspended at one end, with a ball swinging back and forth at the other end. It starts from rest at point 1, where it has only potential energy, P_1. Measure the potential energy from the level of its lowest position (point 2), so that $P_2 = 0$, and it has only kinetic energy there.

Describe the forces on the ball, the work that they do, and the energy changes as the pendulum moves back and forth. Make a bar chart at different points along the path of the pendulum.

Ans.:
There are two forces on the ball. One is the force along the string, called the tension. The motion of the ball is in a circle, with the string as its radius. The circular path is at right angles to the radius. Since the tension is along the radius, it is at right angles to the path, and so does no work.



The second force is the weight. It acts straight down. It is not at right angles to the path (except at one point—do you see where?).

Consider the system of the earth and the pendulum. As the pendulum swings downward, the kinetic energy increases and the potential energy decreases. The pendulum then swings upward, the potential energy increases, and the kinetic energy decreases until it becomes zero. The pendulum comes back, and continues to swing back and forth.

There is also air resistance, and friction at the place where the string is suspended. These are the dissipative forces. If we neglect them there are no dissipative forces and the mechanical energy (the sum of the kinetic and potential energies) remains constant. The pendulum comes back to the same height, with the same potential energy at the top of each swing, and back to the same speed, and the same kinetic energy at the bottom of each swing. It continues forever.

Of course this is not what happens for any real pendulum. The dissipative forces of air resistance and friction cause the mechanical energy gradually to diminish, and the pendulum eventually comes to rest.

If we neglect friction and air resistance, the mechanical energy remains the same. It is *conserved*.

Go to the PhET website (http://phet.colorado.edu) and open the simulation *Pendulum Lab*.

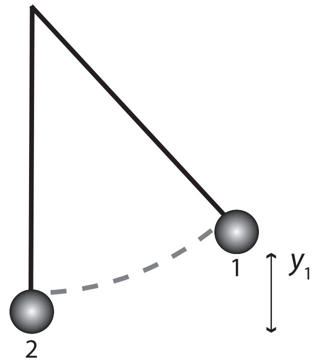
Put the friction slider to the left so that there is no friction. Check to show velocity and acceleration. Set the pendulum into oscillation and watch the two vectors. Slow the motion down to see what happens more clearly. Where is the velocity zero? What is the angle between it and the string of the pendulum? Where is the acceleration tangential to the motion? Where is it perpendicular to the motion and along the pendulum string? Follow what happens to the component of the acceleration parallel to the motion and perpendicular to it.

Click on "Show energy." Follow the kinetic energy, the potential energy, and the total energy. Where is each at its maximum and where are they zero? (Don't let the energy get so large that the bar graph no longer follows it.)

Explore the effect of changing the length and the mass of the pendulum, adding friction, and going to the moon and to other planets.

EXAMPLE 16

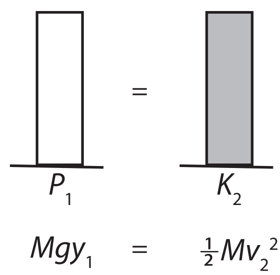
A pendulum starts from rest at the height y_1, 30 cm above its lowest point, where the height is y_2. What is the maximum speed of the pendulum bob as it swings?



Ans.:
The maximum speed is reached at the lowest point. We will assume that the only forces that we need to consider are the tension of the string and the weight of the pendulum bob. Since the tension is at all times perpendicular to the path of the pendulum bob, it does no work and does not change the mechanical energy. In the system that contains the pendulum bob and the earth, the mechanical energy is in part the kinetic energy and in part the potential energy of the pendulum bob.

Take the reference level at the lowest point, so that $y_2 = 0$. The potential energy at that point is then $P_2 = 0$ and $P_1 = Mgy_1$, where $y_1 = 0.3\,\text{m}$. Since the pendulum is at rest at point 1, $K_1 = 0$.

Mechanical energy is conserved, so that $P_1 + K_1 = P_2 + K_2$, which here reduces to $P_1 = K_2$, or $Mgy_1 = \frac{1}{2}Mv_2^2$ or $v_2 = \sqrt{2gy_1} = 2.42\,\text{m/s}$.



EXAMPLE 17

A baseball moves freely through the air, in *projectile motion*. Describe the motion, the forces on the ball, and the changes in energy as it flies through the air from the time just after it leaves the bat to the time just before it returns to the ground. Make a bar chart.

Neglect air resistance. Draw graphs that show the horizontal and vertical components of the velocity at various points along the path of the ball. Also draw a graph that shows the potential, kinetic, and mechanical energies as a function of the horizontal distance along the path.

Ans.:

Once the ball has left the bat the only force on it is the force exerted on it by the earth, i.e., its weight, vertically downward. There is also air resistance, but we will neglect it. This may or may not be a good approximation, but if it is, the only force that does work is the weight, and mechanical energy is conserved.

As long as we neglect all dissipative forces, and the only force is the weight, there is no horizontal force and no horizontal acceleration, and so the horizontal velocity, v_x, remains constant.

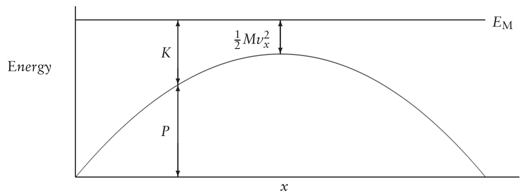
Taking "up" as positive, the vertical acceleration is $-g$, and $v_y^2 = v_{y0}^2 - 2gy$.

Take the reference level at the starting point (point 1), so that $P_1 = 0$. $K_1 = \frac{1}{2}Mv_0^2 = \frac{1}{2}M(v_x^2 + v_{y0}^2)$. These are also the values when the projectile returns to the same height at point 4.

At the highest point (point 3) $v_y = 0$ and $v = v_x$, $K_3 = \frac{1}{2}Mv_x^2$, and $P_2 = Mgy_2$. At intermediate points there are intermediate values for v_y, K, and P.

The figure shows graphs of P, K, and the mechanical energy, E_M, which is equal to $P + K$:

Since $P = Mgy$, a graph of P against x will look just like the graph of y against x. As long as we neglect all dissipative forces, E_M remains constant. K is the difference between E_M and P.



EXAMPLE 18

A stone with mass 0.2 kg is thrown at an angle to the horizontal, with an initial velocity whose horizontal component is 5 m/s and whose vertical component is 3 m/s.

Neglect all dissipative forces. Use the starting point as a reference level for the height and the potential energy.

(a) What is the initial kinetic energy, K_1?

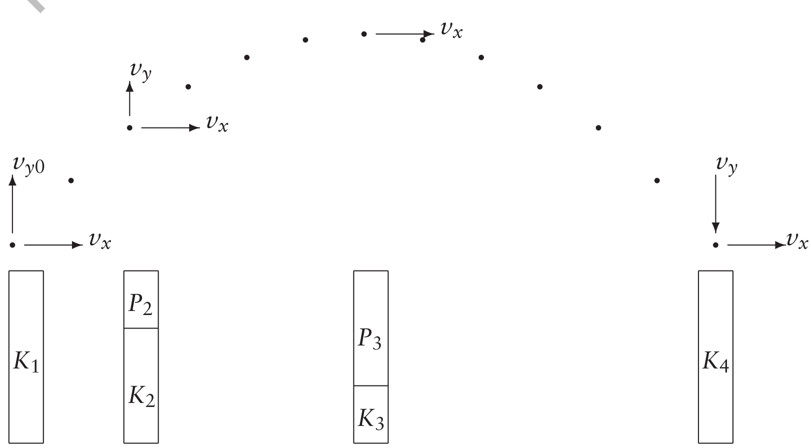
(b) What is the kinetic energy, K_4, at the point where the rock comes back to the height where it started?

(c) What is the kinetic energy, K_3, at the highest point?

(d) What is the potential energy, P_3, at the highest point?

(e) At point 2 the rock has reached half of its maximum height. What are the kinetic and potential energies at this point?

(f) What are the height and the horizontal and vertical components of the velocity at the half-way point?



Ans.:

(a) The initial velocity $\mathbf{v}_0$, has an x component v_x and a y component v_{y0}. The magnitude v_0 is $\sqrt{v_x^2 + v_{y0}^2}$ and the initial kinetic energy is $\frac{1}{2}Mv_0^2$ or $\frac{1}{2}M(v_x^2 + v_{y0}^2)$. This is equal to $(\frac{1}{2})(0.2)(25 + 9) = 3.4$ J.

(b) Since we are neglecting all dissipative forces, the mechanical energy is conserved. The height at the starting and ending points is the same, and so is the potential energy. The kinetic energy is therefore again 3.4 J.

(c) At the highest point the vertical component of the velocity is zero. In the absence of any horizontal forces the acceleration has no horizontal component, and the horizontal component of the velocity remains constant, at 5 m/s. The kinetic energy at that point is therefore $(\frac{1}{2})(0.2)(25) = 2.5$ J.

(d) Let the potential energy be zero at the starting point. $P_1 = 0, K_1 = 3.4$ J, so that the mechanical energy is 3.4 J. In the absence of dissipative forces the mechanical energy is conserved and remains constant at this value. The potential energy at the highest point is therefore $3.4 - 2.5$, or 0.9 J.

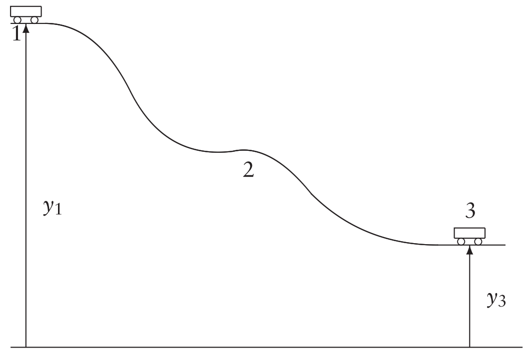
(e) At a point half-way to the highest point, the height and the potential energy are half of what they are at the highest point. $P_2 = 0.45$ J. The mechanical energy is still 3.4 J, so that the kinetic energy at this point is 2.95 J.

(f) $P_2 = Mgy_2$, so that $y_2 = \frac{P_2}{Mg} = \frac{0.45}{(0.2)(9.8)} = 0.23$ m.
v_x remains constant at 5 m/s.
$K_2 = 2.95\text{ J} = (\frac{1}{2})(0.2)(5^2 + v_y^2)$, so that $5^2 + v_y^2 = 29.5$. Hence $v_y^2 = 4.5$, and $v_y = 2.12$ m/s.

EXAMPLE 19

A cart rolls along a track as in the figure, starting from rest at point 1 at the top. Neglect friction and other dissipative forces and neglect any internal motion in the cart. There is no engine, and we will neglect any complication from the fact that the wheels are turning and therefore have *rotational kinetic energy*.

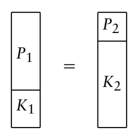
Draw energy bar charts and write mathematical descriptions for the total energy at point 1, point 2 in the middle, and point 3 at the end.



Ans.:

We can use the system containing the cart and the earth. The force of the track on the cart consists of two components: one is the component perpendicular (or normal) to the track, which does no work. The other is the component parallel to the track. This is the force of friction. We are assuming it to be zero.

With our assumptions the sum of the potential energy (P) and the kinetic energy (K) (of the system containing the cart, the track, and the earth) remains constant. We can look at any two points along the path of the cart, characterized by subscripts 1 and 2 at a difference in height $y_1 - y_2$, and write $P_1 + K_1 = P_2 + K_2$, just as in the previous examples.



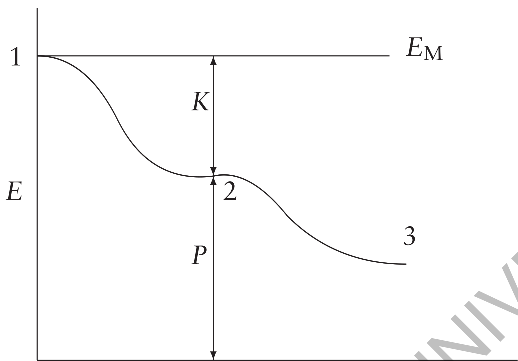
Because $K_1 = \frac{1}{2}Mv_1^2, K_2 = \frac{1}{2}Mv_2^2$, and $P_1 - P_2 = Mg(y_1 - y_2)$, we can also write

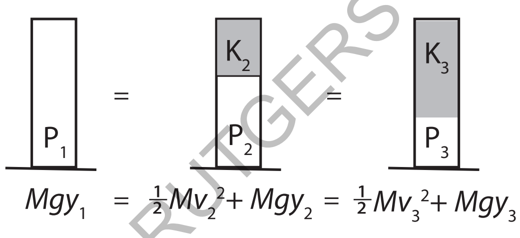
$$Mgy_1 + \tfrac{1}{2}Mv_1^2 = Mgy_2 + \tfrac{1}{2}Mv_2^2$$

There are several interesting aspects of this relation. First, the mass, M, appears in each term, and can be cancelled. This shows that the relation between the heights and the speeds is the same, regardless of the value of the mass, just as was true for the pendulum.

Second, the vertical distance appears only as $y_1 - y_2$, the difference in the height of the two points. As before, changing the reference level changes y_1 and y_2, P_1 and P_2, as well as the mechanical energy at both points, but the differences, $y_2 - y_1$ and $P_2 - P_1$, remain the same. The mechanical energy is different, but the kinetic energies and the speeds at any point remain the same. That's why we say that it is only differences in the potential energies that matter, and not their values.

Finally, the path of the car as it moves between the two points does not affect the result. As long as we stay within our self-imposed limitation that the only force that does work is the gravitational force, we don't have to know anything about what happens between points 1 and 2. Whatever the path, the sum of the kinetic and potential energies is the same at the two points. Whenever the cart gets back to the same height (y_1, y_2, or any other), its potential energy and therefore also its kinetic energy will be the same as when it was at that height earlier.

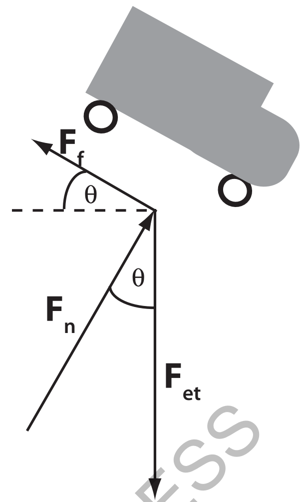




The total force on the cart includes the force that the road exerts on the car. Its component parallel to the road is the force of friction, which we are neglecting. Its component perpendicular to the road does no work and therefore we don't have to know it. All we need to know is that the mechanical energy at point 1 is the same as the mechanical energy at point 2.

A graph of P against x again looks like the graph of y against x. The mechanical energy, E_M, is constant, and $K = E_M - P$.

The figure shows the force diagram of the cart, with the friction that we have been neglecting, but will include in Example 21.



EXAMPLE 20

The cart, on the same hill as in the previous example, starts from rest at a height of 20 m above the ground. (Take the ground as the reference level.) What will be its speed and kinetic energy when it is at a height of 5 m? What is the total mechanical energy? For the kinetic energy we need to know its mass: it is 50 kg.

Ans.:
We have $y_1 = 20\,\text{m}, y_2 = 5\,\text{m}$, and $v_1 = 0$. We are trying to find v_2. We start with $P_1 + K_1 = P_2 + K_2$, where $P_1 = Mgy_1$ and $K_1 = \frac{1}{2}mv_1^2$ at point 1, with similar relations at point 2. M is in each term and can be cancelled. The rest can be rearranged to give $v_2^2 = v_1^2 + 2g(y_1 - y_2)$, or, since $v_1 = 0$, $v_2^2 = 2g(y_1 - y_2)$. All the quantities are already in SI units, and we know that if we put all of them in these units the result will also be in the same system of units, so that $v_2^2 = (2)(9.8)(15) = 294$ and $v_2 = 17.15$ m/s. If we assume that we know all quantities to three significant figures, the result should also have three significant figure, and we round it to 17.2 m/s.

The corresponding kinetic energy is $(\frac{1}{2})(50)(17.2)^2 = (\frac{1}{2})(50)(296) = 7400$ J.

[There is a small problem. We saw earlier that $v_2^2 = 294$ SI units. By rounding 17.15 to 17.2 we have changed its square from 294 to 296! As you can see, it is best to keep an extra digit, and decide only at the end how many figures are significant and should be kept. Here the better value for the kinetic energy is $(\frac{1}{2})(50)(294)$, or 7350 J.]

What is the total mechanical energy? We don't have to specify when, since it is constant. At point 1 there is only potential energy. As long as we continue to assume the reference level to be at the bottom of the track, it is Mgy_1, or (50)(9.8)(20), or 9800 J. At point 2 the potential energy is 2450 J, and the kinetic energy is 7350 J, adding up to the same 9800 J.

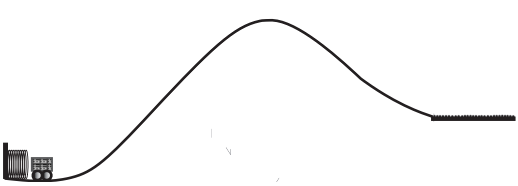
EXAMPLE 21

Now let's change the problem so that it is more realistic. Let's say that the speed at point 2, at a height of 5 m, is only 10 m/s, because some mechanical energy has been lost as a result of dissipative forces, like friction and air resistance. How much mechanical energy has been lost, and what happened to it?

Ans.:
There is no doubt about what happened to it. It has been transferred as heat, and changed to internal energy. This internal energy is partly in the road, partly in the car, and partly in the air. This time we include all of these as part of the system, and also the earth. Let's see how much the internal energy has increased in these various places. K_2 is now $(\frac{1}{2})(50)(100) = 2500$ J. P_2 is, as before, Mgy_2 or 2450 J for a mechanical energy of 4950 J. Since the mechanical energy started out at point 1 ($h = 20$ m) as 9800 J, 4850 J of mechanical energy has been dissipated and changed to internal energy.

EXAMPLE 22

You are an engineer investigating the rollercoaster shown on the figure, and wish to determine the spring constant of the launching mechanism shown there. You may assume that the track is frictionless except for the rough patch, where the force of friction on a cart is 0.2 times the weight of the cart. (The coefficient of sliding friction is 0.2.)



You have a scale and a tape measure.

Describe a procedure to determine the spring constant, k. List the quantities that you need to measure and describe how you will measure them.

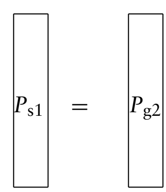
Ans.:
Choose the system that includes the track, cart, spring, air, and the earth.

Energy is conserved: $K + P_g + P_s + U$ is constant, where P_g is the gravitational potential energy, P_s is the elastic potential energy of the spring, and U is the internal energy. Let $P_g = 0$ for the initial position of the cart, let $P_s = 0$ when the spring is not compressed, and let the height of the cart on the rough patch be h_1 above the beginning position.

Measure the mass, M, of the cart with the scale. Push the cart against the spring and measure the distance, x, by which the spring is compressed with the tape measure. Let go. The cart moves either (a) up the track to a maximum height, h, and returns, or (b) it moves up the track, over the hump, and comes to rest after moving a distance L along the rough patch. Measure h for case (a) or L for case (b).

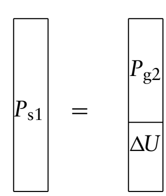
(a) At the initial position $P_{g1} = 0, P_{s1} = \frac{1}{2}kx^2, K_1 = 0$; At the final position $P_{g2} = Mgh, P_{s2} = 0, K_2 = 0, U_1 = U_2$.

Hence $P_{s1} = P_{g2}$, so that $\frac{1}{2}kx^2 = Mgh$. Do this for several values of x and h, and solve for k in terms of the measured quantities.



(b) The initial position is same as in part (a). At the final position is $P_{g2} = Mgh, P_{s2} = 0$. The energy transformed to internal energy is $2MgL$.

Hence $P_{s1} = P_{g2} + 0.2MgL$, or $\frac{1}{2}kx^2 = Mgh_1 + 0.2MgL$. Solve for k in terms of the measured quantities.



6.7 Power: not what the power company sells

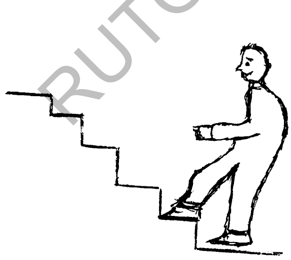
Power is another word that is loosely used in everyday language but has a precise meaning in science, and not necessarily what you might have expected. We talk about the electric company as the *power company*, and many of them have the word *power* in their name. But what they sell is energy.

We use the word *power* to refer to something different, namely the rate at which energy is transferred, i.e., transformed from one kind to another. It is the quantity that tells us how much energy is transformed per second from electric potential energy in a lightbulb, by a household per month, or by the whole country per year. It is the energy divided by the time during which the transfer takes place.

The SI unit for energy is the joule. The unit for power is the *joule/second*. It is given its own name, the watt (W): 1 J/s = 1 W.

While it is turned on, a 40-W bulb uses energy at the rate of approximately 40 W, or 40 J in each second. In an hour it uses (40 J/s)(3600 s) or 144,000 J. A device that uses energy at the rate of 1000 W, or one kilowatt (kW), will consume (1 kW)(1 h) or one kilowatt-hour of energy in one hour. That's equal to (1000 W)(3600 s) or 3.6×10^6 J, and that's what you pay the electric company for.

EXAMPLE 23



As an example, look at a person whose mass is 75 kg, running up a flight of stairs, through a vertical height of 15 m in 12 s. What is the change in the potential energy, and what is the corresponding power?

Ans.:
The change in P (ΔP), is $Mg\Delta y$, or (75)(9.8)(15) J, or 11,025 J. The power is $\Delta P/t$, or $(\frac{11{,}025}{12})$ W, or 919 W.

6.8 Summary

The law of conservation of energy is very simple: *energy is conserved*, i.e., the energy of a closed system remains constant. If the system is not closed, i.e., if energy enters the system or leaves it, you have to count the amount of the change of energy. The energy of the system increases by the net amount that is transferred to the system. You have to know what the boundaries of your system are. You have to make sure that you count all the energy that is transferred in or out. Apart from obvious points that's it for one of the most important principles of science.

There are various kinds of energy:

Kinetic energy or energy of motion: $\frac{1}{2}Mv^2$; this is the difference in energy that an object or system has compared to what it has when it is at rest.

Potential energy is energy of position. There are several kinds. The *gravitational potential energy* changes by the amount of work that is done against the gravitational force.

The *electric potential energy* changes by the amount of work that is done against the electric force. The *elastic potential energy* changes by the amount of work that is done against the elastic force (like the force of a spring). A spring that follows Hooke's law ($F_s = -kx$) has an elastic potential energy $\frac{1}{2}kx^2$ when it is stretched by a distance x from its equilibrium position.

The gravitational potential energy increases by Mgy when a mass M is moved in the direction opposite to the gravitational force through a distance y. The electric potential energy increases when a charge Q is moved in the direction opposite to the electric force.

An object has various kinds of internal energy. It is the kinetic and potential energy of the object's constituents. The atoms and molecules are in motion and have electric potential energies with respect to each other. The sum of these microscopic energies is called *thermal energy*. In addition, the electrons and nuclei of each atom have kinetic and mutual potential energies. So do the nucleons in the nucleus.

Energy can be transferred by doing work and by heating.

Power is the rate at which work is done or energy changes. The SI unit of energy is the joule (J); the SI unit of power is the watt (W): $1\,\text{W} = 1\,\text{J/s}$.

6.9 Review activities and problems

Guided review

1. A box is pushed horizontally by a force of 4 N. A frictional force of 1 N acts in the opposite direction. The box moves forward through 2 m.

(a) What is the work done by the 4 N force?

(b) What is the total work done on the box?

(c) What is the work done by the frictional force?

2. A box whose mass is 2 kg starts from rest, is pushed to the right by a force of 4 N, and moves through 3 m. There are no other horizontal forces.

(a) Set up a mathematical description that relates work and energy.

(b) What is the speed of the box after it has moved through the 3 m? (Use work and energy relations.)

(c) Repeat parts (a) and (b), but with the box initially moving with a speed of 3 m/s in the direction opposite to that of the force.

3. You do 60 J of work lifting a box 2 m. The box is at rest initially and again after it has been lifted.

(a) Set up a mathematical description that relates work and energy.

(b) How much energy does the box gain? What kind of energy does the box gain?

(c) Determine the mass of the box.

4. A stone has a mass of 0.5 kg and is lifted through 1.5 m without gaining any kinetic energy. Answer the following questions:

(a) How much potential energy does it gain?

(b) It is then dropped from rest. How much kinetic energy does it gain on the way down?

(c) What is its speed when it gets back to the starting point?

5. Go to the website phet.colorado.edu and open the simulation *Energy Skate Park*. Put the reference level at the lowest point on the path of the skater. Slow down the motion as much as possible. Open the bar graph.

(a) Click on "energy vs. position." Check "kinetic" and uncheck the others. Use the results of the example to calculate the maximum speed of the skater.

(b) At what height, as a fraction of the maximum, is K equal to half the total energy? At what height, as a fraction of the maximum, is the speed half of the maximum speed?

(c) Pause. Change the track by dragging on the blue circles. Experiment with the following:

(i) Make the track much steeper on the left than on the right. What remains constant about the points where K is zero?

(ii) Lower the right end of the track below the height of the starting point. What happens? Why?

(iii) Make a hump in the middle of the track. What does it take for the skater to overcome this obstacle?

6. A single proton ($M_p = 1.66 \times 10^{-27}$ kg) is released from rest near a large positively charged surface. It loses 4×10^{-18} J of electric potential energy after moving 0.1 m.

(a) Set up the mathematical description that relates the energies.

(b) Determine the proton's velocity after it has moved the 0.1 m.

7. A rock whose mass is 0.5 kg is shot horizontally from a spring gun. The spring of the spring gun is initially compressed with an elastic energy of 11 J.

Use energy bar charts to set up the problem, and determine how fast the rock is moving just after it leaves the spring gun.

8. A ball whose mass is 0.2 kg is dropped from rest at a height of 1.5 m above the floor. It bounces and comes back to a height of 1.2 m. How much mechanical energy is lost? What happened to it?

9. A force of 10 N acts on a box, which moves horizontally with constant velocity through 2 m. What is the total work done on the box? Make a force diagram, showing all forces on the box, an energy bar chart description, and a mathematical description that relates work and energy.

10. For two moles of oxygen at room temperature the internal kinetic energy is about 7500 J. Each mole consists of 6×10^{23} molecules, each of which has a mass of 2.7×10^{-26} kg.

(a) What is the typical value of the random speed of an oxygen molecule?

(b) How long would it take the molecules to move 20 m across a room with this speed?

(c) Why is this not likely to be the actual time that it takes a molecule to get from one side of the room to the other?

11. Go to the website phet.colorado.edu and open the simulation *Energy Skate Park*. Check "potential energy reference" and drag the reference level to the lowest point on the path of the skater.

(a) Click on "track friction." Put the slider marked "coefficient of friction" at the point two spaces from the left end. "Pause," and put the skater back up on the track. "Resume." How many complete cycles does it take for the skater to stop?

Increase the friction by a factor of two and repeat this.

(b) Click on "Bar graph." Observe the changes in K, P, and U. What are the beginning and end values of K, P, U, and of the mechanical energy and the total energy (E) as a fraction of the initial potential energy?

Lower the reference level. What are the beginning and end values of K, P, U, E, and the mechanical energy now in terms of the initial potential energy?

12. A person whose mass is 70 kg runs up a flight of stairs through a vertical height of 6 m. If she uses her food with an efficiency of 10%, how many food calories does she have to consume to do this work?

13. A force of 12 N acts at an angle of 28° to the horizontal. What are its horizontal and vertical components? If it moves an object vertically through 2 m, how much work does it do?

14. A proton moves at constant speed in a circular path in a cyclotron. Explain how you know what the total work done on the proton must be during this motion.

15–16. A pendulum bob whose mass is 0.25 kg is attached to a string whose mass is so small that it may be neglected. As the pendulum swings, its bob moves from a starting height of 18 cm to a height of 12 cm, measured from the lowest pont of the motion.

(a) What is the work done by the tension in the string?

(b) What is the speed at the height of 12 cm?

(c) Use energy bar charts to show the changes in the energy of the pendulum during this segment of its motion.

17–18. A ball whose mass is 0.15 kg has an initial velocity whose horizontal component is 3 m/s and whose vertical component is 2.5 m/s. Neglect dissipative forces.

(a) What is its kinetic energy at the starting point and at the ending point when it returns to the same height?

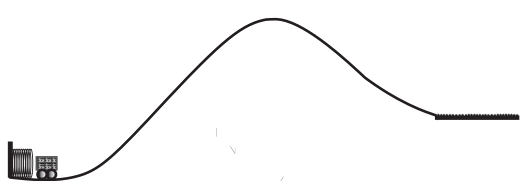
(b) What are its kinetic and potential energies at the highest point?

(c) At a point $\frac{1}{3}$ of the way up, what are the potential and kinetic energies, the height, the speed, and the components of the velocity?

19–21. A car whose mass is 200 kg moves on a rollercoaster, starting from rest at a height of 20 m above ground. If we neglect dissipative forces, to what maximum height can it return? At what height would it be moving with a speed of 5 m/s?

However, when we measure its actual speed we find that it moves with a speed of only 4 m/s at that height. Describe all energy changes, and calculate their amount, from the starting point.

22. You are part of a team of amusement park engineers. You want to know precisely how much the spring in the figure should be compressed so that the cart moves up the ramp, down on the other side, and comes to rest at the exit, which is to be placed half-way along the rough patch. You may assume that the track is frictionless except for the rough patch.



(a) Describe the energy relationships. Bar charts may be helpful to set up the mathematical relationship.

(b) What is the minimum distance through which the spring must be compressed to reach the rough patch?

(c) Calculate how much the spring needs to be compressed so that the cart comes to rest halfway along the rough patch.

List the quantities that need to be known, the quantities that need to be measured, and how they are to be measured.

23. An 80 kg person runs up a flight of stairs whose vertical height is 12 m. In how much time does she have to do that to use energy at the rate of one horsepower? Is this a realistic possibility?

Problems and reasoning skill building

1. Two forces with equal magnitude pull on a box: F_1 to the right and F_2 to the left. The box moves to the right with constant velocity, through a distance x. There are no other horizontal forces or forces with horizontal components.

(a) How much work is done by F_1?

(b) How much work is done by F_2?

(c) What is the total (or net) work, W_T, done on the box?

2. Two forces pull on a box, F_1 to the right and F_2 to the left. The magnitude of F_1 is twice the magnitude of F_2. There are no other horizontal forces or forces with horizontal components. The box moves to the right through a distance x.

In terms of the magnitude F_2:

(a) How much work is done by F_2?

(b) How much work is done by F_1?

(c) What is the total work, W_T, done on the box?

(d) Does the kinetic energy change as the box moves through the distance x, and if so, by how much?

3. An alpha particle (a positively charged helium nucleus) is emitted from a radium nucleus with a kinetic energy of 3.6 MeV. It moves straight toward a gold nucleus (also positively charged). Assume that the gold nucleus remains at rest.

Describe the motion and energy changes of the alpha particle with the help of energy bar charts. (This is the experiment in Rutherford's laboratory that established the existence of the atomic nucleus.)

(a) Sketch a graph of the speed of the alpha particle as a function of its distance, r, from the center of the gold nucleus.

(b) Sketch a graph of the velocity of the alpha particle as a function of r.

4. A ball is released from rest, bounces four times, and then stops.

Sketch graphs of the following quantities as a function of the height of the ball from the ground:

(a) potential energy,

(b) kinetic energy,

(c) mechanical energy,

(d) internal energy.

(e) What happens to the mechanical energy?

5. A vertical spring is held fixed at one end, and a weight is attached at the other end. The weight is pulled down, then released, and the spring oscillates up and down as the weight moves from its minimum height to the equilibrium position and then to its maximum height and back.

At what positions of the weight are the following quantities at their maximum, at zero, and at their minimum:

(a) gravitational potential energy,

(b) elastic potential energy,

(c) kinetic energy.

6. A baseball is batted, hits the ground, and is then caught.

(a) Describe the sequence of energy transformations of the ball that takes place from the time it leaves the bat.

(b) Start at some point in the past (before the player's breakfast) and describe the energy transformations leading up to the moment when the ball leaves the bat.

7. Describe the energy transformations of a pendulum from the time when it is released, neglecting dissipative forces.

8. Describe the energy transformations of a pendulum from the time when it is released (not neglecting dissipative forces) until it stops and remains at rest.

9. Describe each of the lines on the energy diagram (P, E_M, and K) of Example 17.

10. A ton of coal is hoisted up at constant speed through a height of 15 m by a crane.

(a) What are the forces on the load of coal?

(b) What is the total (net) force on the load?

(c) What is the work done by each of the forces on the load?

(d) What is the total work done on the load?

11. An elevator whose mass is 200 kg moves up at a constant speed through a distance of 20 m.

(a) What is the gravitational force on it?

(b) What other forces act on it?

(c) What is the work done on the elevator against the gravitational force?

(d) What is the work done on the elevator by the gravitational force?

(e) What is the total work done on the elevator?

(f) What are the energy changes of the elevator?

12. A stone falls from rest and hits the ground with speed v. Neglect air resistance.

(a) What is its speed as it passes a point halfway to the ground?

(b) What is its speed as it passes a point three quarters of the way to the ground?

13. A ball whose mass is 0.25 kg is thrown straight up. It reaches a height of 16 m before falling back to where it started. Neglect air resistance, and use the starting point as the reference level where $y = 0$ and the potential energy, P, is zero.

(a) Draw a graph of P against y.

(b) On the same coordinate system, draw graphs of the mechanical energy, E_M, and the kinetic energy, K.

(c) Use the graphs to set up mathematical expressions for P, E_M, and K in terms of y.

14. A block whose mass is 2.4 kg slides down an inclined plane, starting from rest. It slides to the bottom through a height, y, of 1.2 m, while the distance along the plane, s, is 2 m.

(a) For this part neglect friction and air resistance, and take the reference level to be at the bottom. Draw graphs of the potential energy (P), the kinetic energy (K), and the mechanical energy (E_M) in terms of the distance (s). (Take $s = 0$ at the starting point.)

(b) Now assume that there is a constant frictional force of 3 N and no other dissipative forces. What is the mechanical energy when the block reaches the bottom?

(c) Again draw graphs of P, E_M, and K, and also the dissipated energy, Q, in terms of s.

15. A rock whose mass is 0.2 kg is dropped from a height of 10 m.

(a) First assume that there is no air resistance, and take the reference level to be at the bottom. Draw graphs of P, K, and E_M as a function of the height until the rock hits the bottom.

(b) Use the graphs to set up mathematical expressions for the three energies.

(c) Now assume that there is some air resistance. Again draw graphs of P, K, E_M, and the dissipated energy, Q.

Use the graphs to write the mathematical expressions for each of the energies.

(d) Repeat parts (a) and (b), but this time use the starting point as the reference level.

(e) Still using the starting point as the reference level, write down the relation between y and the time, t. Use it to get an expression for K in terms of t, and draw a graph of it.

16. A spring hangs vertically, with one end fixed. At the other end a weight whose mass is 0.15 kg hangs from it. The spring constant is 12 N/m. Neglect all dissipative forces, and also the mass of the spring. Assume that the spring obeys Hooke's law.

(a) The system is at rest. What is the elastic potential energy?

(b) You pull the weight down with a force of 1.2 N until it is again at rest. Which energies change? By how much?

(Sketch a figure with the spring and weight. Where have you chosen the reference level? Mark it on your figure. You can only choose one! In this problem there is really only one good choice, namely, where the elastic energy is zero.)

What is the mechanical energy now?

(c) You then release the 0.15 kg weight. The spring returns to the equilibrium point of part (a). What are the various energies now?

(d) The weight continues upward. What is the criterion for how far it goes? Find the value of y at the highest point, measured from the reference level that you chose in part (b).

17. A ball is thrown straight up from ground level, and eventually returns to the same height N. It has an initial velocity v_{0y}. Neglect all dissipative forces.

Write expressions in terms of these two quantities for the following:

(a) The largest kinetic energy. Where does it occur?

(b) The smallest kinetic energy. Where does it occur?

(c) The maximum potential energy. (Be sure to state what reference level you have chosen.)

(d) The maximum height.

(e) The mechanical energy.

(f) What is the amount of work that had to be done on the ball as it was thrown?

18. Repeat the previous problem, but with an additional component of the initial velocity in the horizontal direction, v_{0x}.

19. For each of the listed situations, neglecting all dissipative forces, do the following:

(i) Identify the system that you wish to consider; describe the initial and final states.

(ii) Describe the energy transformation processes in words and with bar charts.

(iv) Draw a force diagram showing all the forces on the system.

(a) A pendulum bob swings down, starting from rest.

(b) An elevator, moving up, slows down until it stops.

(c) A block is launched up an inclined plane by a compressed spring.

(d) Another system of your choice.

20. A bucket hangs from a rope. The other end of the rope is held by a woman. The tension in the rope is 50 N. All movements take place with constant velocity. What is the work done by the rope in each of the following cases?

(a) She moves 5 m horizontally without changing the height of the bucket.

(b) She lifts the bucket 0.5 m.

(c) She lowers the bucket 0.5 m.

(d) She pulls the bucket up an incline, moving it 1 m vertically and 0.5 m horizontally.

(e) She lowers the bucket down the same incline by the same distance as in part (d).

21. (a) Corresponding to each of the following mathematical relations, describe a possible process, in words, with a sketch, and with bar charts:

$K_1 + W = P_2$ and

$P_1 + W = K_2$.

(b) Write a problem with numerical values for each of these situations.

22. A tennis ball has a mass of 0.057 kg. It falls 18 m vertically, from rest, and then has a speed of 12 m/s.

(a) Identify the system that you wish to consider.

(b) What are the initial and final states?

(c) What are the energy transformations?

(d) Draw the corresponding energy bar chart and draw a force diagram.

(e) Write a mathematical statement that describes the energy transformations.

(f) What is the magnitude of the average resistive (dissipative) force?

23. A spring has a spring constant of 1.2×10^4 N/m. A 10 kg block is placed against it, and the spring is compressed 6 cm. The spring is then released, shooting the block forward along a horizontal surface against a force of friction of 15 N.

Follow steps (a) to (e) of the previous problem.

(f) How far does the block travel before coming to rest?

24. A block is pulled up an inclined plane with constant speed against a constant frictional force.

(a) Identify the system that you wish to consider.

(b) Draw a force diagram, showing all forces on the system.

(c) Describe all energy transformations.

(d) What quantities do you need to know to calculate the dissipated energy.

25. A 900 kg car starts from rest, with its engine off, and rolls a distance of 50 m down a hill. A 400 N frictional force opposes the motion. When the car reaches the bottom it is moving with a speed of 8 m/s.

(a) Describe the energy transformations using energy bar charts.

(b) Use your charts to write a mathematical statement that describes the energy transformation processes.

(c) What is the vertical height through which the car moves?

26. A slide is 42 m long and has a vertical drop of 12 m. A 60 kg man starts down the slide with a speed of 3 m/s. A frictional force opposes the motion, causing 20% of the kinetic energy to be dissipated.

(a) Identify the system that you wish to consider and its initial and final states.

(b) Describe the energy transformation processes using energy bar charts.

(c) Use your charts to write a mathematical statement that describes the energy transformations.

(d) What is his speed at the bottom?

(e) What is the magnitude of the average frictional force?

27. Two children ride a sled down a hill, starting from rest. Neglect all dissipative forces.

(a) Identify the system that you wish to consider, and its initial and final states.

(b) Use the following representations to describe what happens: a sketch of the hill, sled, and children; a force diagram; energy bar charts; a mathematical statement of the relation between the various initial and final energies.

(c) The mass of the sled with the children in it is 150 kg. Their speed at the bottom is 13 m/s. What is the vertical height through which they move?

28. Repeat parts (a) and (b) of the previous problem, but with an initial velocity at the top of the hill, and some dissipated energy.

(c) Make up a problem for this situation by giving some numerical quantities and asking for two others. Show the solution of your problem.

29. A ball ($M = 0.60$ kg) is dropped off a cliff, starting from rest. What is its kinetic energy just before it hits the ground 30 m below? (Neglect air resistance.) What is its speed?

30. A ball ($M = 0.50$ kg) is thrown vertically, with an initial kinetic energy of 50 J. (Neglect air resistance.)

(a) What is its kinetic energy when it comes back to the starting height?

31. (a) When a ball ($M = 0.50$ kg) is thrown at an angle of 24.6° with an initial energy of 50 J it is observed that its kinetic energy as it hits the ground is only 90% as high as when air resistance is neglected. How much mechanical energy has been lost during the flight?

(b) When the ball is thrown at a different time, it hits the ground with a speed that is 90% of that calculated when air resistance is neglected. How much mechanical energy is lost during the flight this time?

32. A ball ($M = 0.40$ kg) drops from a height of 5 m. After it bounces it rises only to a height of 4.0 m. How much mechanical energy has been lost?

33. A block ($M = 1.5$ kg) slides down an incline at a constant speed. Its vertical height decreases at a rate of 0.50 m/s. How much internal energy is produced in eight seconds? What is the rate of energy dissipation in watts?

34. A toaster is marked 500 W, indicating that this is its power consumption when it is turned on. It takes three minutes to toast two pieces of bread. How many joules are used during that time? What is the cost if the electric energy costs 20 cents per kwh?

35. A ball ($M = 0.50$ kg) is thrown straight upward with an initial kinetic energy of 75 J. Neglect dissipative forces, assume that the potential energy, P, is zero at the starting point, and find the following:

(a) K at the highest point,

(b) P at the highest point,

(c) the distance between the lowest and the highest point,

(d) K when the ball falls back to the starting point.

36. A block whose mass is 2 kg slides along a table, starting with a speed of 4 m/s. It gradually comes to a halt as a result of friction. How much internal energy (in J and in cal) is produced during this process?

37. A child is on a swing that starts from rest at a point 1.8 m above the lowest point. The mass of the child together with the swing is 30 kg. If you neglect all dissipative forces, what are the maximum kinetic energy and speed of the swing with the child?

38. Consider the child and swing of the previous problem, but this time do not neglect dissipative forces. Without a push the swing does not

come back to the same height, but stops at a point 60 cm lower. How much energy does the father have to supply with each push if the swing is to come back to its original height of 1.8 m above the lowest (central) position?

39. The unit called the horsepower is equal to 550 $\frac{\text{ft-lb}}{\text{s}}$. Calculate how many W this corresponds to.

40. A proton ($M_p = 1.66 \times 10^{-27}$ kg) is 10 cm from a large, positively charged plane, as in the figure accompanying Example 6. The charges on the plane give rise to a constant force of 3×10^{-10} N on the proton, away from the plane. Use energy bar charts to set up the problems and answer the following questions.

(a) The proton is initially at rest, and then moves through a distance of 1 cm. What is its kinetic energy after it has moved through the 1 cm? What is its speed?

(b) The proton is initially moving toward the plane with a kinetic energy of 5×10^{-12} J. Describe its subsequent motion. What is the distance of its closest approach to the plane? How fast is it moving when it is 11 cm from the plane?

41. In the ground state of the hydrogen atom the electron has a kinetic energy of 13.6 eV. (1 eV = 1 electron volt = 1.6×10^{-19} J.) The electric potential energy is less by 27.2 eV than when the electron is far removed from the proton.

Use as reference level for the potential energy the state where the electron and the proton are infinitely far from each other. (This is then the state where the potential energy is zero.) With this reference level, what are the potential energy and the total energy of the atom in its ground state?

How is this amount of energy related to the amount that has to be supplied to the atom to separate the proton and the electron to a point where each is at rest far from the other? (This amount is called the ionization energy of the electron, or its binding energy.)

42. A satellite moves around the earth at a height that is small compared to the radius of the earth. An object or person in the satellite is often described as "weightless."

(a) What has happened to the weight of the object or person, defined, as we do in this book, as the gravitational attraction of the earth?

(b) Which force on the object or person, sometimes defined as the weight, is zero?

43. How much energy does a diet of 2000 Calories per day supply to the human body in one day? (These are food calories, or kilocalories, each equal to 4186 J.) Use the number of seconds in a day to find the rate of energy use in watts corresponding to this diet.

44. You are told that a car has 10^6 J of kinetic energy and 2×10^6 J of potential energy. In what respect is this an incomplete statement?

45. When you go up two floors, approximately how much does your potential energy increase? If you run up in 10 seconds, what is the rate at which you expend energy (in W)?

46. A pendulum consists of a mass (M) and a string of length L. How fast must the mass be moving at the lowest point to be able to just move in a full circle?

Multiple choice questions

1. A rock falls from rest and hits the ground with speed v. (Neglect air resistance.) Halfway along its path to the ground its speed is

(a) $2v$
(b) $\sqrt{2}v$
(c) $\frac{1}{2}v$
(d) $\frac{1}{\sqrt{2}}v$

2. A ball loses a third of its kinetic energy as it bounces off the floor. What is the fraction of its original height, H, to which it rises before coming to rest? (Neglect air resistance.)

(a) $\frac{1}{3}$
(b) $\frac{2}{3}$
(c) $\frac{1}{\sqrt{2}}$
(d) $\sqrt{\frac{2}{3}}$

3. A ball loses a third of its speed as it bounces off the floor. What is the fraction of its original height, H, to which it rises before coming to rest? (Neglect air resistance.)

(a) $\frac{4}{9}$
(b) $\frac{2}{3}$

(c) $\sqrt{\frac{2}{3}}$

(d) $\frac{1}{3}$

4. Two astronauts sit in a spaceship that orbits the earth at a height of 20 km. Which of the following forces is zero?

(a) The gravitational force of one astronaut on the other.

(b) The gravitational force of the earth on one astronaut.

(c) The force of the floor of the spaceship on one astronaut.

(d) The centripetal force on one astronaut.

5. Assume the approximation that the moon (mass m) travels around the earth in a circular orbit of radius R with speed v. The work done on the moon in one complete revolution is

(a) mgR
(b) $\frac{1}{2}mv^2$
(c) $\frac{mv^2}{R}$
(d) zero

Synthesis problems and projects

1. Fill in the missing steps at the end of Section 6.3 leading to $T = \frac{2\pi}{\omega}$ and $T = 2\pi\sqrt{\frac{m}{k}}$.

2. Go to the PhET website and open the simulation Masses and Springs.

(a) Hang one of the masses on one of the springs and set it into oscillation without friction. Measure the period, T. Calculate the spring constant, k from T and m.

(b) Put different known masses on the same spring and measure the equilibrium position for each. (It is easiest to do this after putting on a lot of friction to stop the motion.) Draw a graph of F against x and find the spring constant from your measurements.

(c) Find the masses of the unmarked weights. Calculate T for one of these weights. Check it with a direct measurement of T.

3. You have the following equipment:

(a) a spring whose spring constant you know,

(b) a track that can be inclined and whose friction is negligibly small,

(c) a glider whose mass you know,

(d) a motion detector,

(e) a ruler.

Describe an experiment that you can do with this equipment to test the law of conservation of energy.

4. On a pool table the "cue" ball is given a velocity of magnitude v in the y direction. It hits another equal ball. After the collision the two balls move with velocities whose magnitudes are v_1 and v_2 at an angle of 90° with respect to each other.

(a) Draw a vector diagram of the momenta before and after the collision.

(b) After the collision, what is the relation between the components in the x and y direction of the velocity of each ball?

(c) What is the relation between v, v_1, and v_2?

5. A block with mass M hangs at rest from a string whose upper end is fixed. A pistol shoots a bullet (mass m and velocity v) into the block. The block with the bullet embedded in it then begins to move with a horizontal velocity V.

(a) What law of physics governs the collision of the bullet and the block and determines the velocity V?

(b) What is the relation between v and V?

(c) Following the collision the mass rises. What law of physics determines the height, H, to which the block (with the bullet in it) rises?

(d) Derive the relation that gives v in terms of the quantities V, m, M, and H.

(This arrangement is called a ballistic pendulum. It can be used to measure the speed of a bullet.)

6. The amount of energy used each day by primitive people was roughly what they ate, of the order of 2000 kcal. Look up the total energy used per year in the United States and use this number to calculate the yearly consumption per person, and the consumption per person, per second, in watts. By what factor is this larger than the amount used by primitive people?

7. What, approximately, is the average speed (in m/s) corresponding to the world record of running 100 m? One mile? How much kinetic energy does each of these correspond to? Answer the same questions for your walking fairly fast for one mile.

What, roughly, are the corresponding kinetic energies? Make an assumption of how long it takes the 100 m sprinter to reach a kinetic energy equal to his average kinetic energy, and calculate the required power in watts. How many horsepower is that equivalent to?

8. You are sliding quarters down a ramp. You want to know how large the effect of friction is, and whether it can be ignored.

You have a motion detector and a meterstick. Show how you can use them to answer these questions.

9. A spring, sitting vertically on a table, is compressed. It is released, and jumps up in the air. You want to check whether mechanical energy is conserved as this happens. You have a scale, a ruler, and a set of weights. Describe a possible procedure.

10. Describe an experiment that you can use to convince a classmate that $\cos\theta$ is important in the definition of work.