September, 2005

Theoretical Calculations and Measurements of the DEE Voltage in the Rutgers 12 Inch Cyclotron

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The Rutgers 12 inch cyclotron is a student built and student maintained particle accelerator. Its sole intended purpose is to promote education of an experimental research nature in the relatively obscure field of accelerator physics. Radio Frequency, RF, problems and calculations are relatively obscure within the accelerator field. This document, in the most general of terms, describes some of the RF aspects of the student built cyclotron. It is for reference by future students finding themselves perplexed by the arcs and sparks they may generate in their investigations with the Rutgers 12 inch cyclotron.

I. SIMPLISTIC DEE VOLTAGE THEORY:

The first issue I want to address is the calculation of the expected peak DEE voltage, but before diving right into the details, a review of the cyclotrons RF components should prove useful.

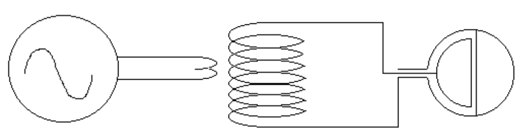


Fig.1 Simple RF system schematic.

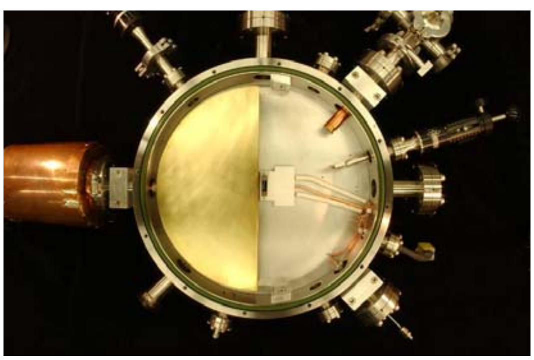


Fig.2 Capacitance of the DEE is about 78.1pF

The Cyclotron's DEE makes up a lumped capacitance value of an L-C tank circuit. Using the parallel plate capacitor approximation a very close estimate of the cyclotron's capacitance can reached, of course simple verification can be made with the use of an L-C meter. By breaking the DEE-chamber geometry into 3 separate sections: top, bottom, and side, you can treat the total capacitance as a sum of the three parallel capacitances. The geometry of the top and bottom is identical, which allows multiplication by a factor of two. The area of the Top or Bottom of the DEE is

$$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(0.127m)^2 = \frac{1}{2}0.0506m^2$$

Then using the parallel plate approximation we determine the top+bottom contributions:

$$C_{top+bottom} = 2C = \frac{2A\varepsilon}{d} =$$

$$= \frac{(0.0506m^2)(8.85\times10^{-12}F/m)}{0.00635m} = 70.5\,pF$$

And the contribution from the edge is

$$C_{edge} = \frac{A\varepsilon}{d}$$

$$= \frac{(0.0101m^2)(8.85\times10^{-12}F/m)}{0.0127m} = 7.04\,pF$$

For a total C of:

$$77.5pF$$

Measurement of the capacitance with an L-C meter yields a value of 78.1*pF*. Nice agreement seen!

As we know, the RF must oscillate at the Cyclotron frequency of:

$$f_0 = \frac{qB}{2\pi m} \approx 15MHz$$

In order to bring the tank circuit into resonance at f_0, an external inductance, L, must be placed in parallel with the chambers capacitance. Using the capacitance value of 78.1 pF, and working backwards from:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow$$

$$L = \frac{C}{(2\pi f_0)^2} = \frac{(78.1pF)}{(2\pi\cdot 15MHz)^2} = 1\mu Hy$$

Thus, a coil was wound to achieve this value. Five turns of copper refrigeration tube was wound on a two inch diameter mandrel. Flat copper strap connects the coil to the DEE stem, which in turn makes electrical contact with the DEE. Indeed, using a network analyzer with weakly coupled loop probes, resonance was measured to be near the desired frequency of 15MHz.

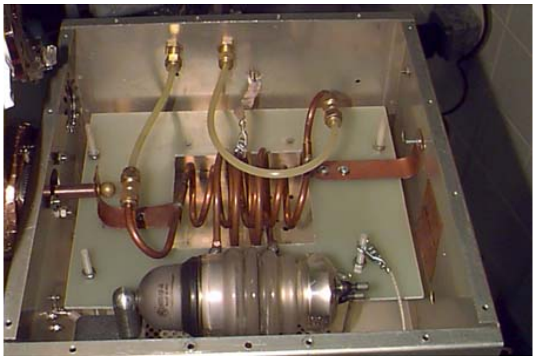


Fig.3 Cyclotron coupling and tank coils.

A simplistic theoretical model of the tank circuit is used to make an estimation of the peak DEE voltage. Again, treat the cyclotron as a tank circuit. As mentioned above, a half turn copper loop feed by a 50Ω coaxial line concentrically positioned about the cyclotrons tank circuit coil inductively couples the power needed to excite the tank. The location of the "tap" points were empirically adjusted to find the (50+*j*0)Ω point at resonance. The coupling is too "weak" to treat the problem as a simple transformer based on primary and secondary turns ratio, and determination of the mutual inductance from a pure geometrical point of view is too difficult to consider. However, following from Terman's section on Inductively Coupled Circuits, the cyclotron RF circuit follows the case of an "untuned" primary driving a "tuned" secondary tank. The secondary's effective impedance coupled into the primary circuit is as if an impedance $Z_{s\text{-coupled}}$ is placed in series with the primary:

$$Z_{s-coupled} = \frac{(\omega M)^2}{R_s + j(\omega L_s - \frac{1}{\omega C_s})}$$

Again, I mention that the mutual inductance M is unknown; however, we do know that the coupled impedance to the coaxial line was empirically set to (50+j0)Ω. We are also only concerned about the voltage developed only at resonance, so the term $(\omega L_s - \frac{1}{\omega C_s})$ is very small compared to R_s and we are left with:

$$Z \approx \frac{(\omega M)^2}{R_s}$$

Where R_s is the "AC" resistance of the tank circuit. Since the current density in the capacitor is small compared to the current that develops in the coil, the AC resistive losses in the DEE can reasonably be neglected. The next estimation is to determine the AC resistance of the copper tubing used to wind the coil. There is approximately 38 inches of refrigeration tubing wrapped up in the coil. Referring to ITT's Reference Data for Radio Engineers, we estimate the AC resistance of 0.25inch refrigeration tubing to be 1.3mΩ/inch, giving us a total of 50mΩ.

Next, using Ohm's power law, $P = I^2 R_s$, and by making several assumptions, we make our attempt at a *simple* model the L-C circuit at resonance. The first assumption is that we apply a forward power P with the taps on the primary coil and it's geometry with respect to the secondary coil is such that the coupled impedance to the coaxial line is a pure resistive 50Ω. Then we can assume that there is zero reflected power to the source. This is an assumption that is achievable in practice. Then, with the assumption that the DEE is an ideal capacitor, we model the inductor as an ideal inductance with a pure resistance, R_s, in series, we can say that the entire forward power is dissipated by R_s. Thus, for a power of P, into impedance R_s, the I_{rms} is: $I_{rms} = \sqrt{\frac{P}{R_s}}$. From conservation of charge, we know that I_{rms} through the resistance must be the same for the I_{rms} through the inductor. Then, using conservation of energy, $\frac{1}{2}LI^2 = \frac{1}{2}CV^2$, re-arrange to solve for $V_{rms} = I_{rms}\sqrt{\frac{L}{C}} = \sqrt{\frac{PL}{R_s C}}$. To get the peak voltage, we multiply V_{rms} by $\sqrt{2}$, or $Vpeak = \sqrt{\frac{2PL}{R_s C}}$, and to get the peak-to-peak voltage we multiply by another 2 or $Vp - p = 2\sqrt{\frac{2PL}{R_s C}}$. Now, plugging in the known inductance of 1.1μHy, capacitance of 78.1pF, and the estimated AC resistance of 50mΩ. We plot $V_{peak}(Power)$ based on these values:

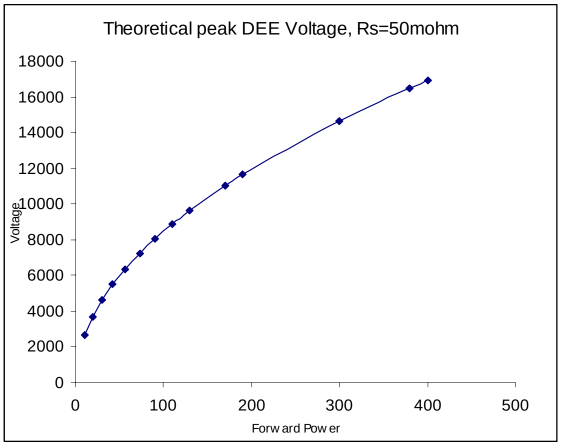


Fig.4 Theoretical DEE Vp-p curve

Next we will compare actual DEE Voltage measurements to give validation to our simple model. It will be seen that the trend of DEE voltage to follow the square root law of the input RF power is accurate over all measured power ranges, but the estimation of the 50mΩ is about an order of magnitude low.

II. VOLTAGE MEASUREMENTS: There have been several attempts to measure the DEE voltage. The first method was to use a GE vacuum tube diode rectifier to charge a HV capacitor to the peak RF voltage, then we would measure this DC high voltage through a high impedance voltage divider circuit.

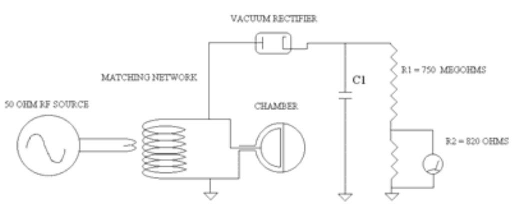


Fig.5 Rectifier measurement schematic.

Initially, the data from the rectifier measurement was not trusted. It did follow the $x^{0.5}$ power trend; however, the actual measured voltage was considerably low compared to the expected theoretical values. This was not totally surprising as there were no available data sheets on the rectifier; we felt that perhaps we were not operating it properly. However, as will be shown later, the rectifier measurement setup was operating properly, and produced accurate DEE voltage measurements.

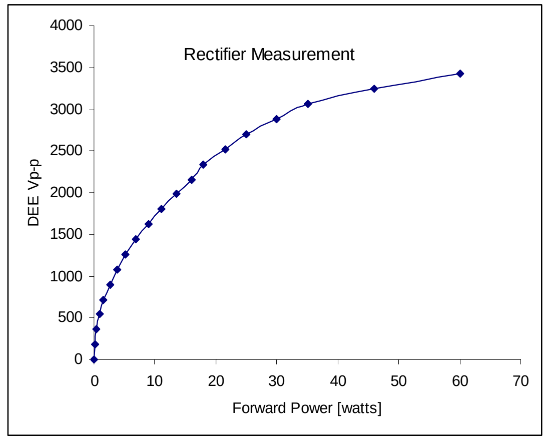


Fig.6 Rectifier measurement data

Another attempt to measure the DEE voltage used a Tektronix Probe model P6015 HV high frequency probe connected directly to the DEE Stem inside the RF matching box. Data from the Tektronix P6015 probe shows that the developed voltage indeed follows the theoretical $x^{0.5}$ power trend, but with two exceptions. The first exception from the theoretical trend was, again, as if R_s had the value of 800mΩ – sixteen times that of the expected coil R_s. This may be understood if you take into account the stainless steel vacuum chamber return, the stainless steel Conflat DEE stem support and RF feed throughs.

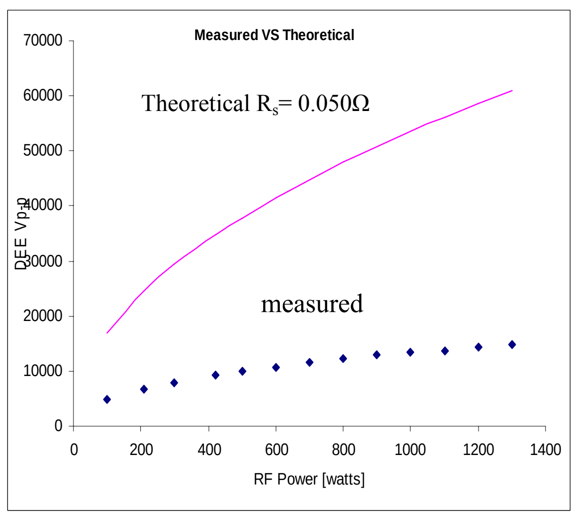


Fig.7 Comparing theory with measured data

The second exception is that after a power level of 200 watts, the measured voltage of the P6015 probe departed from the trend and dropped below the expected value. It is as if an additional resistance is introduced. The behavior was similar to that of a high-resistance break-down, most likely this occurred in the probe. The Tektronix's probe requires a charge of a Freon based insulator which was not available to us. Furthermore, while the P6015 probe's value deviated from the trend, the induced voltage on the chamber's capacitively coupled pickup continued to followed the trend which was consistent with the theoretical model through the higher input power.

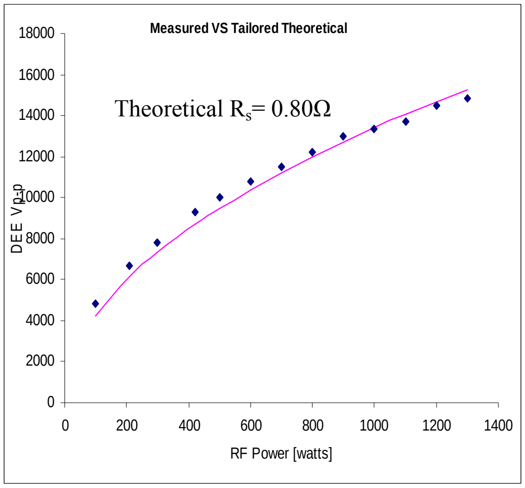


Fig.8 R_s adjusted to match measured

It should be noted that the P6015 probe introduced 3.0pF of capacitance; the tank circuit was indeed reduced in frequency corresponding to 3 pF (details in log book page 142)

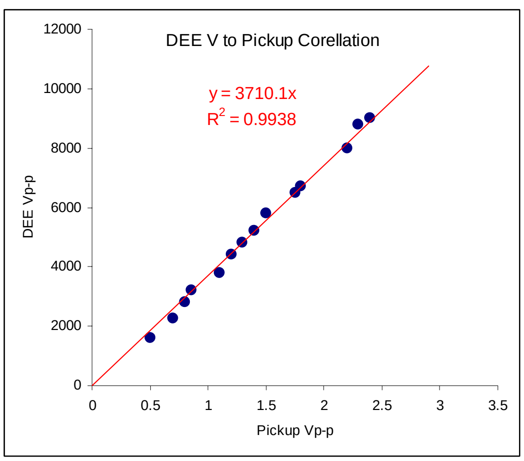


Fig.9 DEE $V_{p\text{-}p}$-Pickup $V_{p\text{-}p}$ calibration

As mentioned above, the Tektronix's P6015 gives unreliable data for forward power in excess of 200 watts. In order to continue measurements of yet higher power, the induced voltage on the capacitive pickup facing the DEE was calibrated against forward power at lower levels. Extrapolation allowed us to measure forward power levels up to 1300 watts, and there should be no reason that we cannot go even higher in power if desired.

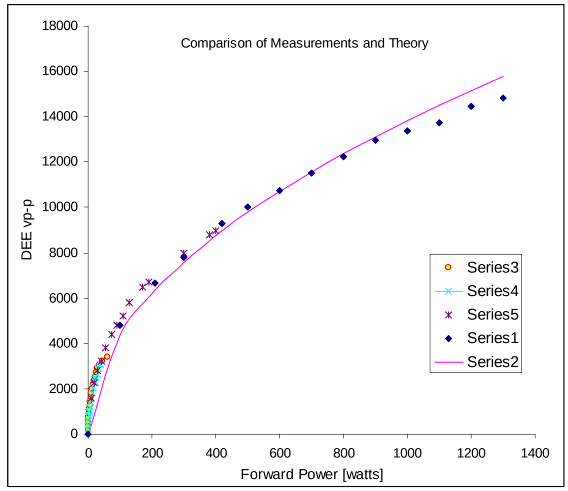


Fig.10 Plot of all measurements and theoretical trend

Finally, we see good agreement when making a comparison of *all* DEE $V_{p\text{-}p}$ measurements taken to date. Additionally, plotting the theoretical curve, assuming an R_s of 0.8Ω, one can feel confident in the RF systems characterization and operation.

We can finally attempt to say something about the mutual inductance constant *M*. Recall, that for a un-tuned primary inductance the effective impedance is:

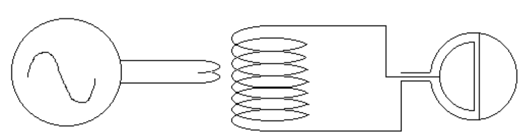
$$Z = \frac{(\omega M)^2}{R_s}$$

We empirically know Z=50Ω and have made a measurement of R_s to be 0.8Ω, rearrangement of the above gives

$$M = \frac{\sqrt{R_s Z}}{2\pi f}$$

for a value of 6.7E-8 μH.

III A RIGOROUS THEORETICAL treatment of the untuned-primary, tuned secondary circuit gives confirmation of our more simple approach in section I. Again consider the circuit:



Let us write Kirchoff's equations for the two circuits, the primary and the secondary are denoted by *1* and *2* subscripts respectively.

$$R_1 i_1 + L_1 \frac{di_1}{dt} - M\frac{di_2}{dt} = Ee^{j\omega t} \quad (1)$$

$$R_2 i_2 + L_2 \frac{di_2}{dt} + \frac{\int i_2 dt}{C_2} - M\frac{di_1}{dt} = 0 \quad (2)$$

Let

$$i_1 = Ae^{j\omega t} \quad (3)$$

$$i_2 = Be^{j\omega t} \quad (4)$$

We'll guess the solutions for (1) and (2) as:

$$e^{j\omega t}[A\{R_1 + j(\omega L_1)\} - jM\omega B] = Ee^{j\omega t} \quad (5)$$

$$e^{j\omega t}[A\{R_2 + j(\omega L_2 - \frac{1}{\omega C_2})\} - jM\omega A] = 0 \quad (6)$$

Next, we'll designate:

$$X_1 = (\omega L_1) \text{ and } X_2 = (\omega L_2 - \frac{1}{\omega C_2})$$

Divide (5) and (6) through by $e^{j\omega t}$ as well as make the above substitutions for the reactance:

$$A\{R_1 + jX_1\} - jM\omega B = E \quad (7)$$

$$B\{R_2 + jX_2\} - jM\omega A = 0 \quad (8)$$

Now, $\{R_1 + jX_1\}$ and $\{R_2 + jX_2\}$ are constants of circuits 1 and 2 respectively, and we define these complex impedances as:

$$z_1 = R_1 + jX_1 \quad (9)$$

$$z_2 = R_2 + jX_2 \quad (10)$$

which have magnitudes:

$$Z_1 = \sqrt{R_1 + jX_1} \quad (11)$$

$$Z_2 = \sqrt{R_2 + jX_2} \quad (12)$$

Re-writing (7) and (8) we have:

$$Az_1 - jM\omega B = E \quad (13)$$

$$Bz_2 - jM\omega A = 0 \quad (14)$$

and with sufficient algebra we find that:

$$B = \frac{jM\omega A}{z_2} \quad (15)$$

$$A = \frac{E}{z_1 + \frac{M^2\omega^2}{z_2}} \quad (16)$$

We arrive at the currents i_1 and i_2:

$$i_1 = \frac{Ee^{j\omega t}}{z_1 + \frac{M^2\omega^2}{z_2}} \quad (17)$$

$$i_2 = \frac{jM\omega Ee^{j\omega t}}{z_1 z_2} \quad (18)$$

From (17) it is seen that coupled impedance of the secondary circuit is indeed as if $\frac{M^2\omega^2}{z_2}$ is placed in series with z_1, validating our simple approach in **I**. The overall effective impedance, denoted as $z_1^{'}$, can be written as:

$$z_1^{'} = z_1 + \frac{M^2\omega^2}{z_2} = R_1 + jX_1 + \frac{M^2\omega^2}{R_2 + jX_2}$$

After rationalizing the last term we write:

$$z_1^{'} = R_1 + \frac{M^2\omega^2}{z_2^2}R_2 + j\{X_1 - \frac{M^2\omega^2}{z_2^2}X_2\}$$ We can separate the effective pure resistive and reactive terms the signal source would see:

$$R_1^{'} = R_1 + \frac{M^2\omega^2}{z_2^2}R_2 \quad (19)$$

$$X_1^{'} = X_1 - \frac{M^2\omega^2}{z_2^2}X_2 \quad (20)$$

In our case, R_1 would be the 50Ω of the RF power source in series with the pure resistance of the primary coupling loop. Using the fore mentioned estimation of R_{ac} of ¼ inch copper we find that the resistance of the primary coupling loop is a negligible 0.010mΩ. We know from power theory that maximum power is transferred when the load resistance equals the source impedance, thus forcing $\frac{M^2\omega^2}{z_2} = 50\Omega$ which sets a value for $M^2\omega^2$ or M:

$$M = \frac{\sqrt{(50)(0.100)}}{9.42x10^7} = 2.4x10^{-8} \ \mu\text{H} \quad (21)$$

Likewise, because of the signal source's 50+j0Ω impedance, the total reactance coupled from into the primary circuit from the secondary must cancel the reactance of the primary, namely ωL_1. We can impose:

$$X_1 = \frac{M^2\omega^2}{z_2^2}X_2 \quad (22)$$

allowing another way to determine M. However, because of the very small X_1 value and almost zero X_2 values near resonance, one must exercise extreme caution in using the reactance terms to infer M.

If one has access to the inductors L_1 and L_2 a measurement M is fairly easy made. From Professor Terman's tome: *when two coils of inductance L_1 and L_2, between which a mutual inductance M exists, are connected in series, the equivalent inductance of the combination is*

$$L_1 + L_2 \pm 2M$$

The term 2M takes into account the flux linkages in each coil due to the current in the other coil. These mutual linkages may add to or subtract from the self-linkages, depending upon the relative direction in which the current passes through the two coils. When all linkages are in the same direction, the total inductance of the series combination exceeds by 2M the sum of the individual inductances of the two coils; while if the mutual linkages are in opposition to the other linkages, the inductance of the combination is less than the sum of the two coil inductances by 2M. This property can be taken advantage of to measure the mutual inductance between the two coils. The procedure is to connect the two coils in series and measure the equivalent inductance of the combination. The connections to one of the coils are then interchanged and the equivalent inductance is measured again. The difference between the two measured inductances is then 4M.

IV CYCLOTRON RF SIMULATOR It was for this measurement of M that a "cyclotron RF simulator" was initially built. The simulator consists of an adjustable vacuum capacitor in parallel with a 4 turn coil wound of ¼ inch copper tubing. The assembly is mounted on a copper ground plane. To the side of the coil is mounted a ½ turn loop of ¼ inch copper tubing. The distance between the coupling loop and the tank circuit coil is made adjustable by being mounted onto a linear motion stage.

Fig.11 Cyclotron RF simulator

The extremely small inductance of the primary, or coupling loop, made the measurement of M difficult. Positioning of the test leads had to be routed in such a manner as to minimize their effect between measurements. The best measurement value for M of the test setup was 0.02μH at a coupling loop-tank coil distance that had a 50+j0Ω impedance – i.e. perfectly matched to the 50Ω RF source. This compared reasonably well to the estimated value of 0.024μH from (21) and is in the same order of magnitude of as the cyclotron's M of 0.068 μH estimated in section **II**. Intuitively the higher value of the cyclotron's M makes sense for as the load increases, namely from 0.100Ω in the "cyclotron RF simulator" to 0.800Ω in the actual cyclotron, the need for power coupling should increase.

V CRITICAL COUPLING using Q measurements. The relationship between $Q_{measured}$ and Q_o of the tank circuit, assuming $R_1 >> \omega L_1$, is:

$$\frac{Q_{measured}}{Q_o} = \frac{1}{1 + \frac{M^2\omega^2 / R_2}{R_1}}$$

The $Q_{measured}$ of the response curve approaches Q_o of the tank when the coupling is very small, and is exactly ½ of Q_o when the primary is critically coupled corresponding to the value giving maximum response, denoted as $Q_{critical}$.

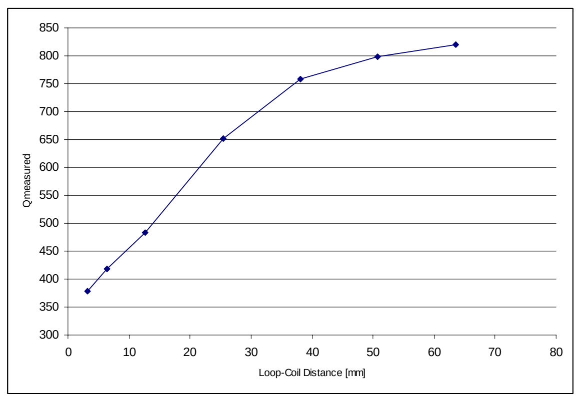


Fig.12 $Q_{measured}$ vs. loop-coil distance of simulator

Again, the "cyclotron RF simulator" was put into service to verify this claim. Note that R_1=50Ω while ωL_1=14.3Ω does not exactly satisfy $R_1 >> \omega L_1$. The coupling loop was initially adjusted to ensure that the loop could be "tuned" to 50Ω at resonance into the network analyzer. Then the coupling loop was positioned close to the tank coil. A small capacitive pickup placed near the vacuum capacitor served as the very weakly coupled pickup probe, akin to the probe mounted on the cyclotron chamber. Making S_{21} type measurements with the network analyzer, FWHM values of the response curves were measured as the coupling loop's position was varied. Using the definition of Q as $\frac{f_o}{\Delta f_{FWHM}}$, the FWHM measured Q values were plotted in Fig.12. As can be seen, $Q_{measured}$ does approach Q_o as the loop is moved further away from the tank coil, i.e. weaker coupling. Additionally, the transfer function, or S_{21} scattering value through the "cyclotron RF simulator" setup was measured and plotted – see Fig.13. A peak in the tank circuit's response was noted at a loop-coil distance of 11mm - this location was verified to be the matched (50+*j*0)Ω point.

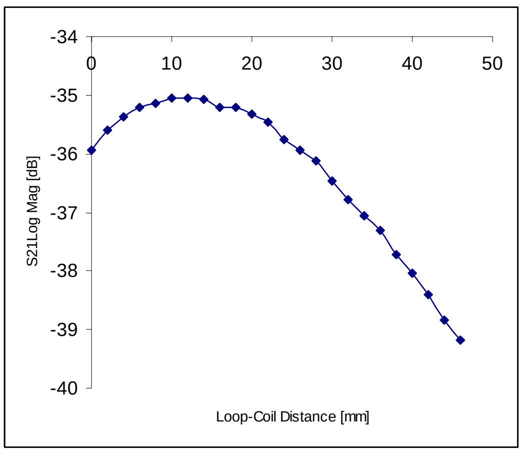


Fig.13 S_{21} vs. coupling of RF simulator.

From Fig.12 we determine $Q_{measured}$ at a distance of 11mm to be 460. This implies a Q_o of 920. We compare our extrapolated Q_o of 920 to a Q_o from pure theory:

$$Q_o = \frac{\omega L_2}{R_s} = \frac{(9.42x10^7)(1.1x10^{-6})}{0.100} = 1036$$

and we find that our measured Q_o comes in a little low. Remember however, that the value of R_s of 0.100Ω was an estimate, if we adjust R_s in the above equation to 0.113Ω we arrive at a Q_o 920.

This network analyzer measurement should provide the most precise technique for determining critical coupling in the cyclotron. However, a simple reflected RF power meter will suffice to show a perfect match – indicating critical coupling.

It was curious to note that (50+*j*0)Ω could be arrived at with different loop-tank coil distances within a small band of distances. This was accomplished by adjusting the locations of the "alligator clips" taping into the coupling loop – thus varying L_1. As one would intuitively expect, to achieve (50+*j*0)Ω when the coupling loop is closer to the tank, less of the coupling loop needed to be in the circuit, hence lowering the value of L_1. As the coupling loop was distanced from the tank coil, progressively more of the coupling loop needed to be invoked.

As a final inquiry with the "cyclotron RF simulator" the P6015 probe was used to directly measure the voltage developed on the capacitor as a function of input power.

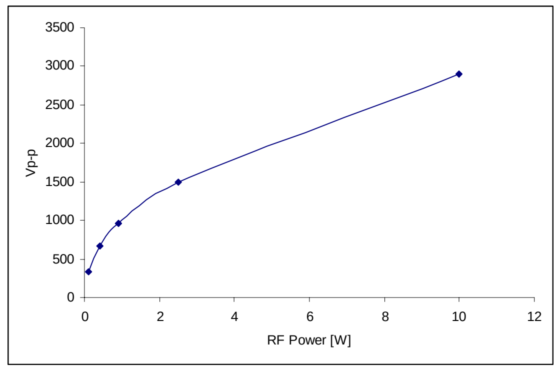


Fig.14 Measured capacitor Vp-p

Using the P6015 direct measurement of Vp-p different (50+*j*0)Ω setups, thus different critically coupled coupling loop-tank coil distances, were tested for preference in achieving the absolute maximum possible voltage across the vacuum capacitor. Empirically it was found for a given forward power into each of the (50+*j*0)Ω points, the peak capacitor voltage was always the same.

This measurement provides further verification of our simplistic model in section **I** of this paper. Visually, we again see the $x^{0.5}$ power law trend. Re-writing the equation for Vp-p we have:

$$Vp-p = 2\sqrt{\frac{2PL_2}{R_s C_2}}$$

Since we have measured Vp-p and have measured values for all parameters except R_s, where we only have an estimate of 100mΩ, let us solve for R_s and obtain the inferred value from measurement:

$$R_s = \frac{2PL_2}{C_2}\left(\frac{2}{V_{p-p}}\right)^2$$

Using the five data points shown in Fig.14 R_s was determined to be 0.107Ω with a standard deviation of 0.013.

This nice agreement between the handbook's estimation and the extrapolation from measurement gives us the needed confidence in the measurement of 800mΩ of the actual cyclotron RF system. It is clear from:

$$Vp-p = 2\sqrt{\frac{2PL_2}{R_s C_2}}$$

that minimizing R_s, or increasing L2 while simultaneously decreasing C2 (to maintain the resonant frequency) are the only parameters that can be adjusted to increase the DEE voltage for a given amount of RF power.

Once more, we estimate Q_o with the new value of R_s:

$$Q_o = \frac{\omega L_2}{R_s} = \frac{(9.42x10^7)(1.1x10^{-6})}{0.107} = 968$$

Closer to the Q_o of 920 extrapolated from Fig.12 and Fig.13.

VI 2500 WATT UPGRADE: Recently an external metal anode-ceramic sealed tube based RF power supply capable of 2,500 watts has become available. Its manufactured purpose was for plasma generation at 13.560 MHz, however, with only slight modification the unit was retuned to operate at 14.900 MHz. During the retuning procedure, the machine was tested as high as 15.100 MHz, and undoubtedly can be pushed further.

Preliminary tests with the new generator show that the cyclotron chamber is capable of withstanding 2000 watts of input power. The tank, tank housing, cyclotron chamber and DEE stem become very warm. It is not necessary to operate at 2000 watts, as shown above 1000 watts produces a peak-to-peak DEE voltage of approximately 15kV.

Fig.15 2.5kW generator in operation


Fig 16 Opened view of RF amplifier

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