

# Detecting String-Scale QCD Axion Dark Matter



Blas Cabrera  
Scott Thomas

## Strong CP Problem :



$$\mathcal{L} \supset \theta \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

Violates Parity and Time Reversal – (Renormalizable)

No Violation of Parity or Time Reversal Has Ever  
Been Observed in Strong Interactions !!!

Bounds on Electric Dipole Moments of  
Neutron and Atoms

$$\theta \lesssim 10^{-10}$$

## Strong CP Problem :



$$\mathcal{L} \supset \theta \frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

$$\theta \lesssim 10^{-10}$$

- CP Symmetry – Spontaneously Broken

$$\theta_{\text{CKM}} \sim \text{O}(1) \quad \text{while} \quad \theta < 10^{-10}$$

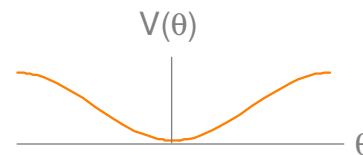
- Massless up-Quark  $e^{i\theta p} = e^{i\theta \text{Det}(m_q)}$

- Peccei-Quinn Mechanism (Peccei, Quinn; Weinberg; Wilczek)

Spontaneously Broken  $U(1)_{\text{PQ}}$  -

QCD Anomaly

Goldstone – Axion  $\theta = \theta(x)$



Dynamical Symmetry Restoration

# Relic Dark Matter Axions : (Preskill, Wise, Wilczek; Abbott, Sikive; Dine, Fischler)



## A COSMOLOGICAL BOUND ON THE INVISIBLE AXION

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and

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Received 14 September 1982

The production of axions in the early universe is studied. Axion models which break the  $U(1)_{PQ}$  symmetry above  $10^{12}$  GeV are found to produce an unacceptably large axion energy density.

$$d^2\phi_A/dt^2 + 3H(t)d\phi_A/dt + m_A^2(T)\phi_A = 0$$

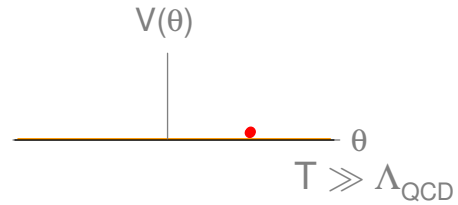
## Relic Dark Matter Axions :



- Coherent Production

Axion Exists as a Goldstone During and After Inflation

$\theta_i(x) \simeq \text{Constant over Observable Universe} + \text{Random}$



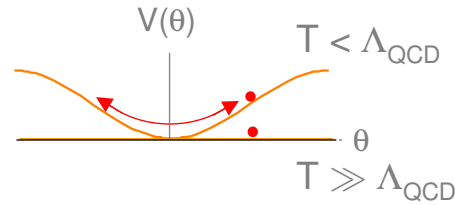
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$$\Omega_a h^2 \sim 0.4 \left( \frac{f_a/N}{10^{12} \text{ GeV}} \right)^{7/6} \theta_i^2$$

$$\Omega_{\text{CDM}} h^2 \simeq 0.11$$

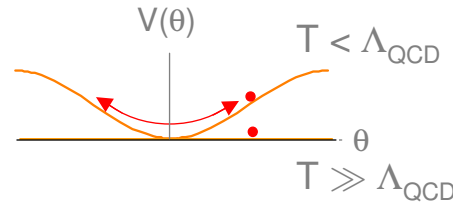
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$\uparrow$                        $\uparrow$                        $\langle \theta_i^2 \rangle = \pi^2/3$   
 Fixed                  Distribution

$$\langle \Omega_a h^2 \rangle \simeq 0.1 \Rightarrow f_a/N \sim 10^{11-12} \text{ GeV}$$

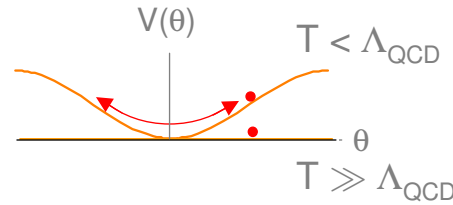
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$$\Omega_{\text{CDM}} h^2 \simeq 0.11$$

$$\Omega_a h^2 \simeq 0.1 \Rightarrow f_a/N > 10^{11-12} \text{ GeV}$$

$\rho_{\text{DM}}$  Selection Effects !?



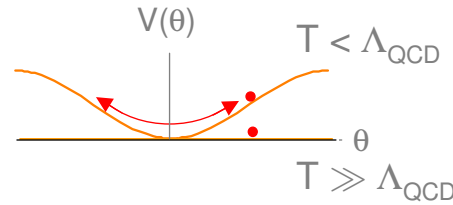
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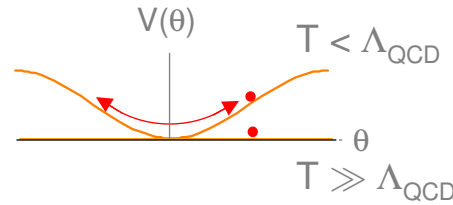
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- Large Classes of String Vacua  $f_a/N \sim 10^{16+-1} \text{ GeV}$  (Svrcek, Witten)

Moduli - p-Form Fields on Cycles – Shift Symmetry

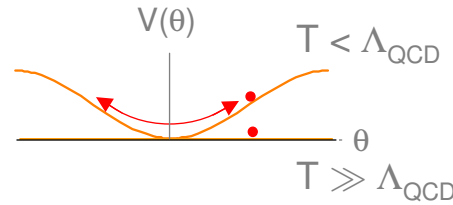
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Moduli - p-Form Fields on Cycles – Shift Symmetry

Possible to Detect ?

# Axion Electrodynamics : (Sikive)



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## Experimental Tests of the “Invisible” Axion

P. Sikivie

*Physics Department, University of Florida, Gainesville, Florida 32611*

(Received 13 July 1983)

Experiments are proposed which address the question of the existence of the “invisible” axion for the whole allowed range of the axion decay constant. These experiments exploit the coupling of the axion to the electromagnetic field, axion emission by the sun, and/or the cosmological abundance and presumed clustering of axions in the halo of our galaxy.

PACS numbers: 14.80.Gt, 11.30.Er, 95.30.Cq

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^2 N}{12\pi^2} \frac{a}{v} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 [1 + O(a^2/v^2)]$$

$$\nabla \cdot \vec{E} = \frac{e^2 N}{3\pi^2 v} \vec{B} \cdot \nabla a, \quad \nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \frac{e^2 N}{3\pi^2 v} \left[ \vec{E} \times \nabla a - \vec{B} \frac{\partial a}{\partial t} \right], \quad \square a = \frac{e^2 N}{3\pi^2 v} \vec{E} \cdot \vec{B} - m_a^2 a$$

# Axion Electrodynamics : (Sikive)



$$\mathcal{L} = \frac{1}{2}\epsilon_0\mathbf{E}^2 - \frac{1}{2\mu_0}\mathbf{B}^2 - \frac{3}{4}\xi\frac{\alpha}{2\pi\mu_0c}\frac{a}{f_a/N}\mathbf{E}\cdot\mathbf{B}$$

$$\xi \simeq \frac{4}{3}\left(\frac{E}{N} - \frac{2}{3}\frac{4+z}{1+z}\right) \quad z = m_u/m_d$$

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla\times\mathbf{B} = \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t} + \mu_0\mathbf{j}$$

$$\rho = \frac{3}{4}\xi\frac{\alpha}{2\pi\mu_0c}\frac{a}{f_a/N}\mathbf{B}\cdot\nabla a$$

$$\mathbf{j} = \frac{3}{4}\xi\frac{\alpha}{2\pi\mu_0c}\frac{a}{f_a/N}\left(\mathbf{E}\times\nabla a - \mathbf{B}\frac{\partial a}{\partial t}\right)$$

	E/N	$\xi$
DFSZ	8/3	0.97
KSVZ	0	-2.59

Galactic Dark Matter Axions

$$\frac{\partial a}{\partial t} \simeq m_a a \quad \nabla a \simeq m_a v a \quad v \sim 10^{-3}c$$

Constant Fields in Laboratory

$$\mathbf{B}_{\text{lab}} \gg \frac{1}{c}\mathbf{E}_{\text{lab}}$$

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$$\begin{aligned}\nabla\cdot\mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla\times\mathbf{B} &= \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t} + \mu_0\mathbf{j}\end{aligned}$$

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Constant Fields in Laboratory

$$\mathbf{B}_{\text{lab}} \gg \frac{1}{c}\mathbf{E}_{\text{lab}}$$

# Galactic Dark Matter Axions :



$$\rho_{\text{DM}} c^2 \simeq 0.3 \text{ GeV cm}^{-3}$$

$$z = m_u / m_d \simeq 0.56$$

$$\rho_a c^2 \simeq \frac{1}{2} \frac{m_a^2 a_0^2}{(\hbar c)^3} \quad m_a \simeq \frac{\sqrt{z} f_\pi m_\pi}{1 + z f_a / N}$$

$$\theta = \frac{a}{f_a / N}$$

$$\theta_0 \simeq 3.6 \times 10^{-19}$$

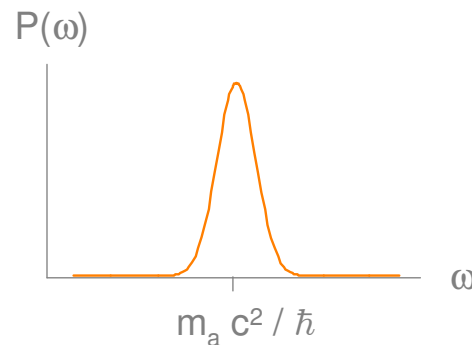
- On length scales  $D < \hbar / (m_a v)$

$$\theta(\mathbf{x}, t) \simeq \theta_0 e^{i\omega t} \quad \omega = \frac{m_a c^2}{\hbar}$$

$$\mathbf{j}(x, t) \simeq \frac{3}{4} \xi \frac{\alpha}{2\pi \mu_0 c} \theta_0 \omega \mathbf{B}(x) e^{i\omega t}$$

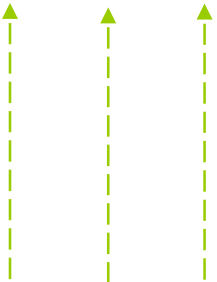
$f_a / N$	$\nu = \omega / 2\pi$
$10^{12} \text{ GeV}$	$1.5 \text{ GHz}$
$10^{16} \text{ GeV}$	$150 \text{ KHz}$

- Stochastic Spectrum



$$\frac{\Delta\omega}{\omega} \sim \frac{v^2}{c^2} \sim \frac{1}{Q_a} \sim 10^{-6}$$

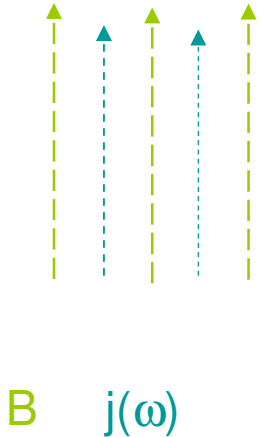
Dark Matter Axion Detection :



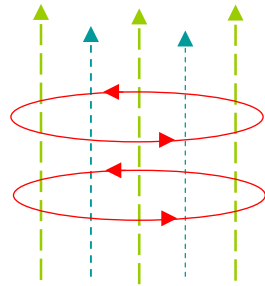
B



Dark Matter Axion Detection :



# Dark Matter Axion Detection :



B     $j(\omega)$     B( $\omega$ )

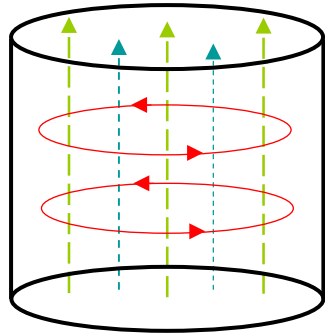
$$\nabla \times \mathbf{E} = -i\omega\mathbf{B}$$

$$\text{For } D \sim \hbar / m_a \quad \frac{E}{c} \sim B$$

# Dark Matter Axion Detection :



- Resonant Cavity  $TM_{010}$

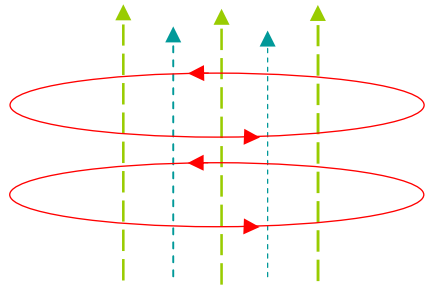


B  $j(\omega)$  B( $\omega$ )

$$\omega_{010} \simeq 4.8 c / D$$

$f_a / N$	D
$10^{12}$ GeV	15 cm
$10^{16}$ GeV	1.5 km

# Dark Matter Axion Detection – Large $f_a/N$ :



B     $j(\omega)$      $B(\omega)$

$$\nabla \times \mathbf{E} = -i\omega\mathbf{B}$$

$$\text{For } D \ll \hbar / m_a = c / \omega$$

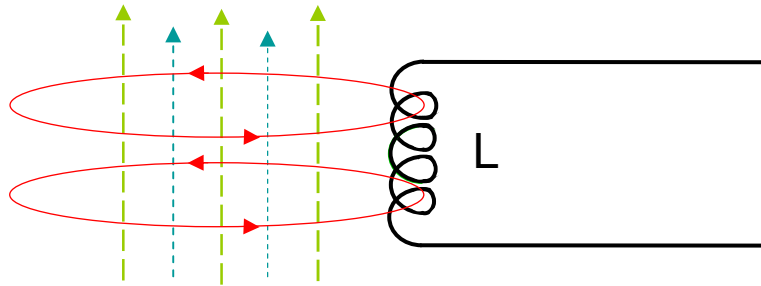
$$\frac{E}{c} \ll B$$

Adiabatic Limit

# Dark Matter Axion Detection – Large $f_a/N$ :



- Inductor L



Link Slowly Changing  
Magnetic Flux with Inductor

$B$   $j(\omega)$   $B(\omega)$

$$L\dot{I} = \mathcal{E} = M\dot{I}_a$$

$$I_a = \int \mathbf{j} \cdot d\mathbf{A}$$

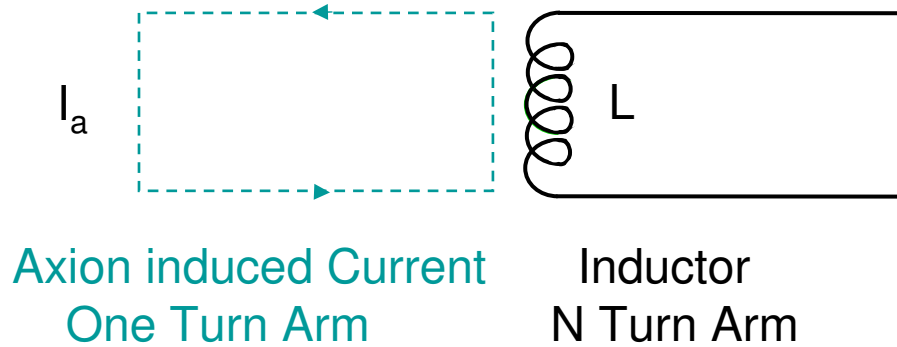
$$I = \frac{M}{L} I_a$$

$$\Phi = LI = MI_a$$

# Dark Matter Axion Detection – Large $f_a/N$ :



- Inductor L



Transformer

$$LI = \mathcal{E} = MI_a$$

$$I = \frac{M}{L}I_a$$

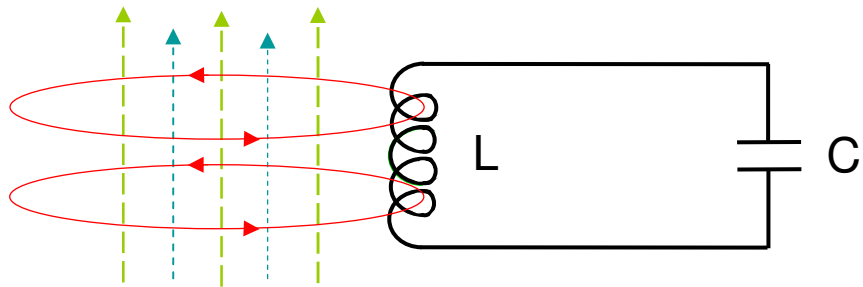
$$I_a = \int \mathbf{j} \cdot d\mathbf{A}$$

$$\Phi = LI = MI_a$$

# Dark Matter Axion Detection – Large $f_a/N$ :



- Resonant LC Circuit



$$\omega_0^2 = 1 / LC$$

$$\gamma = R/L = \omega_0/Q$$

B  $j(\omega)$  B( $\omega$ )

$$\left(-\omega^2 L - i\omega R + \frac{1}{C}\right) q = \mathcal{E}$$

$$I = \frac{i\omega \mathcal{E} / L}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

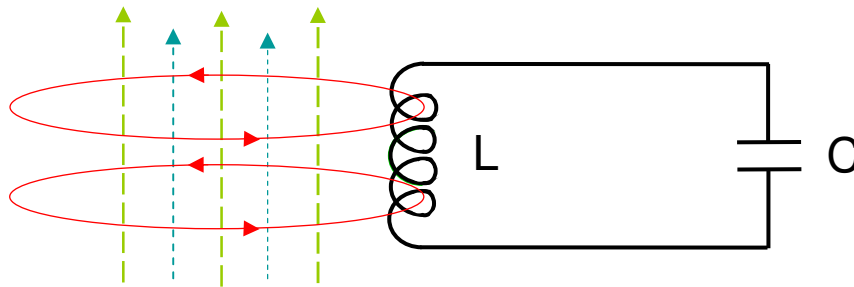
On Resonance

$$U = \frac{1}{2} L |I|^2 = \frac{1}{2} Q^2 \frac{M^2}{L} |I_a|^2$$

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On Resonance

$$U = \frac{1}{2} L |I|^2 = \frac{1}{2} \left( Q^2 \frac{M^2}{L} \right) |I_a|^2$$



## Inductance :



- Number of Turns  $N$

$$M \propto N$$

$$L \propto N^2$$

$$M^2 / L \propto N^0$$

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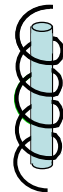
$$M^2 / L \propto N^0$$

- Permeability

$$M^2 / L \sim \mu h$$

$$\mu = \mu_r \mu_0$$

$$\mu_r \sim 10^{4-5-6}$$



Overcome Grain Cohesive Forces at  
Low Temperature with  $B=B(\omega')$

Large Permeability Resonant Transformer

## Quality Factor:



- Core Losses

Eddy Currents → Joule Heating

Lamination, Powder

\* Hysteresis

$$\frac{U_{\text{loss}}}{V} = \oint H dB$$

$$B = \mu H$$

$$Q \sim 10^{2-3}$$

- Radiation Resistance

Capacitor – Electric Dipole Antenna

Inductor – Magnetic Dipole Antenna

Small Antennas Inefficient Radiators

- Resistive Losses

## Axion Induced Current :



$$|I_a| = \int |\mathbf{j} \cdot d\mathbf{A}| \simeq \frac{3}{4} \xi \frac{\alpha}{2\pi\mu_0 c} \omega\theta_0 \Phi$$

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A}$$

## Magnetic Flux :

	D (m)	h (m)	B (T)	$\Phi$ (Weber)
LLNL Axion	0.6	1	8	2
ANL 12 ft Bubble Chamber	4.8	3	1.8	30
CMS Solenoid	6.5	12	4	130

Maximum B ~ Limited



Compact Muon Solenoid:

$$D = 6.5 \text{ m}$$

$$h = 12 \text{ m}$$

$$B = 4 \text{ T}$$

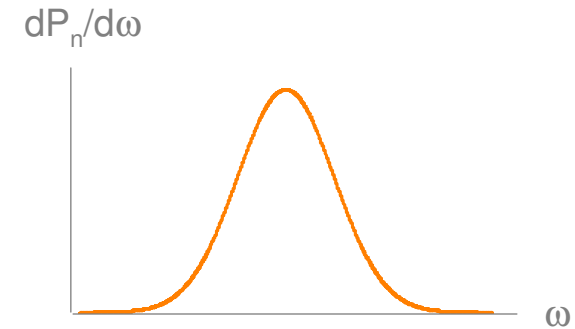
## Signal and Noise Temperature :



- Noise Power and Temperature

$$P_n = \frac{1}{2} |I_n|^2 R = \frac{U_n R}{L} = U_n \gamma = \frac{U_n \omega_0}{Q}$$

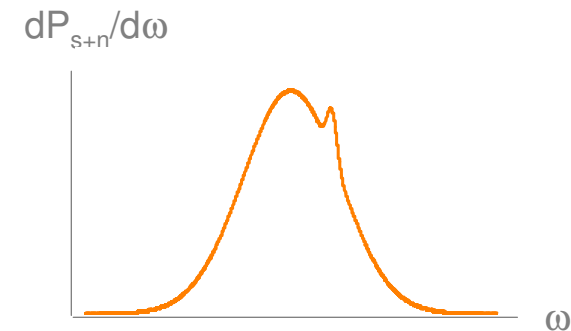
$$\left. \frac{dP_n}{d\omega} \right|_0 = \frac{2U_n}{\pi} \equiv \frac{2kT_n}{\pi}$$



- Signal Temperature

$$\frac{2kT_s}{\pi} \equiv \left. \frac{dP_s}{d\omega} \right|_0$$

$$kT_s = \frac{1}{2} Q Q_a \frac{M^2}{L} |I_a|^2 = \frac{Q_a}{Q} U_s$$



## Scanning Time :



### Noise Limited

- Time for Significance  $S$  in Signal Bandwidth  $\Delta \omega = \omega / Q_a$

$$t \sim S^2 \frac{T_n^2 Q_a}{T_s^2 \omega}$$

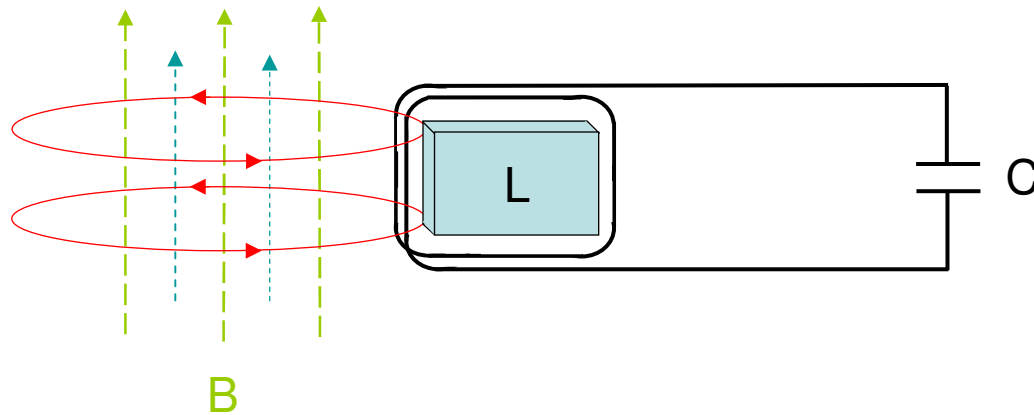
- Time to Scan an Octave  $t_{\text{oct}} \sim Q t$

$$t_{\text{oct}} \sim S^2 \frac{T_n^2 Q_a Q}{T_s^2 \omega} = S^2 \left( \frac{L k T_n}{M^2 |I_a|^2} \right)^2 \frac{1}{Q_a Q \omega}$$

- For fixed  $S$  and  $Q_a$

$$t_{\text{oct}} \propto \frac{T_n^2 f_a^5}{Q \mu_r^2 h^2 \Phi^4}$$

$$\frac{M^2}{L} \propto \mu_r h$$

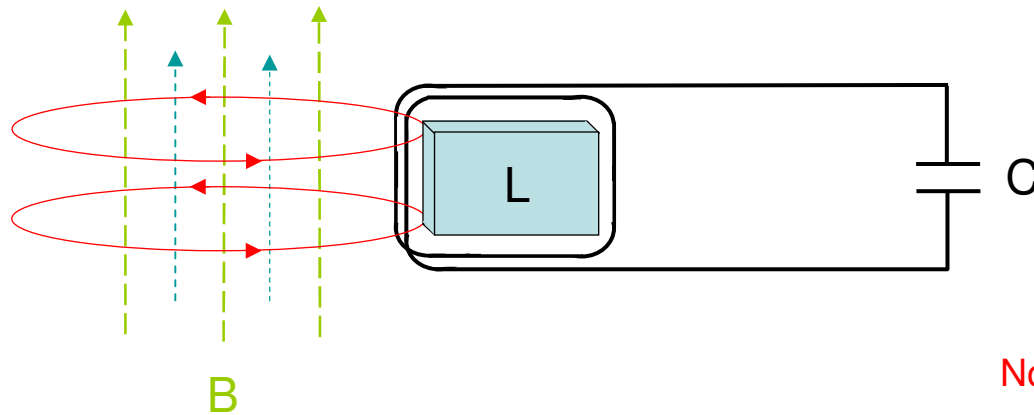


$L \sim 100-1000 \text{ mH}$   
 $Q \sim 10^2$        $C \sim 10-100 \text{ pF}$   
 $T_n \sim 2 \text{ K}$        $R \sim 1-10 \text{ k}\Omega$

- For  $t_{\text{oct}} \sim 1 \text{ yr}$  with  $S \sim 5$       DFSZ

	$\Phi$	$h$	$\mu_r$	$f_a/N$	$\nu$	$T_s$	$T_Q$
ANL	30 Weber	3 m	$3 \times 10^4$	$10^{15} \text{ GeV}$	1.5 MHz	7 mK	350 $\mu\text{K}$
CMS	130 Weber	12 m	$10^5$	$10^{16} \text{ GeV}$	150 kHz	18 mK	35 $\mu\text{K}$





Note Strong Dependence

$$t_{\text{oct}} \propto \frac{T_n^2 f_a^5}{Q \mu_r^2 h^2 \Phi^4}$$

$L \sim 100\text{-}1000 \text{ mH}$

$Q \sim 10^2$

$C \sim 10\text{-}100 \text{ pF}$

$T_n \sim 2 \text{ K}$

$R \sim 1\text{-}10 \text{ k}\Omega$

Sizeable Permeability  $\mu_r$   
and  
Large Flux  $\Phi$  Required

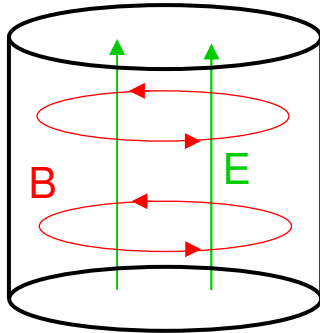
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# Resonant Cavity :



- Cylindrical  $TM_{010}$



$$C \sim \frac{\epsilon_0 A}{h}$$

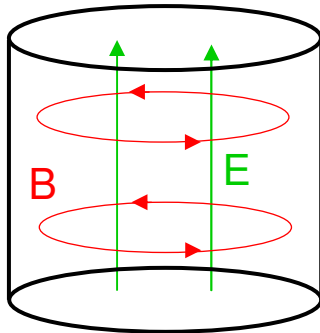
$$L \sim \frac{M^2}{L} \sim \frac{\mu_0 h}{2\pi}$$

$$\omega_0 \sim \frac{4c}{D}$$

# Resonant Cavity :



- Cylindrical  $TM_{010}$

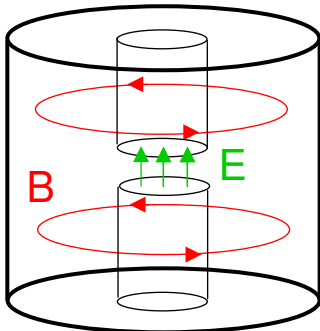


$$C \sim \frac{\epsilon_0 A}{h}$$

$$L \sim \frac{M^2}{L} \sim \frac{\mu_0 h}{2\pi}$$

$$\omega_0 \sim \frac{4c}{D}$$

- Lower  $\omega_0$  with Geometry + Permittivity



$$C \sim \frac{\epsilon A'}{h'}$$

$$L \sim \frac{M^2}{L} \sim \frac{\mu_0 h}{2\pi}$$

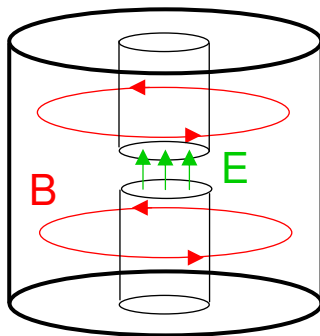
$$\omega_0 \sim \frac{4c}{D'} \sqrt{\frac{h'}{\epsilon_r h}}$$

Adiabatic limit - E and B modes small overlap

## Resonant Cavity :



- Lower  $\omega_0$  with Geometry + Permittivity +  
Increase  $M^2/L$  with Permeability



$$C \sim \frac{\epsilon_0 A'}{h'}$$

$$L \sim \frac{M^2}{L} \sim \frac{\mu h}{2\pi}$$

$$\omega_0 \sim \frac{4c}{D'} \sqrt{\frac{h'}{\mu_r \epsilon_r h}}$$

Identical to LC Circuit with one turn  
Toroidal Inductor

- **Detector:** SQUID Internal Antenna Coupled to B

$$Q^{-1} = Q_0^{-1} + Q_D^{-1} \quad \text{Match } Q_0 \text{ and } Q_D$$

Conclusion :



Axion Dark Matter Detection  $f_a/N \sim 10^{13} - 10^{15-16}$  GeV  
(Low  $f_a/N$  Covered Rapidly)

LC Resonant Circuit  
||  
Modified Resonant Cavity

- Large Flux Magnet \*
- Modified LC Resonant Cavity \*
- Large Permeability Core \*  
(Cool Large Mass)