WALL-CROSSING FORMULAE FOR BPS STATES & SOME APPLICATIONS

ETH LECTURE I

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BASED ON WORK DONE WITH

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OUTLINE

1. INTRODUCTION: BPS STATES
2. THREE WALL-CROSSING FORMULAE
3. SUPERGRAVITY DERIVATION OF PRIMITIVE AND SEMI-PRIMITIVE WCF
4. D6-D2-D0 SYSTEM + Z_{TOP}.
5. D4D2D0 SYSTEM: MODULAR GENERATING FUNCTION
6. ROUTE TO OSV: ENTROPY ENIGMA & DEGENERACY DICHOTOMY
7. OPEN PROBLEMS

1. INTRODUCTION

THE "SPACE OF BPS STATES" HAS BEEN A CENTRAL CONCEPT IN SUSY GAUGE THEORY & STRING THEORY FOR ALMOST 30 YEARS.

LECTURES I & II FOCUS ON RECENT PROGRESS IN UNDERSTANDING PHENOMENA ASSOCIATED TO MARGINAL STABILITY.

A. DEFINING THE "SPACE OF BPS STATES"

FOR DEFINITENESS, WE FOCUS ON THEORIES WITH $d=4$, $\mathcal{N}=2$ SUSY IN (ASYMPTOTIC) MINKOWSKI SPACE M_4

HILBERT SPACE OF ONE-PARTICLE STATES, $\mathcal{H}$, IS A REP. OF THE $d=4$, $\mathcal{N}=2$ ALGEBRA.

RECALL THE $\mathcal{N}=2$, $D=4$ SUSY ALGEBRA

$$\mathfrak{s} = \mathfrak{s}_0 \oplus \mathfrak{s}_1$$

$$\mathfrak{s}_0 = \underset{\hat{M}_{\mu\nu}}{\left(\mathrm{spin}(1,3)\right.} \ltimes \underset{\hat{P}_\mu}{\left.\mathbb{R}^4\right)} \oplus \mathfrak{u}(2) \oplus \underset{\hat{Z}}{\mathbb{R}}$$

$$\mathfrak{s}_1 = \left[\mathrm{Spinor} \otimes \mathbb{C}^2\right]_{\mathbb{R}}$$

$$Q_{\alpha I}, \ \bar{Q}^{\dot{\alpha} I}$$

$$\{Q_{\alpha I}, \bar{Q}_{\dot{\beta}}{}^{J}\} = 2\hat{P}_\mu \sigma^\mu_{\alpha\dot{\beta}} \delta_I{}^J$$

$$\{Q_{\alpha I}, Q_{\beta J}\} = 2\hat{Z} \epsilon_{\alpha\beta} \epsilon_{IJ}$$

DECOMPOSE $\mathcal{H} = \bigoplus_{z \in \mathbb{C}} \mathcal{H}_{\hat{Z}=z}$

LEMMA: $E \geq |Z|$ ON $\mathcal{H}_Z$

PROOF: $\mathcal{N}=2$ IS A 6D SUSY ALGEBRA DIMENSIONALLY REDUCED TO 4D:

$$Spin(1,3) \times Spin\, 2 \hookrightarrow Spin(1,5)$$

$$2_{1/2} \oplus 2^*_{-1/2} = 4$$

$\{Q_{rA}, Q_{sB}\} = \Gamma^M_{rs} P_M \, \varepsilon_{AB} \Longrightarrow Z = P_4 + i P_5$

UNITARY REP. $\Longrightarrow P^M P_M \geq 0$.

$\Longrightarrow E^2 - \vec{P}^2 - |Z|^2 \geq 0$ ■

DEF'N: $\mathcal{H}^{BPS}$ IS THE SUBSPACE OF $\mathcal{H}$ WHERE $E = |Z|$.

B. GENERAL FEATURES

THE THEORIES WE WILL DISCUSS ($d=4, \mathcal{N}=2$ FIELD THEORIES & COMPACTIFICATIONS OF TYPE II STRINGS ON A CALABI-YAU X) HAVE TWO GENERAL FEATURES LEADING TO EXTRA STRUCTURE

(1.) MODULI SPACE OF VACUA $\tilde{\mathcal{M}}$

(2.) AT A GENERIC POINT $u \in \tilde{\mathcal{M}}$ THERE IS AN UNBROKEN ABELIAN GAUGE SYMMETRY.

AS A REP. OF $\mathcal{N}=2$, $\mathcal{H}$ DEPENDS ON u, SO WRITE $\mathcal{H}_u$

MOREOVER,

ABELIAN GAUGE GROUP $\Rightarrow$ LATTICE Λ OF ELECTRIC & MAGNETIC CHARGES

- Λ IS A SYMPLECTIC LATTICE

$\langle \cdot , \cdot \rangle : \Lambda \times \Lambda \to \mathbb{Z}$

DUALITY INVARIANT D-S-Z PRODUCT

- $\mathcal{H}_u$ CAN BE GRADED BY CHARGE SECTORS:

$$\mathcal{H}_u = \bigoplus_{\gamma \in \Lambda} \mathcal{H}_{\gamma, u}$$

ON THE SUBSPACE $\mathcal{H}_{\gamma,u}$ THE CENTRAL CHARGE OPERATOR $\hat{Z}$ IS A SCALAR:

$$\hat{Z} = Z(\gamma;u),$$

DEFINING THE CENTRAL CHARGE FUNCTION

SO ON $\mathcal{H}^{BPS}_{\gamma,u}$

$$E = M = |Z(\gamma;u)|$$

MOREOVER, $Z : \Lambda \times \mathcal{M} \longrightarrow \mathbb{C}$,

IS LINEAR ON Λ

$$Z(\gamma_1+\gamma_2;u) = Z(\gamma_1;u) + Z(\gamma_2;u)$$

KEY PHYSICS POINT:

*** SOME BPS STATES ARE BOUNDSTATES OF OTHER BPS STATES. ***

BINDING ENERGY:

$$E(u) = |Z(\gamma_1+\gamma_2;u)| - |Z(\gamma_1;u)| - |Z(\gamma_2;u)| \leq 0$$

$\Longrightarrow$ THERE ARE VALUES $u \in \widetilde{\mathcal{M}}$ WHERE SUCH BOUNDSTATES MIGHT DECAY: POINTS u WHERE

$$|Z_1 + Z_2| = |Z_1| + |Z_2|$$

$\Longrightarrow$ Z_1 & Z_2

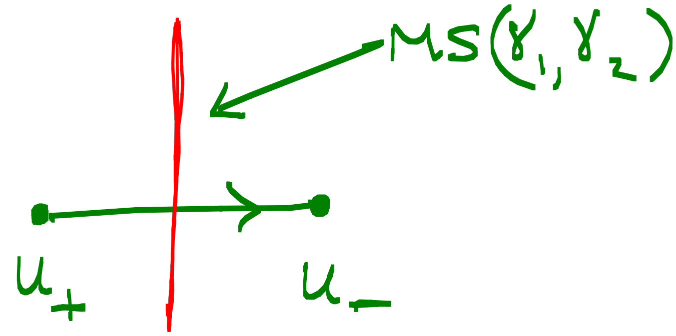
<u>HAVE THE SAME PHASE.</u>

WALLS OF MARGINAL STABILITY:

$$MS(\gamma_1,\gamma_2) := \{u \mid Z(\gamma_1;u) = \lambda Z(\gamma_2;u),\ \lambda \in \mathbb{R}_+\}$$

$MS(\gamma_1,\gamma_2)$ IS CODIMENSION ONE IN $\widetilde{\mathcal{M}}$

A BOUNDSTATE OF PARTICLES WITH CHARGES γ_1, γ_2 CAN DECAY



[CECOTTI, FENDLEY, INTRILIGATOR, VAFA ; SEIBERG & WITTEN]

WE WANT TO SAY HOW MANY STATES DECAY.

$$\dim \mathcal{H}^{BPS}_{\gamma,u_+} - \dim \mathcal{H}^{BPS}_{\gamma,u_-} = ?$$

$\Longrightarrow$ WALL-CROSSING FORMULA

$\dim \mathcal{H}^{BPS}_{\gamma,u}$ CAN ALSO CHANGE FOR "UNINTERESTING REASONS"

SO DEFINE AN INDEX

$$\Omega(\gamma;u) = -\frac{1}{2}\mathrm{Tr}_{\mathcal{H}^{BPS}_{\gamma,u}}(2J_3)^2(-1)^{2J_3}$$

TECHNICAL POINT:

$$\mathcal{H}^{BPS}_{\gamma,u} = \underbrace{\mathcal{H}_{\frac{1}{2}HM}}_{\text{1/2 hyper}} \otimes \mathcal{H}(\gamma;u)$$

$2(0)+(\frac{1}{2})$ as spin rep$^{\underline{n}}$

$$\Omega(\gamma;u) = \mathrm{Tr}_{\mathcal{H}(\gamma;u)}(-1)^F$$

HENCEFORTH FOCUS ON $\mathcal{H}(\gamma;u)$

C. TYPE II STRINGS

NOW - SPECIALIZE TO TYPE II STRING THEORY ON $M_4 \times X$.

1. X = STATIC, COMPACT, CY 3-FOLD
2. FLAT B-FIELD: $B \in H^2(X, \mathbb{R})$
3. FLAT RR FIELDS

$\Rightarrow$ $\mathcal{N}=2$, $d=4$ SUGRA ON M_4

M_4 IS NONCOMPACT $\Rightarrow$ TO DEFINE THE HILBERT SPACE AS A REP. OF $\mathcal{N}=2$ WE MUST SPECIFY BOUNDARY COND'S FOR THE MASSLESS FIELDS:

$$u = \lim_{\vec{x} \to \infty} \left(g_{MN}, \phi, B_{MN}, RR \right)$$

MODULI SPACE $\tilde{\mathcal{M}}$ IS A PRODUCT:

HYPERMULTIPLETS × VECTORMULTIPLETS

[CPLX STR., ϕ, RR FIELDS] × [COMPLEXIFIED KÄHLER]

WE WORK AT A GENERIC HYPERMULTIPLET.

Ω IS CONSTANT AS A FUNCTION OF HYPERMULTIPLETS, BUT IT TURNS OUT TO BE ONLY PIECEWISE CONSTANT AS A FUNCTION OF VECTORMULTIPLETS.

RECENT PROGRESS HAS BEEN CONCERNED WITH THE DEPENDENCE ON VECTORMULTIPLETS, IN THIS TALK,

$$\boxed{t = B + iJ}$$

LOW ENERGY ABELIAN GAUGE THEORY FROM RR FIELDS

$\mathcal{H}_u$ IS GRADED BY ELECTRIC/MAGNETIC CHARGE SECTORS:

$$\mathcal{H}_u = \bigoplus_{\Gamma} \mathcal{H}_{\Gamma, u}$$

$\Gamma \in$ (TWISTED) K-THEORY (X)

PHYSICISTS USUALLY WORK WITH COHOMOLOGY AND IN THIS TALK WE REPLACE

$$\mathcal{E} \in K^0(X) \longrightarrow ch(\mathcal{E})\sqrt{\hat{A}} \in H^{ev}(X, \mathbb{Q})$$

$K^0(X)/\text{TORSION} = \text{LATTICE } \Lambda$

$ch(\mathcal{E})\sqrt{\hat{A}} \in$ CORRESPONDING LATTICE IN $H^{ev}(X,\mathbb{Q})$

Λ HAS A $\mathbb{Z}$ SYMPLECTIC FORM

$$\langle \mathcal{E}_1, \mathcal{E}_2 \rangle = \text{Index } \not{D}_{\mathcal{E}_1 \otimes \bar{\mathcal{E}}_2}$$

$$= \int \left(ch\,\mathcal{E}_1 \sqrt{\hat{A}}\right) \wedge \left(ch\,\bar{\mathcal{E}}_2 \sqrt{\hat{A}}\right)$$

IN TERMS OF COHOMOLOGY:

$$\Gamma = (p^0, P, Q, q_0) \in H^0 \oplus H^2 \oplus H^4 \oplus H^6$$

$$\langle \Gamma, \Gamma' \rangle = \int_X -p^0 q_0' + P\cdot Q' - P'\cdot Q + p^{0\prime} q_0$$

WE OFTEN REFER TO THESE COMPONENTS VIA THEIR D-BRANE SOURCES:

D6	D4	D2	D0
p^0	P	Q	q_0
H_6	H_4	H_2	H_0
H^0	H^2	H^4	H^6

SOMETIMES IDENTIFY $H^6(X, \mathbb{Z}) \cong \mathbb{Z}$

D. WHY DO WE CARE?

PHYSICS MOTIVATION

1. THE MAIN MOTIVATION FOR RECENT WORK IS THE PROGRAM, INITIATED BY STROMINGER-VAFA (1995) OF ACCOUNTING FOR BH ENTROPY VIA MICROSTATE COUNTING. THAT GOAL IS STILL NOT FULLY ACCOMPLISHED.

> WE DON'T KNOW BPS DEGENERACY FOR CERTAIN NATURAL CHARGE REGIMES, FOR EXAMPLE:
>
> $$\Gamma \longrightarrow \lambda\Gamma \qquad \lambda \longrightarrow \infty$$

2. OSV CONJECTURE:

   RELATION BETWEEN

   $\Omega(\Gamma)$ & GW / DT / GV INVARIANTS

$\Longrightarrow$ NONPTVE TOPOLOGICAL STRING?

MATH MOTIVATION

1. PHYSICAL STABILITY OF BPS STATES IS RELATED TO MATH. STABILITY IN THE BOUNDED DERIVED CATEGORY OF COHERENT SHEAVES ON A C.Y.: KONTSEVICH, DOUGLAS, BRIDGELAND, THOMAS, PANDHARIPANDE ....

   PHYSICS ⟹ PREDICTIONS/CONSTRAINTS ON WHAT WE EXPECT SHOULD BE TRUE.

2. MANY INTERESTING CONNECTIONS TO AUTOMORPHIC FORMS AND ANALYTIC NUMBER THEORY; SOME RELATIONS TO ARITHMETIC CY'S.

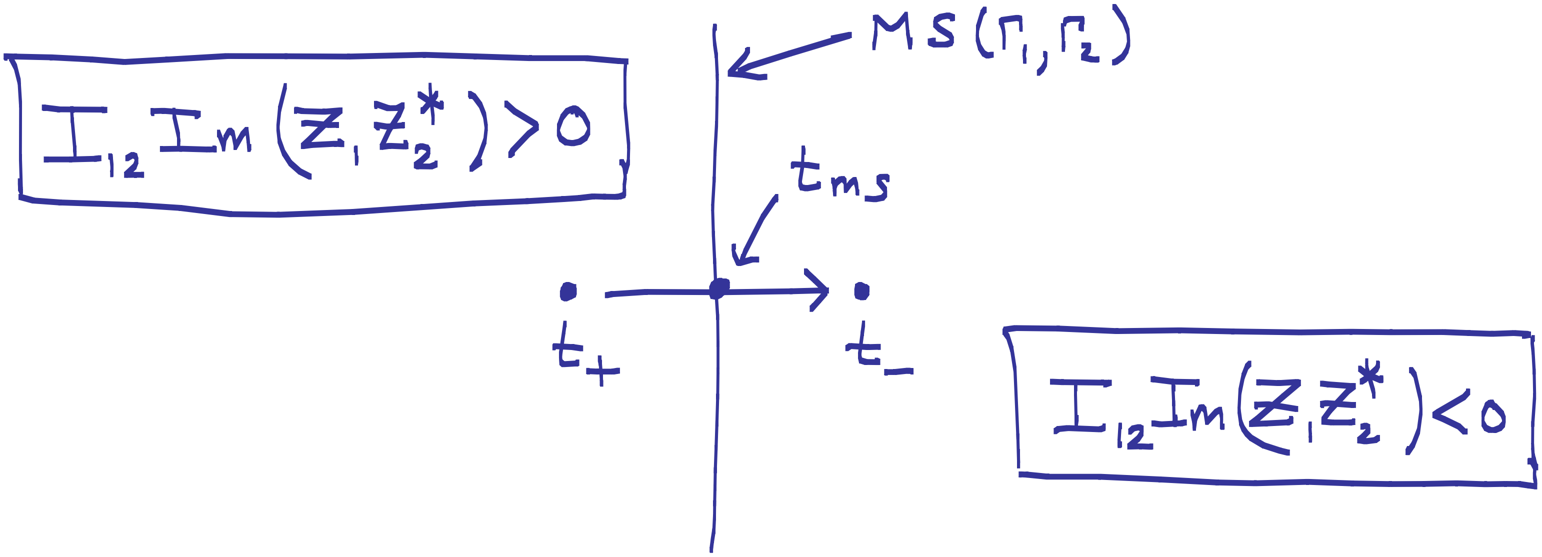
3. THERE ARE SEVERAL OTHER MORE SPECULATIVE APPLICATIONS, E.G. BPS ALGEBRAS: GENERALIZING NAKAJIMA'S WORK AND SUGGESTED BY TYPE II/HET DUALITY SHOULD BE CLOSELY RELATED.

2. WALL-CROSSING FORMULAE: STATEMENT

A. PRIMITIVE WALL-CROSSING FORMULA:

Λ HAS SYMPLECTIC FORM $\langle \cdot , \cdot \rangle$

LET $I_{12} := \langle \Gamma_1 , \Gamma_2 \rangle$



Γ_1, Γ_2 PRIMITIVE, t_{ms} GENERIC $\Longrightarrow$

$$\mathcal{H}_+ - \mathcal{H}_- = (J_{12}) \otimes \mathcal{H}(\Gamma_1 ; t_{ms}) \otimes \mathcal{H}(\Gamma_2 ; t_{ms})$$

$$J_{12} = \frac{1}{2}\left(|I_{12}| - 1 \right)$$

$$\Delta \Omega = (-1)^{I_{12}-1} |I_{12}| \, \Omega(\Gamma_1, t_{ms}) \Omega(\Gamma_2, t_{ms})$$

B. SEMI-PRIMITIVE WALL-CROSSING FORMULA

IN ADDITION TO $\Gamma_1+\Gamma_2$ BOUNDSTATES WE CAN ALSO FORM $N_1\Gamma_1+N_2\Gamma_2$ BOUNDSTATES

$$MS(\Gamma_1,\Gamma_2) = MS(N_1\Gamma_1, N_2\Gamma_2) \quad N_1,N_2\in\mathbb{Z}_+$$

CONSIDER $N_1=1$, $N_2\geq 1$:

$$\bigoplus_{N_2} u^{N_2}\Delta\mathcal{H}\Big|_{\Gamma\to\Gamma_1+N_2\Gamma_2}$$

CLAIM: THIS IS A $\mathbb{Z}_2$-GRADED FOCK SPACE

$$\mathcal{H}(\Gamma_1;t_{ms})\bigotimes_{k=1}^{\infty}\mathcal{F}\Big(u^k\underbrace{\big(J_{\Gamma_1,k\Gamma_2}\big)\otimes\mathcal{H}(k\Gamma_2;t_{ms})}_{\text{GRADED SPACE OF OSCILLATORS}}\Big)$$

IN PARTICULAR:

$$\Omega_1+\sum_{N>0}u^N\Delta\Omega(\Gamma_1+N\Gamma_2) =$$

$$= \Omega(\Gamma_1)\prod_{k>0}\Big(1-(-1)^{\langle\Gamma_1,k\Gamma_2\rangle}u^k\Big)^{|\langle\Gamma_1,k\Gamma_2\rangle|\,\Omega(k\Gamma_2)}$$

C. KONTSEVICH-SOIBELMAN'S MULTIPLICATIVE WCF

THE METHODS I WILL EXPLAIN IN THIS LECTURE DO NOT EASILY GENERALIZE TO DECAYS

$$\Gamma \longrightarrow N_1 \Gamma_1 + N_2 \Gamma_2 \qquad N_1, N_2 > 1$$

BUT K & S HAVE PROPOSED A REMARKABLE WCF FOR Ω WHICH COVERS ALL CASES.

WE WILL GIVE A PHYSICAL EXPLANATION & PROOF OF THE KS WCF IN LECTURE II.

3. PHYSICAL DERIVATION OF WCF

A. THE ATTRACTOR TRICHOTOMY

D-BRANES ARE OBJECTS IN A CATEGORY

IN TYPE IIA/CY, THE SUBCATEGORY OF SUSY BRANES IS PROBABLY THE BOUNDED DERIVED CATEGORY OF COHERENT SHEAVES.

BUT WE WANT TO DESCRIBE THE (PHYSICALLY) STABLE OBJECTS.

AT WEAK STRING COUPLING, AND $J \to \infty$

$\exists$ A BEAUTIFUL DESCRIPTION OF STABLE BPS STATES USING SUGRA.

IN THE SEMICLASSICAL LIMIT

$\psi \in \mathcal{H}_{BPS} \rightsquigarrow$ BPS SOLUTION OF SUGRA EQUATIONS

* SUPERGRAVITY ALLOWS ONE TO IDENTIFY MANY "STABLE OBJECTS" THANKS TO THE ATTRACTOR MECHANISM.

ATTRACTOR MECHANISM: (F.K.S. ; STROMINGER)

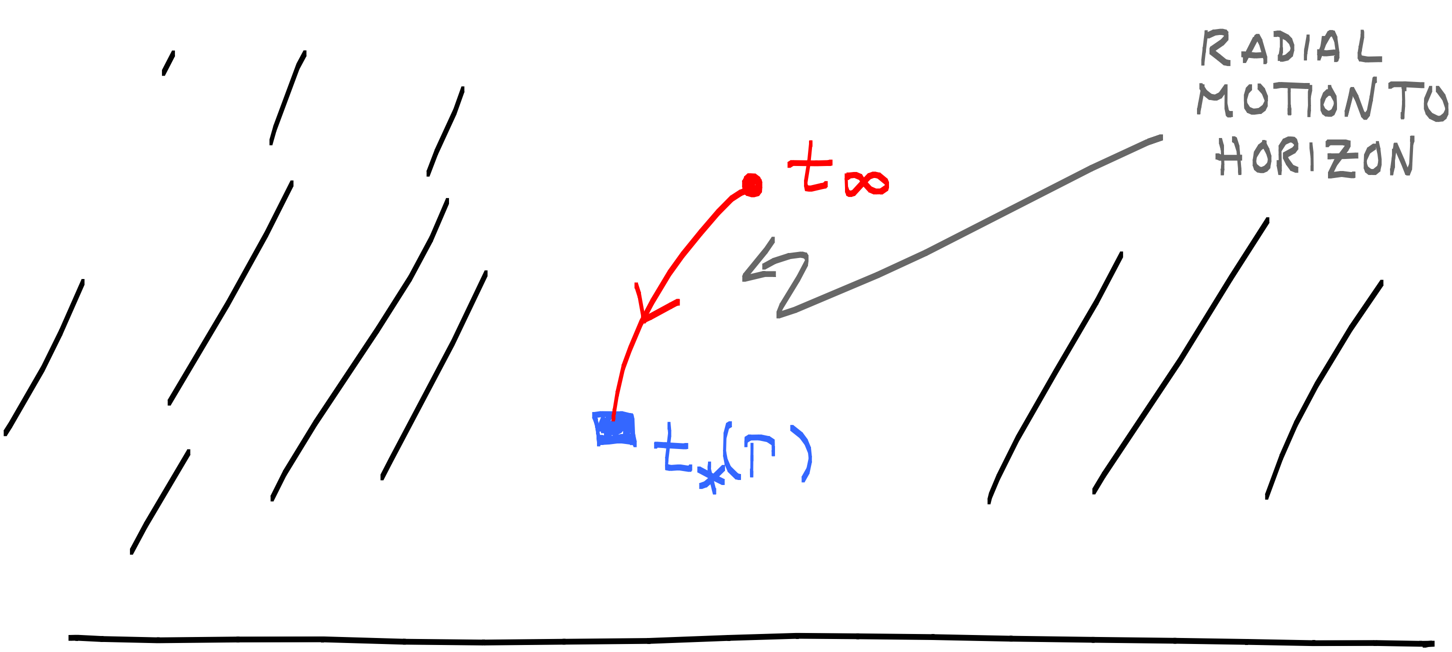
Γ, t_∞ & SPHERICAL SYMMETRY

$\Longrightarrow$ $\exists$ AT MOST ONE BPS SOLUTION OF SUGRA.

IF IT EXISTS....

SCALAR FIELDS $t = t(r)$, AND EVOLUTION FROM $r=\infty$ to $r=0$ APPROACHES AN ATTRACTIVE FIXED POINT $t_*(\Gamma)$:

$\widetilde{\mathcal{M}}_{VM}$



ATTRACTOR FLOW = GRADIENT FLOW FOR

$$\log |Z(\Gamma;t)|^2$$

$$Z = \frac{\langle \Gamma, \omega \rangle}{\sqrt{\langle \omega, \omega^* \rangle}}$$

$$\langle \Gamma, \Gamma' \rangle = \int_X -p^0 q_0' + PQ' - QP' + q_0 p^{0\prime}$$

ω = PERIOD VECTOR

IN LARGE RADIUS APPROXIMATION:

$$\omega = -e^t = -e^{B+iJ}$$

$$Z \approx \frac{\frac{1}{6}p^0 t^3 - \frac{1}{2}P t^2 + Qt - q_0}{\sqrt{(\mathrm{Im}\, t)^3}}$$

ATTRACTOR TRICHOTOMY

1. $t_*(\Gamma) \in \text{Interior}(\widetilde{\mathcal{M}})$

   and $Z(\Gamma; t_*(\Gamma)) \neq 0$

   "REGULAR ATTRACTOR POINT"

2. $\exists$ NONEMPTY SUBVARIETY $\subset \widetilde{\mathcal{M}}$

   $Z(\Gamma; t) = 0$

3. $t_*(\Gamma) \in \partial \widetilde{\mathcal{M}}$

(1.) $\exists$ SPHERICALLY SYMMETRIC BPS BLACK HOLES IN $\mathcal{H}_{BPS}(\Gamma; t)$ FOR ALL t

(2.) $\mathcal{H}_{BPS}(\Gamma; t) = \phi$ IN AN OPEN REGION OF THE ZERO LOCUS.

$\mathcal{H}_{BPS}$ MIGHT BE NONEMPTY FURTHER AWAY

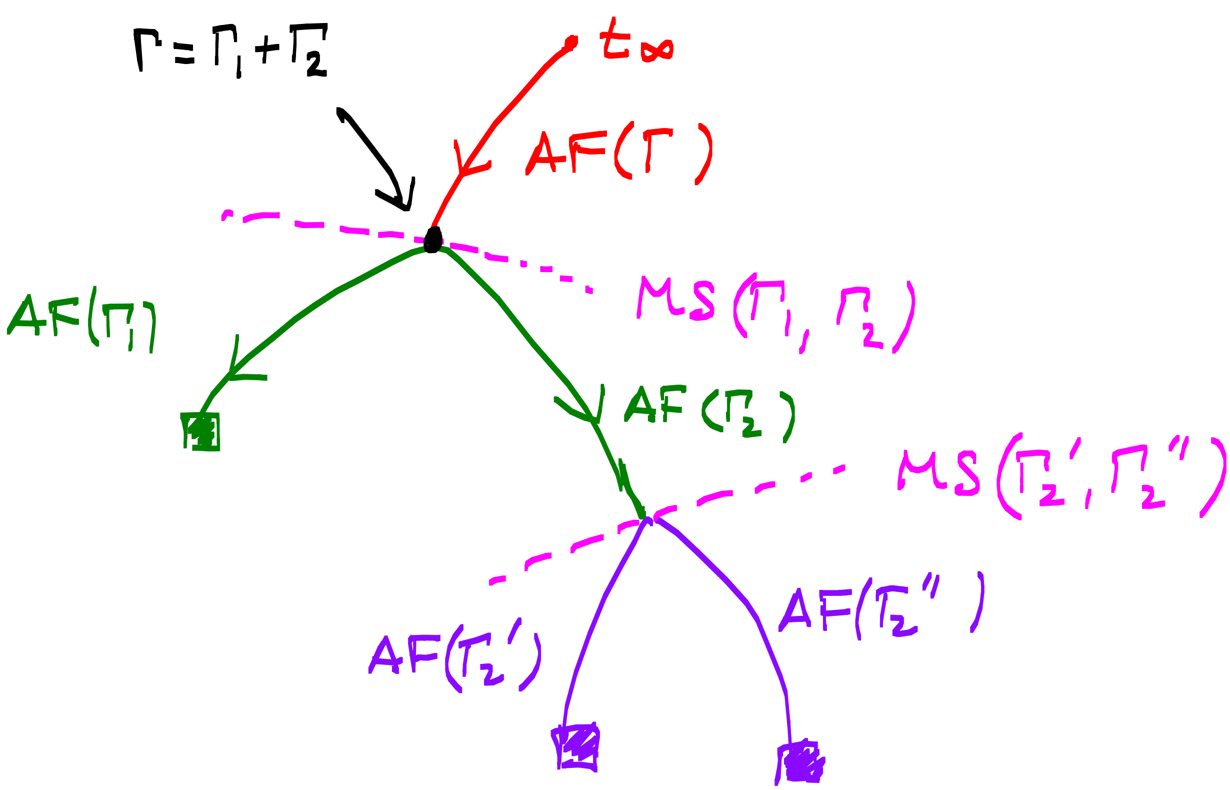
(3.) CANNOT USE SUGRA TO ESTABLISH EXISTENCE: MUST USE MICROSCOPIC ARGUMENTS.

B. SPLIT ATTRACTOR FLOWS

IF $Z(\Gamma; t) = 0$ HAS SOLUTIONS IN THE INTERIOR OF MODULI SPACE THEN USE:

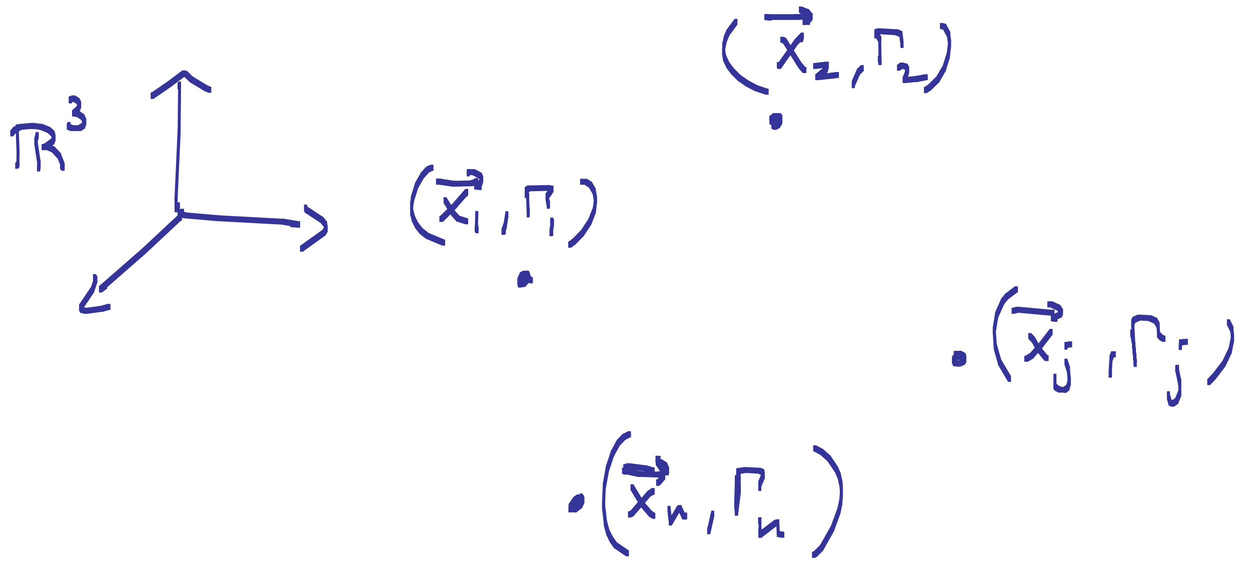
DENEF'S RULE: $\mathcal{H}(\Gamma; t) \neq 0 \iff$ $\exists$ A SPLIT ATTRACTOR FLOW (S.A.F.)

S.A.F.: A PIECEWISE ATTRACTOR FLOW, JOINED ALONG WALLS OF M.S., CONSERVING CHARGE AT THE VERTICES, TERMINATING ON R.A.P.'S:



- IF SUCH A S.A.F. EXISTS WE CAN CONSTRUCT A CORRESPONDING SOLUTION OF SUGRA.

- SPACETIME PICTURE:



- NEAR EACH POINT $\vec{x}_i$ THE SOLUTION LOOKS LIKE THE SINGLE-CENTERED SOLUTION: "BLACK-HOLE MOLECULES"

MULTICENTERED SOLUTIONS:

GENERAL BPS EQUATIONS

(1.) $$ds^2 = -e^{2U}(dt+\omega)^2 + e^{-2U} d\vec{x}^2$$

$$U = U(\vec{x}), \quad \vec{x} \in \mathbb{R}^3$$

(2.) CHOOSE A HARMONIC MAP

$$H : \mathbb{R}^3 \longrightarrow H^{ev}(X, \mathbb{R})$$

$$H(\vec{x}) = \sum_j \frac{\Gamma_j}{|\vec{x} - \vec{x}_j|} + H_\infty$$

$$\boxed{2e^{-U}\,\mathrm{Im}(e^{-i\alpha}\omega) = -H(\vec{x})} \Longrightarrow$$

(a.) $H(\vec{x})$ completely fixed,

(b.) $$e^{-2U(\vec{x})} = S(H(\vec{x}))$$

(3.) $*_3 d \mathcal{H} = \langle dH, H \rangle$

$\Longrightarrow$ INTEGRABILITY CONDITION:

$$\forall i \quad \sum_{\substack{j \\ j \neq i}} \frac{\langle \Gamma_i, \Gamma_j \rangle}{|\vec{x}_i - \vec{x}_j|} = 2 \left. \frac{\operatorname{Im}(\overline{Z(\Gamma)} Z(\Gamma_i))}{|Z(\Gamma)|} \right|_{t_\infty}$$

SUGRA SOLUTION EXISTS $\Longleftrightarrow$

$\forall \vec{x} \in \mathbb{R}^3$:

$t(\vec{x}) \in \mathcal{M}_{VM}$ &

$$\pi e^{-2U(\vec{x})} = S(H(\vec{x})) \geq 0$$

(A VERY NONTRIVIAL CONDITION TO CHECK ...)

SPLIT ATTRACTOR CONJECTURE (DENEF)

(a.) (COMPONENTS OF MODULI OF) MULTICENTERED SOLUTIONS ARE IN $1 \leftrightarrow 1$ CORRESPONDENCE WITH S.A.F.'S.

(b.) FOR A FIXED (t_∞, Γ) THERE ARE A FINITE NUMBER OF S.A.F.'S

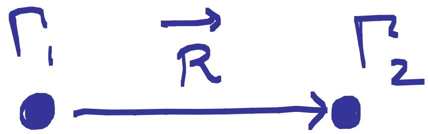
- USEFUL BECAUSE CHECKING $S(H(\vec{x})) > 0$ IS DIFFICULT

- $\mathcal{H}_{BPS}$ IS PARTITIONED BY SPLIT ATTRACTOR FLOWS

- $\exists$ SOME INTERESTING OPEN PROBLEMS HERE ....
  * QUANTUM MIXING BETWEEN DIFFERENT AF TREES
  * USEFUL EXISTENCE CRITERION FOR SCALING SOLUTIONS.

C. DERIVATION OF PRIMITIVE WCF:

CONSIDER BOUNDSTATE OF TWO PRIMITIVE CHARGES:



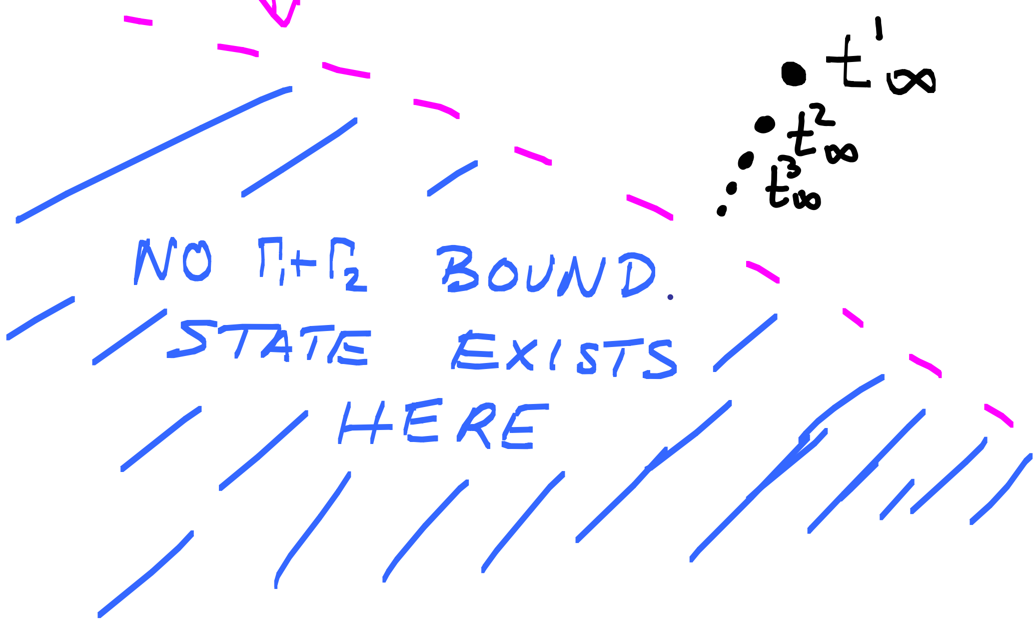
$$R = \frac{1}{2}\langle \Gamma_1, \Gamma_2 \rangle \frac{|Z_1 + Z_2|_\infty}{\mathrm{Im}(Z_1 \bar{Z}_2)_\infty}$$

- NOTE $\Rightarrow$ $\langle \Gamma_1, \Gamma_2 \rangle \mathrm{Im}(Z_1 \bar{Z}_2)_\infty > 0$

- NOTE THAT BY CHANGING t_∞ WE CAN MAKE $\mathrm{Im}(Z_1 \bar{Z}_2)|_{t_\infty} \to 0$ WHILE $|Z_1 + Z_2|_{t_\infty} \neq 0$

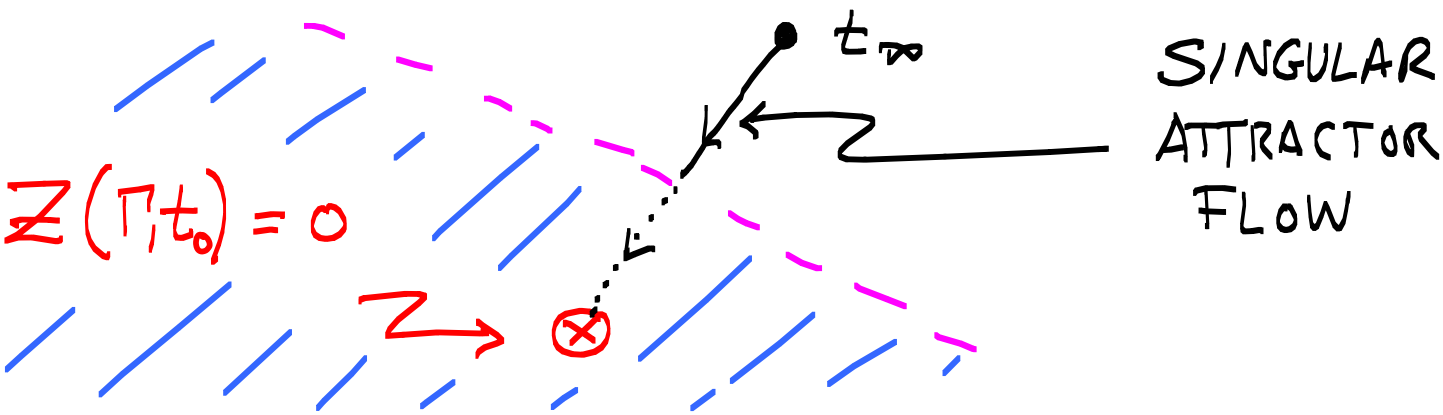
- INDEED $\mathrm{Im}(Z_1 \bar{Z}_2)|_{t_\infty} \to 0$ IF t_∞ APPROACHES A WALL $MS(\Gamma_1, \Gamma_2)$

$$MS(\Gamma_1, \Gamma_2) := \left\{ t \in \mathcal{M}_{VM} \;\middle|\; \frac{Z_1}{Z_2} \in \mathbb{R}_+ \right\}$$



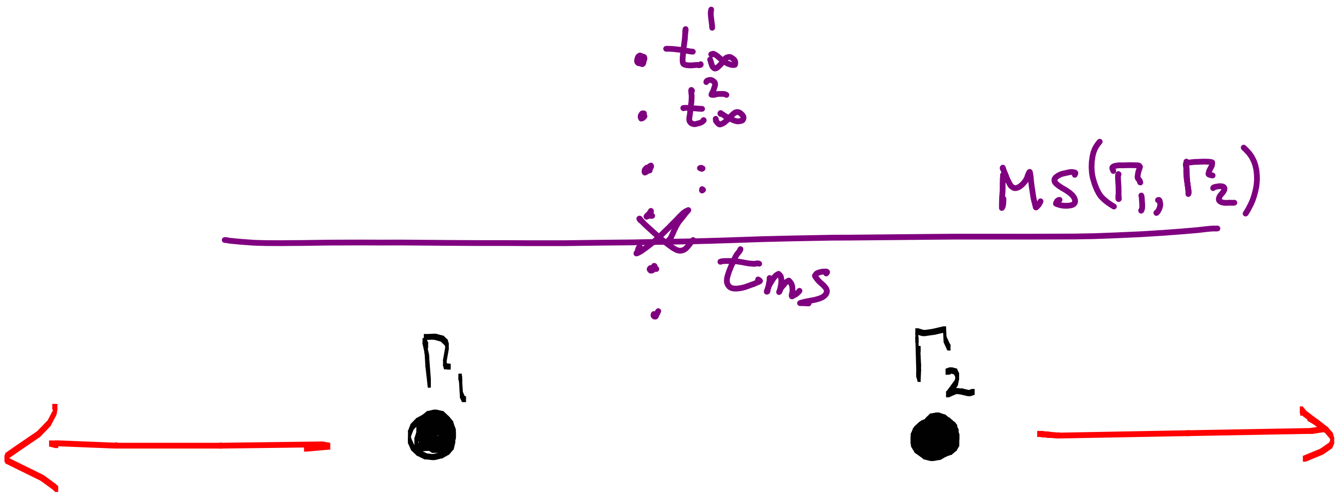
CHANGE BC's @ $r = \infty$ $\Rightarrow$ $R_{12} \longrightarrow \infty$

IF $Z(\Gamma; t_0) = 0$ THEN t_0 IS IN THE BLUE REGION.



MACROSCOPIC ARGUMENT FOR WCF:

$$R_{12} = \frac{1}{2}\langle\Gamma_1,\Gamma_2\rangle \frac{|Z_1+Z_2|_\infty}{\mathrm{Im}(Z_1\bar{Z}_2)_\infty}$$



ELECTROMAGNETIC FIELD OF TWO DYONS HAS SPIN:

$$J_{12} = \frac{1}{2}\left(|\langle\Gamma_1,\Gamma_2\rangle| - 1\right)$$

quantum correction

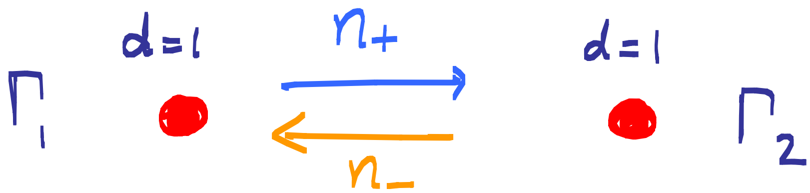
LOCALITY $\Rightarrow$ FOR Γ_1, Γ_2 PRIMITIVE:

STATES LOST FROM $\mathcal{H}(\Gamma; t_\infty)$ ARE

$$(J_{12}) \otimes \mathcal{H}(\Gamma_1; t_{ms}) \otimes \mathcal{H}(\Gamma_2; t_{ms})$$

MICROSCOPIC ARGUMENT FOR WCF:

WHEN $\vartheta = \arg Z_2/Z_1 \longrightarrow 0$, MODEL LIGHT D.O.F. BY A QUIVER GAUGE THRY:



TRANSLATION TO SUPERGRAVITY:

STABILITY DATA: $(\vartheta, -\vartheta)$

$n_+ - n_- = I_{12}$

GENERICALLY $n_+ = 0$ or $n_- = 0$.

SUPPOSE $n_- = 0$:

$\vartheta > 0 \qquad \mathcal{M} = \mathbb{C}P^{n_+ - 1}$

$\vartheta < 0 \qquad \mathcal{M} = \emptyset$

$$\Delta \mathcal{H} = H^*\left(\mathbb{C}P^{n_+ - 1}\right)$$

spin(3) $\approx$ Lefshetz

QUIVER QUANTUM MECHANICS

0+1 SUSY QED WITH

1 VM $(A_0, \vec{x}, \lambda)$

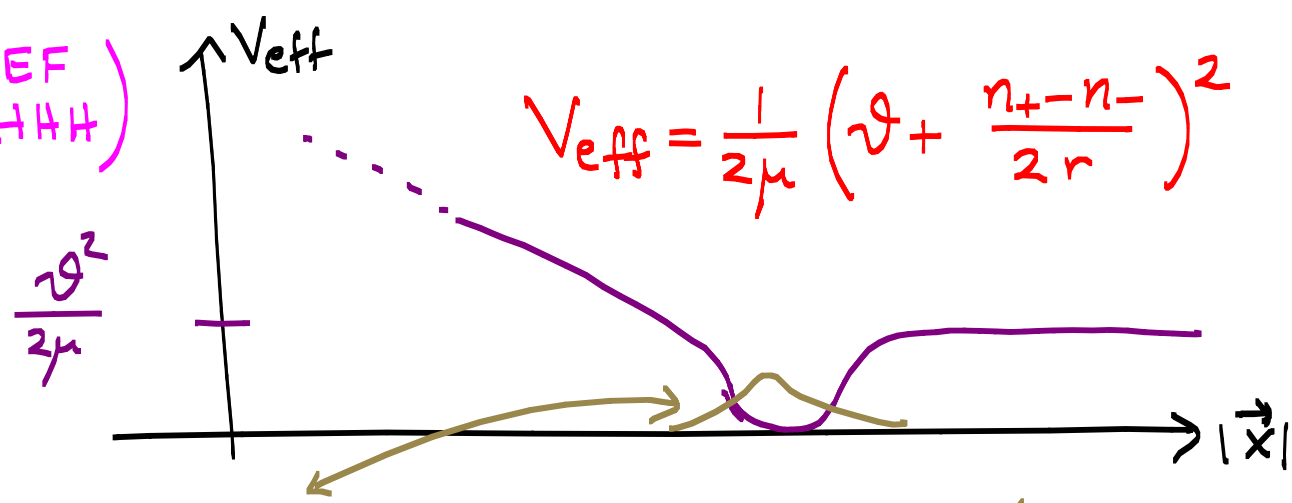
$n_\pm$ CM's $(\vec{\phi}_\pm, \vec{\psi}_\pm)$ CHARGE ± 1

SMALL $|\langle \vec{x} \rangle| \Rightarrow$ HIGGS BRANCH = MODULI OF STABLE QUIVER REP'S

LARGE $|\langle \vec{x} \rangle| \Rightarrow$ INTEGRATE OUT $\vec{\phi}_\pm \Rightarrow$

(DENEF QQHHH)

$$V_{eff} = \frac{1}{2\mu}\left(\vartheta + \frac{n_+ - n_-}{2r}\right)^2$$



$(n_+ - n_-)$ BPS STATES OF SPIN $\frac{1}{2}(n_+ - n_- - 1)$

$\vartheta < 0$	$\vartheta = 0$	$\vartheta > 0$
n_+ HIGGS BR. BPS STATES	$(n_+ - n_-)$ COULOMB BR. $\rightarrow \infty$	n_- HIGGS BR. BPS STATES

D. DERIVATION OF SEMI-PRIMITIVE WCF

HALO STATES

SUPPOSE $\langle \Gamma_1, \Gamma_2 \rangle \neq 0$,

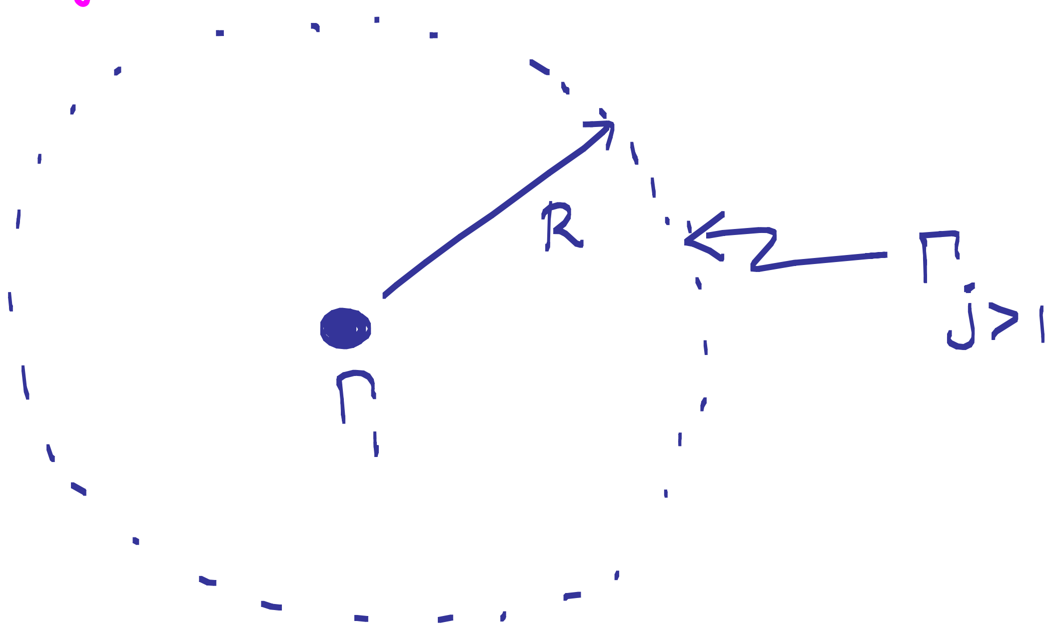
$$\Gamma_j = \lambda_j \Gamma_2 \qquad \lambda_j > 0, j = 2, \cdots, N$$

ARE ALL MUTUALLY LOCAL

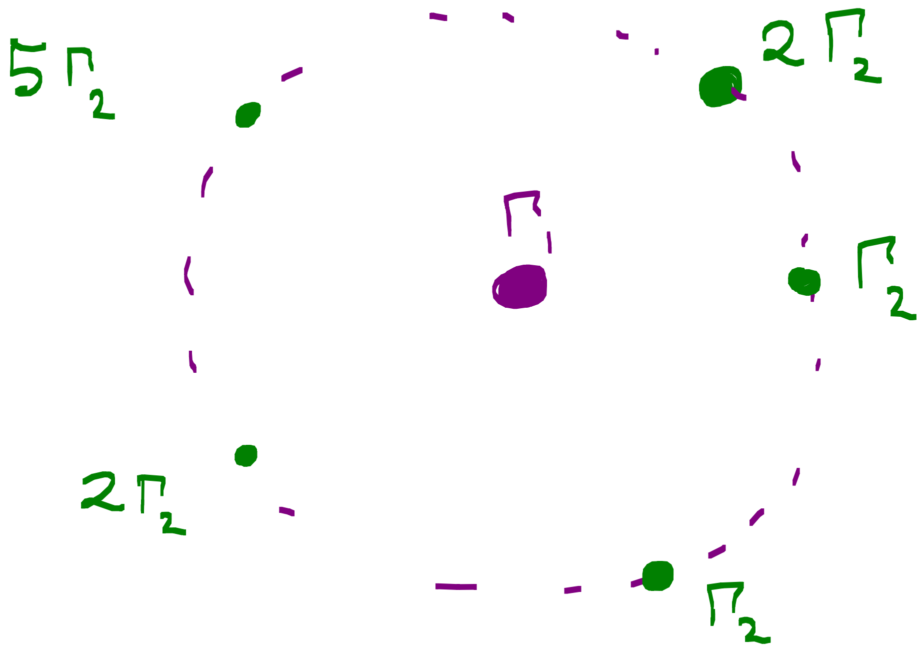
INTEGRABILITY CONDITIONS SAY

$$j \geq 2: \quad \frac{\langle \Gamma_j, \Gamma_1 \rangle}{|\vec{x}_j - \vec{x}_1|} = 2 \frac{\mathrm{Im}\left(Z(\Gamma_j)\overline{Z(\Gamma)}\right)}{|Z(\Gamma)|}$$

$\Rightarrow$ ALL $|\vec{x}_j - \vec{x}_1|$ ARE EQUAL



CROSS $MS(\Gamma_1, \Gamma_2)$: HALO RADIUS $\nearrow \infty$



THE PARTICLES IN THE HALO GENERATE A FOCK SPACE WITH

$(J_{\Gamma_1, k\Gamma_2}) \otimes \mathcal{H}(k\Gamma_2; t_{ms})$ CREATION OPERATORS OF CHARGE $k\Gamma_2$

ALL WALLS $W(\Gamma_1, N\Gamma_2)$ COINCIDE $\Rightarrow$ CROSSING A WALL WE LOSE ENTIRE FOCK SPACE:

$$\Omega(\Gamma_1) + \sum_{N\geq 1} \Delta\Omega(\Gamma \to \Gamma_1 + N\Gamma_2)\, u^N$$

$$= \Omega(\Gamma_1) \prod_{k>0} \left(1 - (-1)^{k\langle\Gamma_1,\Gamma_2\rangle} u^k\right)^{|\langle\Gamma_1, k\Gamma_2\rangle|\Omega(k\Gamma_2)}$$

4. D6D2D0 SYSTEM

AS EXPLAINED AT THE END, A KEY INGREDIENT IN OSV IS THE SYSTEM:

1 D6 BRANE WRAPPING X,

BOUND TO D2 & D0 BRANES IN X.

$$H^0 \oplus H^2 \oplus H^4 \oplus H^6 \ni \Gamma = (p^0, P, Q, q_0)$$

D6 D4 D2 D0

CONSIDER: $\Gamma(\beta, n) := \Gamma = (1, 0, -\beta, n)$

$\beta = P.D.[\sigma] \quad \sigma \subset X$ HOLOMORPHIC CURVE

CHARGE OF (THE DUAL OF) AN IDEAL SHEAF:

$$ch\, \mathcal{I} \sqrt{\hat{A}} = 1 - \beta + n\, dV$$

CONSIDER BINDING THESE

TO D2D0 PARTICLES WITH CHARGE:

$$\Gamma_h = (0, 0, -\beta_h, n_h)$$

PLOT MARGINAL STABILITY CURVE

$$Z(\Gamma(\beta,n);t) = \lambda Z(\Gamma_h;t) \qquad \lambda \in \mathbb{R}_+$$

$$Z(\Gamma,t) = \frac{\langle \Gamma, \omega \rangle}{\sqrt{\langle \omega, \omega^* \rangle}}$$

SUGRA REGIME: $\Omega = -e^{t}$

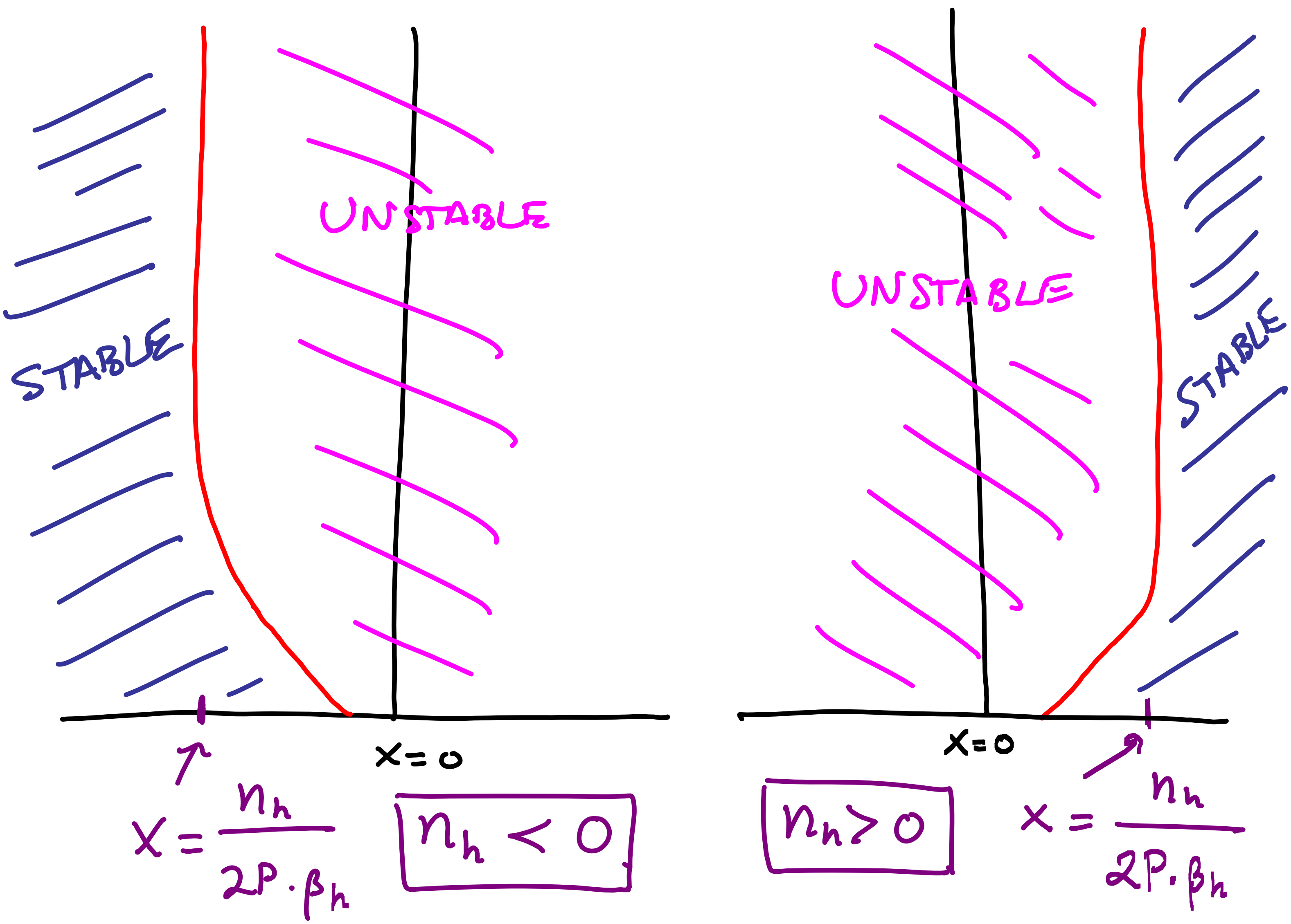
$$t = B + iJ$$

$$\boxed{\frac{t^3}{6} - \beta \cdot t - n = \lambda\left(-\beta_h \cdot t - n_h\right)} \qquad \lambda \in \mathbb{R}_+$$

THESE WALLS EXTEND TO ∞ IN THE KÄHLER CONE!

SET $t = z\underline{P}$ $\quad \underline{P} \in \mathcal{K}$

$z = x + iy$



CONSIDER THE HALO BOUNDSTATES WITH CENTRAL PARTICLE $\Gamma(\beta,n)$ AS WE INCREASE THE B-FIELD

$B = xP \qquad x$ INCREASES

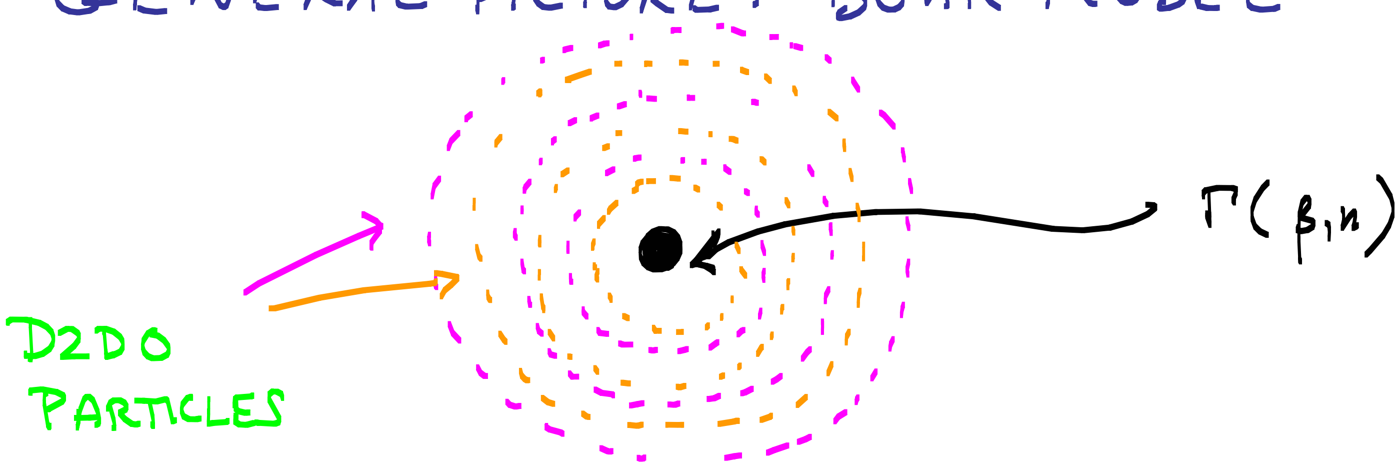
HALOS OF D2D0 PARTICLES $(0,0,-\beta_h,n_h)$. APPEAR & DISAPPEAR.

FOR $x > 0$

ALL $n_h < 0$ STATES HAVE DECAYED.

AS $x \longrightarrow +\infty$ WE MOVE INTO THE STABLE REGION FOR ALL $n_h > 0$, AND EVER LARGER "ATOMS" BECOME STABLE

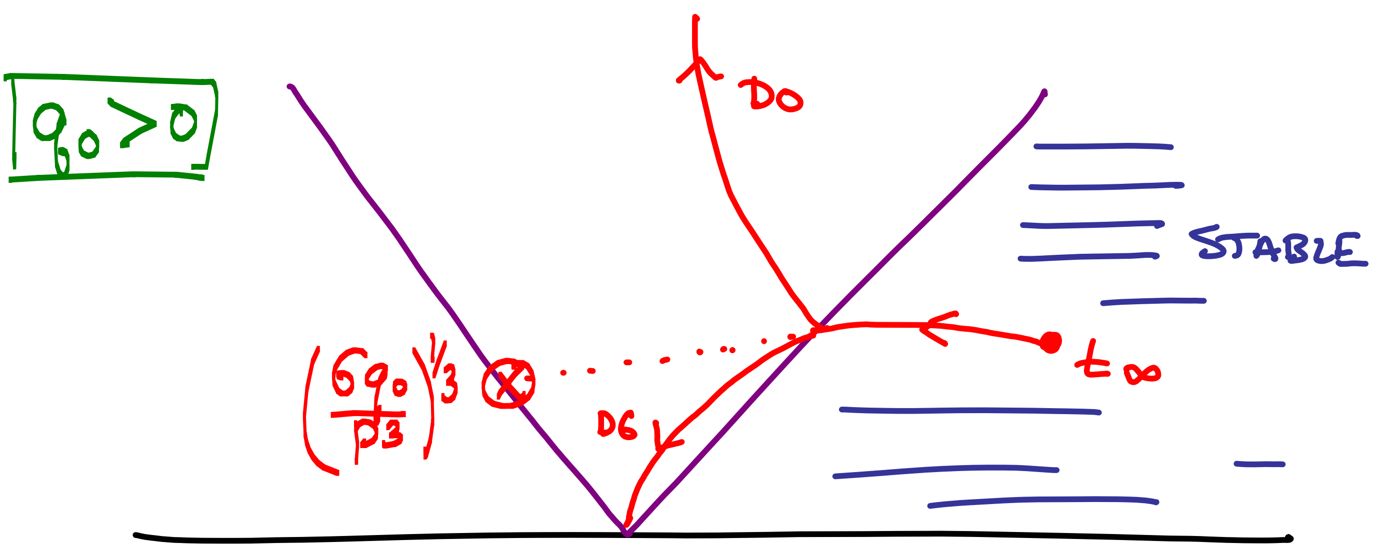
GENERAL PICTURE: BOHR MODEL

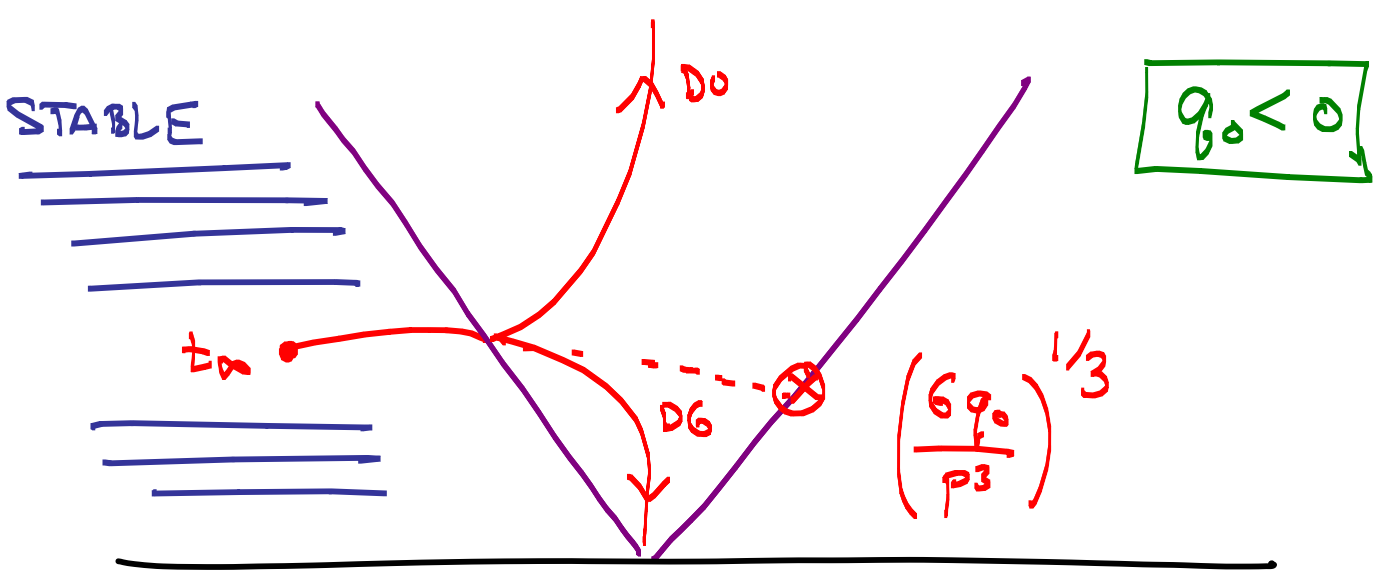


WHEN $\beta_h = 0$ WALLS LOOK DIFFERENT

$$\Gamma = \underbrace{1}_{\Gamma_1} + \underbrace{q_0 \, dV}_{\Gamma_2} \qquad Z = \frac{t^3}{6} - q_0$$

SET $t = (x+iy)P \implies$ ZERO @ $t = \left(\frac{6q_0}{p^3}\right)^{1/3} p$





INTRODUCE GENERATING FUNCTION

$$Z_{D6D2D0}(u,v;t) := \sum_{n,\beta} \Omega(\Gamma(\beta,n);t)\, u^n v^\beta$$

SEMI-PRIMITIVE WALL-CROSSING FORMULA:

CONTRIBUTION OF FOCK SPACE GENERATED BY $\Gamma_h = -\beta_h + n_h dV$ CROSSING INTO STABLE REGION:

$$Z_{D6D2D0} \rightarrow \left(1-(-u)^{n_h} v^{\beta_h}\right)^{|n_h|\, n^0_{\beta_h}} Z_{D6D2D0}$$

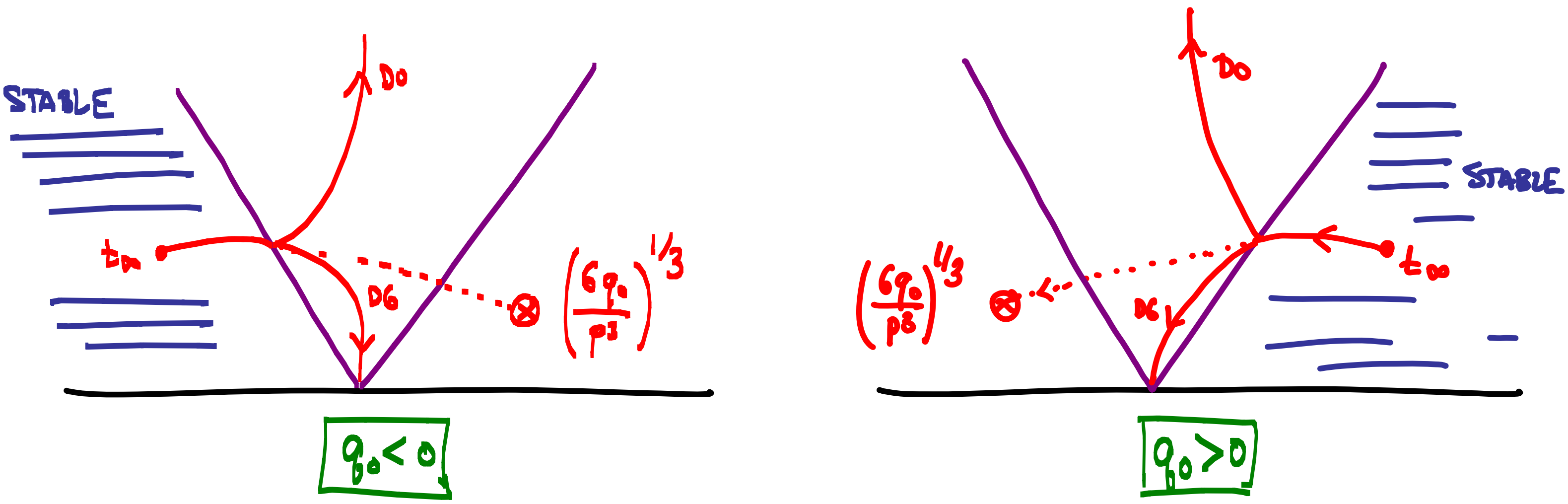
$$\Omega(-\beta_h + n_h dV) = \sum_{m_L, m_R} (-1)^{2m_L+2m_R} N^{m_L m_R}_{\beta_h}$$

$$= n^0_{\beta_h}$$

"SPIN ZERO GV INVARIANT" $(\beta_h \neq 0)$

EXAMPLE: D6D0

$$Z_{D6D0}(u) = \sum \Omega(1+q_0 dV; t)\, u^{q_0}$$



$$\Omega(q_0 dV) = -\chi(X)$$

$$Z_{D6D0}(u) = \begin{cases} \left(M(-u)\right)^{\chi(X)} & \arg z < \frac{\pi}{3} \\ 1 & \frac{\pi}{3} < \arg z < \frac{2\pi}{3} \\ \left(M(-u^{-1})\right)^{\chi(X)} & \frac{2\pi}{3} < \arg z \end{cases}$$

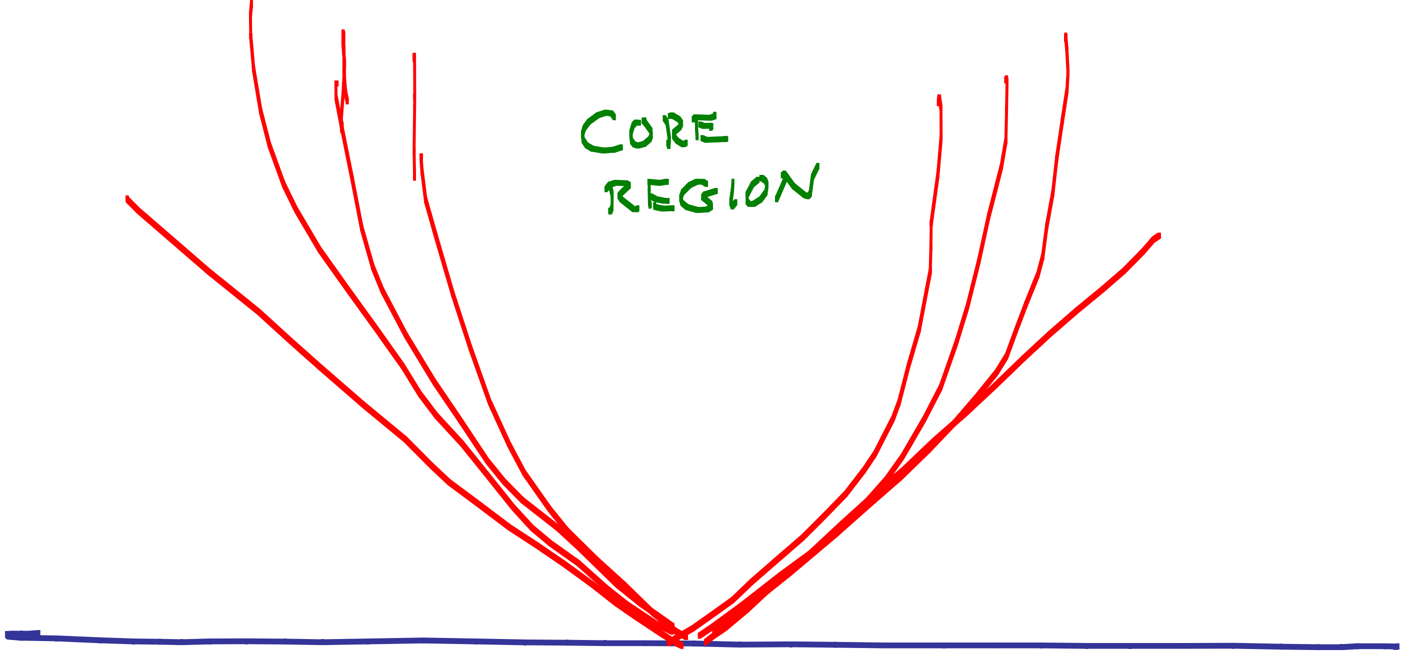
$$M(u) := \prod_{k \geq 1} (1-u^k)^{-k}$$

SIMILARLY, WALL-CROSSINGS FOR THE FULL Z_{D6D2D0} AS $x \to \infty$ BUILD UP AN INFINITE PRODUCT SIMILAR TO THE INFINITE PRODUCT FORM OF $Z_{DT}(u,v)$

ON THE OTHER HAND, AN ARGUMENT FROM M-THEORY [Dijkgraaf, Verlinde, Vafa; Denef, Moore] IMPLIES:

$$\lim_{x \to +\infty} Z_{D6D2D0}(u,v;zP) = Z_{DT}(u,v)$$

$$\lim_{x \to -\infty} Z_{D6D2D0}(u,v;zP) = Z_{DT}(u^{-1},v)$$



- STATES IN CORE REGION ARE COMPLICATED BOUNDSTATES

- PRODUCT OF WALL-CROSSINGS $\Rightarrow$

$$Z_{DT}^{1,r=0}(u,v) = \prod_{\beta>0,k>0} \left(1-(-u)^k v^\beta\right)^{k n_\beta^0}$$

- LIMIT FOR $x \longrightarrow +\infty$:

$$Z_{DT}^{1}(u,v) = \underbrace{Z_{DT}^{1,r=0}(u,v)}_{\text{HALOS}} \; \underbrace{Z_{DT}^{1,r>0}(u,v)}_{\text{CORES}}$$

$$Z_{DT}^{1,r>0}(u,v) = \prod_{\substack{\beta>0,k>0\\ r>0}} \prod_{\ell=0}^{2r-2} \left(1-(-u)^{r-\ell-1} v^\beta\right)^{(-1)^{r+\ell}\binom{2r-2}{\ell} n_\beta^r}$$

APPLICATION TO LOCAL CALABI-YAU

(M. AGANAGIC, G.M, D. JAFFERIS)

B. SZENDROI HAS PROPOSED THAT THE NONCOMMUTATIVE CONIFOLD SHOULD HAVE Z_{DT} GIVEN BY:

$$Z = \left(M(-u)\right)^2 \prod_{k=1}^{\infty}\left(1-(-u)^k v\right)^k \prod_{k=1}^{\infty}\left(1-(-u)^k v^{-1}\right)^k$$

$$\uparrow \qquad\qquad \uparrow$$

$$D2 \qquad\qquad \overline{D2}$$

WE CAN REDERIVE THIS RESULT USING THE SEMI-PRIMITIVE WCF.

5. D4-D2-D0 SYSTEM

THE BEST-STUDIED SYSTEM FOR OSV + B.H. ENTROPY IS $p^0 = 0$.

$$\Gamma = P + Q + q_0 dV$$

A. MACROSCOPIC VIEWPOINT

Γ HAS A REG. ATTRACTOR POINT IFF:

P IN KÄHLER CONE & $\hat{q}_0 < 0$

$$\hat{q}_0 := q_0 - \frac{1}{2}\left(D_{ABC}P^C\right)^{-1} Q_A Q_B$$

SINGLE-CENTERED SOLN'S ARE BLACK HOLES

HORIZON AREA $= 4\, S(\Gamma) = 4\pi |Z_*(\Gamma)|^2$

$$S(\Gamma) = \frac{2\pi}{\sqrt{6}} \cdot \sqrt{-\hat{q}_0 \, \mathcal{X}(P)}$$

$\mathcal{X}(P) := P^3 + c_2 \cdot P > 0$ FOR $P \in$ KÄHLER CONE

WE WOULD LIKE TO COMPARE

$S(\Gamma)$ WITH $\log \Omega(\Gamma; t)$

AS IN STROMINGER-VAFA & M-S-W.

KEY QUESTION: WHICH VALUE OF t?

FOR MICROSCOPIC DERIVATION MUST USE LARGE RADIUS:

$$\text{“}\ \Omega(\Gamma)_\infty := \lim_{J \to \infty} \Omega(\Gamma; B + iJ)\ \text{”}$$

SURPRISE! WHEN $h^{1,1}(X) > 1$ THERE ARE SPLITTINGS @ ∞ :

$$\Gamma = P + Q + q_0 dV$$

$$= (P' + Q' + q_0' dV) + (P'' + Q'' + q_0'' dV)$$

WITH: $\sqrt{-\hat{q}_0''(P'')^3} > \sqrt{-\hat{q}_0 P^3}$

$\Rightarrow$ EVEN THE LEADING ORDER ENTROPY IS CHAMBER DEPENDENT

[E. ANDRIYASH + G.M.]

- FOR $\Gamma = P + Q + q_0\, dV$,

$P \in$ KÄHLER CONE, $\exists$ DISTINGUISHED CHAMBER:

$$\Omega(\Gamma)_\infty := \lim_{\lambda \to \infty} \Omega(\Gamma; B + i\lambda P)$$

CLAIM: LIMIT EXISTS & IS B-INDEPENDENT

(FINITENESS OF ATTRACTOR FLOW TREES)

HENCEFORTH WORK IN THIS CHAMBER.

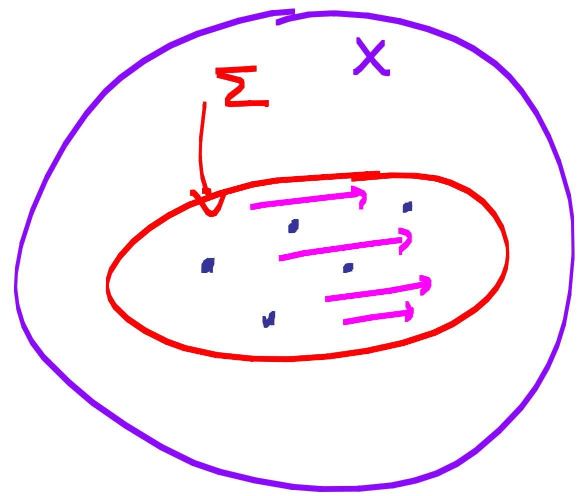
B. MICROSCOPIC VIEWPOINT

FOR LARGE J: SINGLE D4 WRAPS $\Sigma \in |P|$

$\chi(P) = P^3 + c_2 \cdot P$ = EULER CHARACTER OF Σ

FLUX $F \in H^2(\Sigma, \mathbb{Z})$

AND N $\overline{D0}$'s



COMPUTE INDUCED RR CHARGES:

D2: $Q = (\imath_\Sigma)_*(F)$

D0: $$\boxed{\hat{q}_0 = \frac{\chi(P)}{24} + \frac{1}{2}(F^-)^2 - N}$$

SUSY $\Rightarrow$ $N \geq 0$, $F^{2,0} = 0 \Rightarrow (F^-)^2 \leq 0 \Rightarrow$

$$\hat{q}_0 \leq (\hat{q}_0)_{max} = \frac{\chi(P)}{24}$$

N.B. SIGN CONVENTION

$\mathcal{M}(P,F,N) :=$ MODULI OF SUCH D4'S

ROUGHLY:

$$\begin{array}{ccc} \mathrm{Hilb}^N(\Sigma) & \hookrightarrow & \mathcal{M}(P,F,N) \\ \downarrow & & \downarrow \\ \underset{\text{Smooth}}{\Sigma} & \longrightarrow & \{\Sigma \in |P| \ \big| \ F \in H^{1,1}(\Sigma)\} \end{array}$$

"MODULI OF STABLE OBJECTS $\mathcal{E}$ IN THE DERIVED CATEGORY WITH SPECIFIED CHERN CLASSES"

$$\mathrm{ch}\,\mathcal{E}\sqrt{\hat{A}} = P + Q + q_0 \qquad (*)$$

$$= \bigcup_{F,N:(*)} \mathcal{M}(P,F,N)$$

$$d(F,N) := (-1)^{\dim \mathcal{M}} \chi\left(\mathcal{M}(P,F,N)\right)$$

$\Omega(\Gamma)_\infty$ = FINITE SUM OF $d(F,N)$

C. MODULAR GENERATING FUNCTION

NOW WE WANT TO ASSEMBLE "BLACK HOLE" DEGENERACIES IN A MODULAR GENERATING FUNCTION.

SUSY PARTITION FUNCTION FOR D4:

$$Z_{D4D2D0} \sim \mathrm{Tr}(-1)^{2J_3} e^{-\beta H - 2\pi i q_\Lambda C^\Lambda}$$

CAN BE WRITTEN $Z_P(\tau, \bar{\tau}, C) :=$

$$\sum_{F,N} d(F,N) \exp\left\{ -2\pi i \tau \hat{q}_0 - 2\pi i \bar{\tau} \tfrac{1}{2}(F^+)^2 - 2\pi i F \cdot \left(C + \tfrac{P}{2}\right) \right\}$$

$$\tau = \oint C^1 + i\beta \in \mathbb{H}; \qquad C \in \mathbb{Z}^*_\Sigma \left(H^2(X)\right)$$

U-DUALITY $\Rightarrow$

$Z_P(\tau,\bar\tau,C)$ IS A JACOBI FORM $\Rightarrow$

$$Z_P(\tau,\bar\tau,C) = \sum_{\mu\in L^*/L} H_\mu(\tau)\, \underbrace{\Theta_{\mu,L}(\tau,\bar\tau,C)}_{\text{SIEGEL-NARAIN}}$$

$$L := 2^*_\Sigma\left(H^2(X,\mathbb{Z})\right) \subset \underbrace{H^2(\Sigma;\mathbb{Z})}_{\text{SELF-DUAL}}$$

[$\ell\in L$ IS ALWAYS IN $H^{1,1}(\Sigma) \Rightarrow$

$d(F+\ell,N) = d(F,N)\quad \forall \ell\in L$]

- $H_\mu(\tau)$ IS A VECTOR-VALUED NEARLY HOLO. MODULAR FORM OF WEIGHT $w = -1 - \frac{h^{1,1}(X)}{2}$ AND MULTIPLIER SYSTEM M^* DUAL TO THAT OF $\Theta_{\mu,L}$

POLAR STATES

- $w < 0 \implies H_\mu$ IS DETERMINED BY ITS POLAR TERMS.

SUPPRESS μ-INDEX FOR SIMPLICITY:

$$H(\tau) = \sum_{\hat{q}_0} \Omega(\Gamma)_\infty \, e^{-2\pi i \hat{q}_0 \tau}$$

$$= \underbrace{\sum_{0 < \hat{q}_0 \le \frac{\chi(P)}{24}} (\cdots\cdots)}_{\text{POLAR}} + \underbrace{\sum_{-\infty < \hat{q}_0 \le 0} (\cdots\cdots)}_{\text{NONPOLAR}}$$

$\implies$ BLACK HOLE DEGENERACIES ARE COMPLETELY DETERMINED BY THE POLAR DEGENERACIES.

D. MACROSCOPIC POLAR STATES

IF $\Gamma = (0, P, Q, q_0) = P + Q + q_0 dV$

IS POLAR: $0 < \hat{q}_0 \leq (\hat{q}_0)_{max}$

THEN IT CANNOT HAVE A REGULAR ATTRACTOR POINT.

INDEED $S(\Gamma) = \frac{2\pi}{\sqrt{6}} \cdot \sqrt{-\hat{q}_0 \, \chi(P)}$

SO NO SINGLE-CENTERED SOLUTION

BUT $H(\tau)$ HAS $w < 0 \Rightarrow$ SOME POLAR DEGENERACIES ARE NONZERO

$\Rightarrow$ THESE MUST BE REALIZED AS SPLIT ATTRACTOR STATES.

SIMPLE EXAMPLE

PURE D4 : $\Gamma = P + q_0 dV$

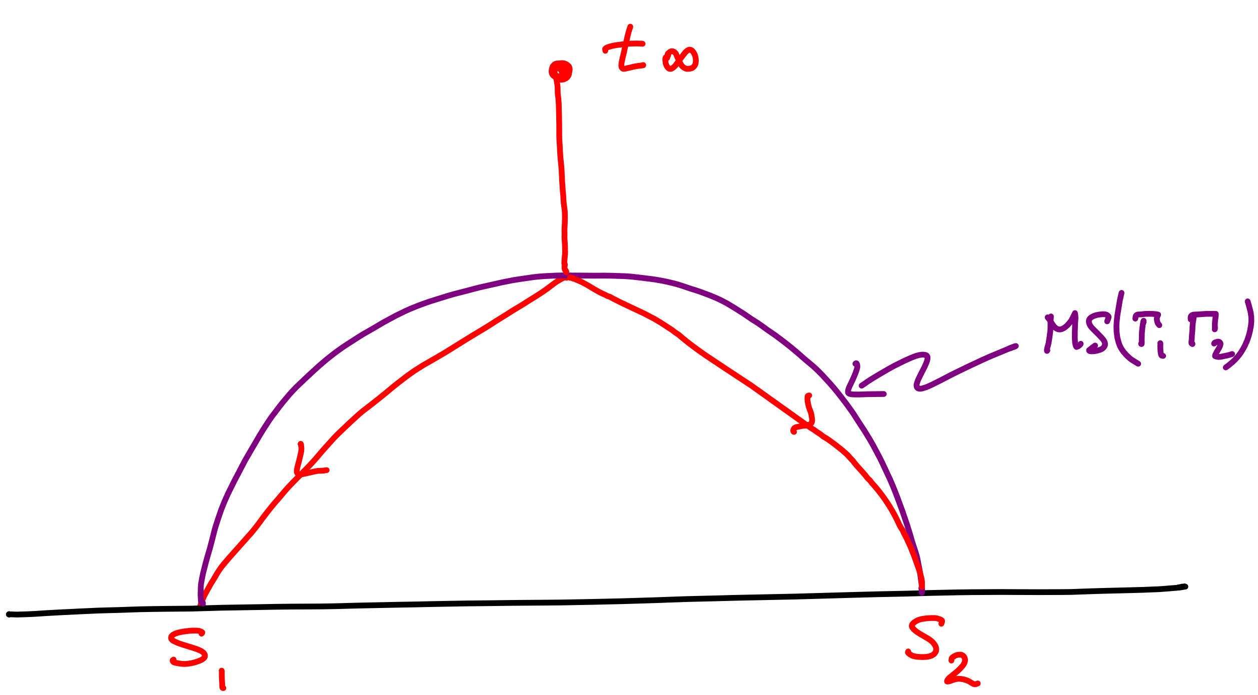
WITH $q_0 = \hat{q}_0 = (\hat{q}_0)_{max} = \frac{\chi(P)}{24}$

FIND ONLY ONE SPLITTING

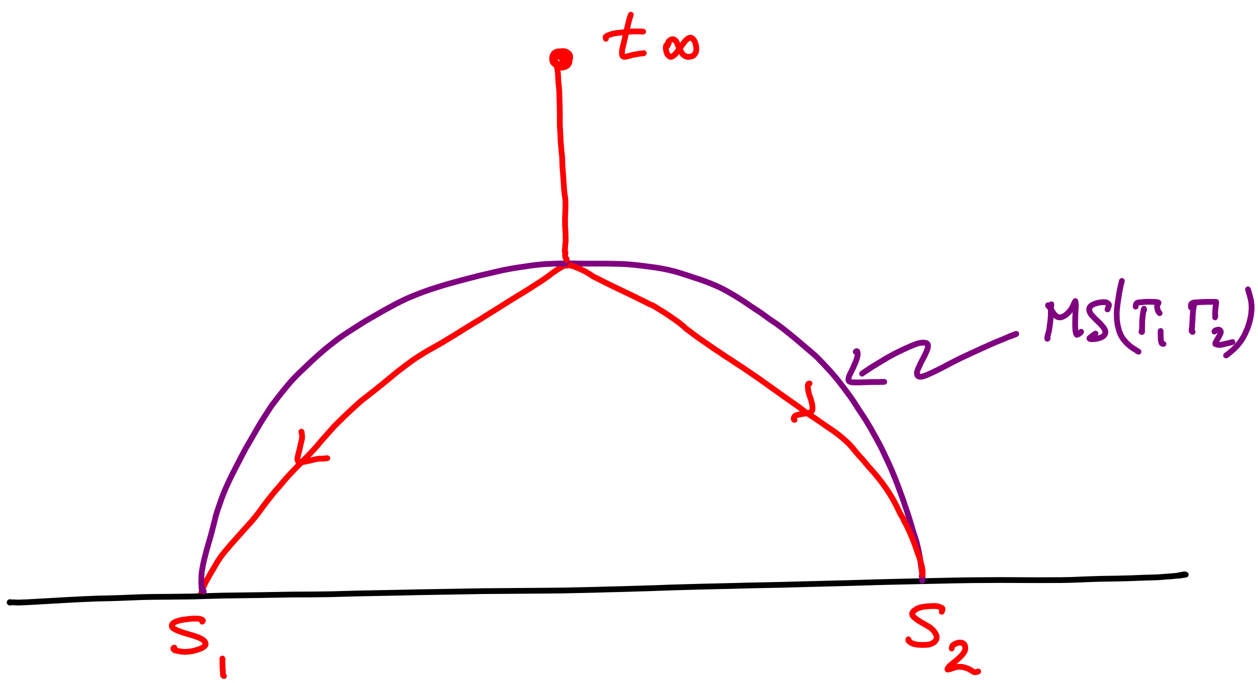
$$\Gamma = P + q_0 dV = \Gamma_1 + \Gamma_2$$

$$= \underbrace{e^{S_1}\left(1 + \frac{c_2(X)}{24}\right)}_{\text{1 D6 WITH FLUX} = S_1} - \underbrace{e^{S_2}\left(1 + \frac{c_2(X)}{24}\right)}_{1\,\overline{\text{D6}}\text{ w/ FLX } S_2}$$

$$S_1 - S_2 = P$$



MOREOVER – YOU CAN COMPUTE THE POLAR DEGENERACY:



$$\Omega(\Gamma, t_\infty) = (-1)^{I_{12}-1} |I_{12}| \, \Omega(\Gamma_1)\Omega(\Gamma_2) = (-1)^{I_{12}-1} |I_{12}|$$

$$I_{12} = \langle \Gamma_1, \Gamma_2 \rangle = \frac{P^3}{6} + \frac{c_2(X)\cdot P}{12}$$

INDEED = THE CORRECT ANSWER FOR

χ (MODULI OF PURE D4) = $\chi(|P|)$

E. EXTREME POLAR STATE CONJECTURE

DESCRIBING THE SPLIT ATTRACTOR FLOWS FOR $0 < \hat{q}_0 < \frac{\chi(P)}{24}$ IS MUCH MORE COMPLICATED...

IN GENERAL, POLAR STATES CAN BE VERY COMPLICATED SPLIT ATTRACTORS, REALIZED IN MANY DIFFERENT WAYS....

BUT IN THE LIMIT $P \rightarrow \infty$ WE CAN SAY SOMETHING

EXTREME POLAR STATES

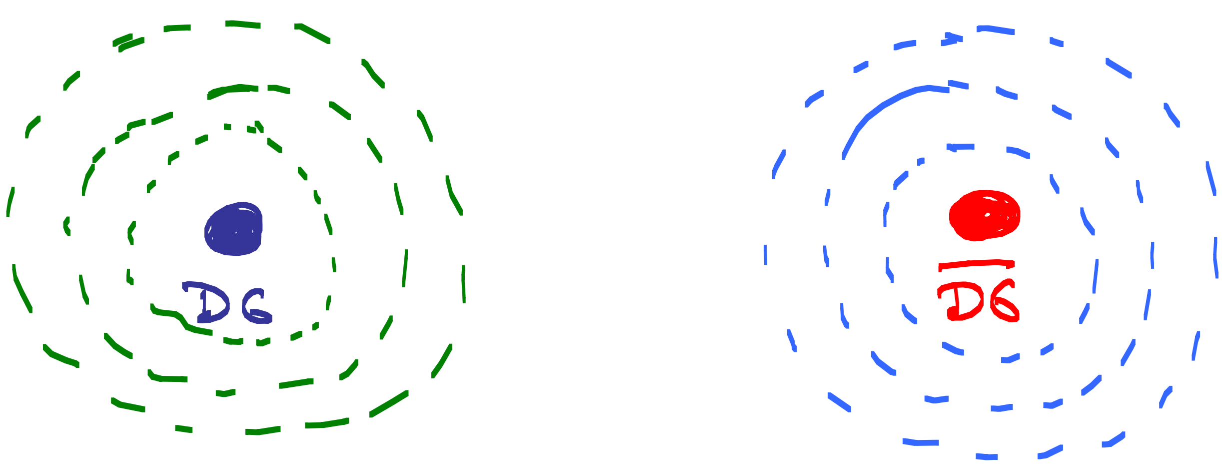
$$H^{POLAR}(\tau) = |I_p|\, e^{-2\pi i \tau \frac{\chi(p)}{24}} + \cdots \quad + O\left(e^{\frac{-2\pi i \tau}{|p|}}\right)$$

"EXTREME POLAR" "BARELY POLAR"

E.P.S. CONJECTURE: $\exists\ \epsilon < 1$ SO THAT

$$\frac{\hat{q}_0^{max} - \hat{q}_0}{\hat{q}_0^{max}} < \epsilon \quad \Rightarrow$$

POLAR STATES SPLIT AS $D6\overline{D6}$ + HALOS:



$$\Gamma_1 = e^{S_1}(1 - \beta_1 + n_1 dV) \qquad \Gamma_2 = -e^{S_2}(1 - \beta_2 + n_2 dV)$$

6. ROUTE TO THE OSV CONJECTURE

A. BY THE W.C.F. THE (EXTREME) POLAR DEGENERACIES GO LIKE

$$\Omega(D6\text{-}D2\text{-}D0) \times \Omega(\overline{D6\text{-}D2\text{-}D0})$$

B. BUT BPS INVARIANTS OF THE D6-D2-D0 SYSTEM ARE RELATED TO GROMOV-WITTEN INVARIANTS COUNTING WORLDSHEET INSTANTONS IN X

SO, BY THE W.C.F. TOGETHER WITH RESULTS ON Z_{D6D2D0} THE EXTREME POLAR DEGENERACIES ARE RELATED TO

$$|Z_{TOP}|^2$$

SUGGESTING A RELATION LIKE THE OSV CONJECTURE

$$\Omega(\Gamma)_\infty = \int d\phi \; |Z_{top}(g_{top}, t)|^2 e^{-2\pi q_0 \phi}$$

- $\exists$ STRONG ARGUMENTS FOR $|\hat{q}_0| \gg P^3$
- $\exists$ POTENTIAL COUNTEREXAMPLES FOR $|\hat{q}_0| \lesssim P^3$: "ENTROPY ENIGMA"

IN THE CHARGE REGIME

$$g_{top} \sim \sqrt{\frac{-\hat{q}_0}{p^3}} \lesssim \mathcal{O}(1)$$

THE DERIVATION IN DENEF-MOORE BREAKS DOWN:

- BARELY POLAR DEGENERACIES BECOME LARGE

- CORRECTIONS TO THE CARDY FORMULA (LEADING TERM IN F.T. EXPANSION) BECOME LARGE.

....

THERE IS A GOOD PHYSICAL REASON THE DERIVATION BREAKS DOWN...

ENTROPY ENIGMA

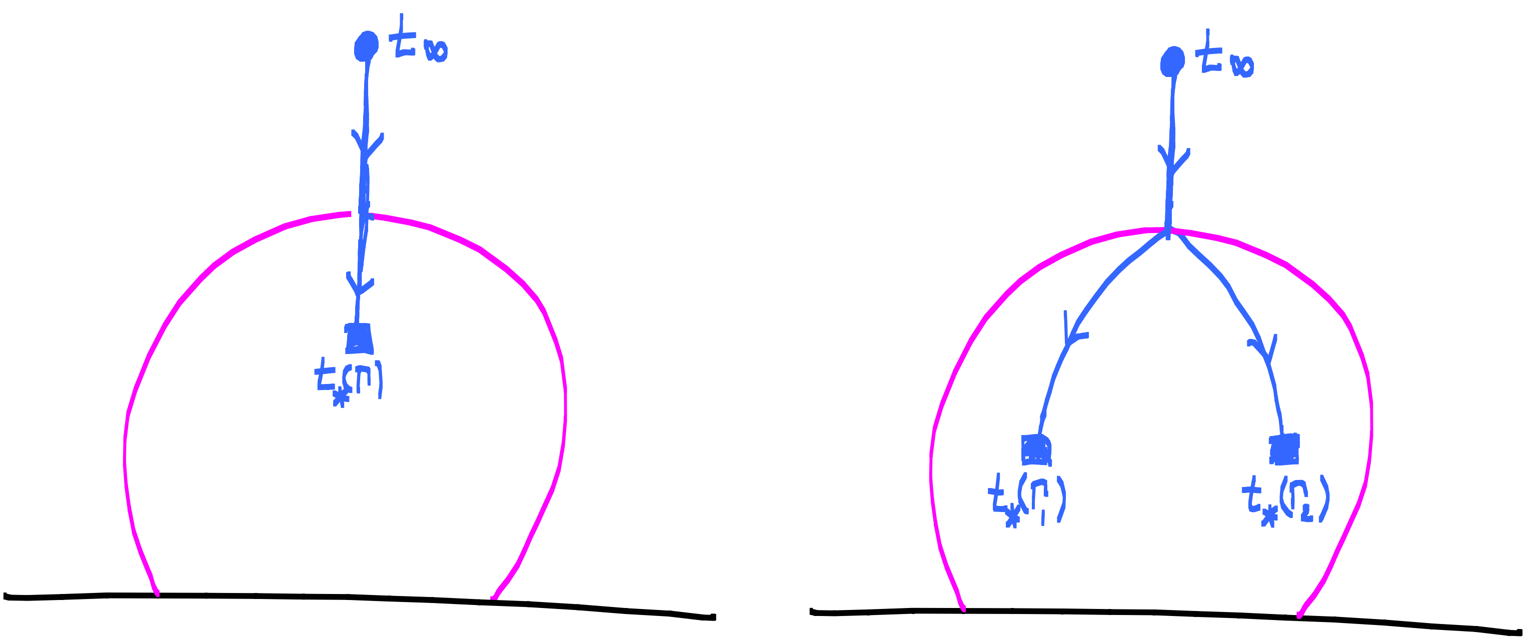
NOW CHOOSE $q_0 < 0$, P AMPLE SO

$$\Gamma = (0, P, 0, q_0)$$

HAS A REGULAR ATTRACTOR POINT

NEVERTHELESS! WE CAN CHOOSE q_0, Q_A SO THAT $\exists$ A TWO-CENTERED SOLUTION WITH $\Gamma = \Gamma_1 + \Gamma_2$

$$\Gamma_1 = (r, \tfrac{1}{2}P, Q, \tfrac{1}{2}q_0) \quad \Gamma_2 = (-r, \tfrac{1}{2}P, -Q, \tfrac{1}{2}q_0)$$



BOTH SOLUTIONS EXIST

SO... COMPARE ENTROPIES

$$S(\Gamma) \quad \text{vs.} \quad S(\Gamma_1) + S(\Gamma_2)$$

IN FACT,

$\exists$ FAMILY OF CHARGES

$$\lambda\Gamma = \lambda(0, P, 0, q_0) = \Gamma_1^\lambda + \Gamma_2^\lambda$$

$$\Gamma_1^\lambda = \left(r, \tfrac{\lambda}{2}P, \lambda^2 Q, \tfrac{\lambda}{2}q_0\right) \quad \Gamma_2^\lambda = \left(-r, \tfrac{\lambda}{2}P, -\lambda^2 Q, \tfrac{\lambda}{2}q_0\right)$$

SCALING OF ENTROPIES:

$$S(\lambda\Gamma) = \lambda^2 S(\Gamma)$$

BUT!

$$\boxed{S(\Gamma_1^\lambda) = S(\Gamma_2^\lambda) \sim \frac{(\lambda P)^3}{r} \sim \lambda^3}$$

$\Rightarrow$ MANY IMPLICATIONS FOR PHYSICS & MATHEMATICS

SOME TECHNICAL DETAILS

1. CONSTRUCT A FAMILY OF 2-CENTERED

$$\tilde{\Gamma}_1^{\lambda} = \left(r, \frac{P}{2}, Q, \lambda^{-2}\frac{q_0}{2}\right)$$

$$\tilde{\Gamma}_2^{\lambda} = \left(-r, \frac{P}{2}, -Q, \lambda^{-2}\frac{q_0}{2}\right)$$

$\tilde{\Gamma}_i^{\lambda}$ CAN BE 1-CENTERED BH'S OR CAN THEMSELVES BE POLAR

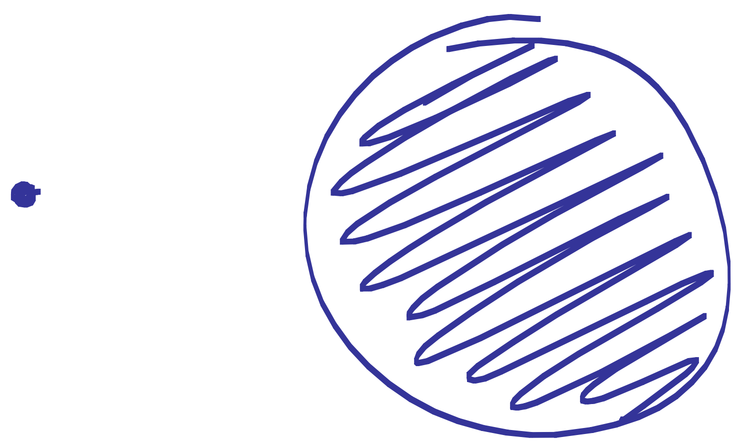
2. ATTRACTOR FORMALISM HAS A SCALING SYMMETRY UNDER

$$T_\lambda(p^0, P, Q, q_0) = (p^0, \lambda P, \lambda^2 Q, \lambda^3 q_0)$$

$$S(T_\lambda \Gamma) = \lambda^3 S(\Gamma)$$

3. APPLY TO $T_\lambda \tilde{\Gamma}_1^{\lambda} + T_\lambda \tilde{\Gamma}_2^{\lambda} = \lambda \Gamma$

RECENTLY, DEBOER ET. AL.
SHOWED THAT IF WE SPLIT
THE D2-D0 CHARGE <u>ASYMMETRICALLY</u>
BETWEEN THE TWO CENTERS
THEN THE COEFFICIENT OF THE
λ^3 GROWTH CAN BE INCREASED:



DOMINATES

BUT BOTH CONTRIBUTIONS SCALE
LIKE λ^3.

DEGENERACY DICHOTOMY

- WE HAVE FOUND CONTRIBUTIONS TO $\Omega(\lambda\Gamma)_\infty$ GROWING LIKE e^{λ^3}

- IF INDEED $\Omega(\lambda\Gamma)_\infty \sim e^{\lambda^3}$ THEN WEAK COUPLING OSV IS WRONG, SINCE OSV $\Rightarrow$ $\Omega(\lambda\Gamma)_\infty \sim e^{\lambda^2}$.

- BUT $\Omega(\lambda\Gamma)_\infty$ IS AN INDEX. IT IS POSSIBLE THAT

$$\Omega(\lambda\Gamma)_\infty = \sum \pm e^{\lambda^3} \sim e^{\lambda^2}$$

- WE ARGUE THAT THIS IS UNLIKELY, BUT IT IS NOT EXCLUDED.

SUPPOSE THAT THERE <u>ARE</u>
" MAGICAL CANCELLATIONS " AND

$$\Omega(\lambda\Gamma)_\infty \sim e^{\lambda^2}$$

- THIS RAISES THE QUESTION OF $\dim \mathcal{H}(\Gamma;t)$ vs. $\Omega(\Gamma;t)$

- PHYSICALLY: THE <u>DIMENSION</u> IS RELEVANT

- BUT ALL TESTS OF THE STROMINGER–VAFA PROGRAM USE THE <u>INDEX</u> (WITH ONE EXCEPTION).

- IT IS [illegible] TO SUPPOSE THAT IN THE EXACT THEORY, NONPTVE STRINGY EFFECTS GIVE:

$$\dim \mathcal{H}(\Gamma;t) = \Omega(\Gamma;t)$$

IF WE GRANT THIS POINT,

AND IF, MORE SPECULATIVELY,

THERE ARE "MAGICAL CANCELLATIONS"

SO THAT

$$\log \Omega(0, \lambda P, 0, \lambda q_0) \underset{\lambda \to \infty}{\sim} \lambda^2$$

THEN THE SPECTRUM OF NEAR-BPS STATES TAKES A REMARKABLE FORM:

$E - |Z| = 0 \qquad \sim e^{\lambda^2}$ states

$E - |Z| \sim e^{-1/g_s} \qquad \sim e^{\lambda^3}$ states

7. SOME OPEN PROBLEMS

S.A.F., E.P.S., ... CONJECTURES

a.) RELATION OF Z_{D4D2D0} TO M5 ELLIPTIC GENUS.

b.) HOW TO COMPUTE POLAR DEGENERACIES EFFECTIVELY?

c.) RESOLVE THE QUESTION OF THE ENTROPY ENIGMA: ARE THERE CANCELLATIONS BRINGING $e^{\lambda^3} \to e^{\lambda^2}$?

d.) IN MANY WAYS $\Omega(\Gamma, t_*(\Gamma))$ SEEMS LIKE A MORE NATURAL QUANTITY

- MICROSCOPIC COMPUTATION?
- AUTOMORPHIC PROPERTIES?
- OSV-LIKE RELATION?