

MATHEMATICAL FOUNDATIONS OF ORIENTIFOLDS

ETH LECTURE III, AUGUST 14, 2008

1. INTRODUCTION

A: MOTIVATION & MAIN MATH. THEMES

B: SUMMARY OF RESULTS

2. COMMENT ON WORLDSHEET FORMULATION

3. SPACETIMES AS GROUPOIDS & STACKS

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6. SELF-DUAL FIELDS

7. THE CHARGE OF O-PLANES

8. CONSISTENCY CONDITIONS FOR D-BRANES

9. CONCLUSION

1. INTRODUCTION

THIS TALK IS A PROGRESS REPORT
ON AN EXTENSIVE PROJECT WITH
JACQUES DISTLER & DAN FREED

OUR ORIGINAL GOAL WAS TO GIVE A
K-THEORETIC FORMULA FOR THE RR CHARGE
OF O-PLANES.

WE SUCCEEDED IN A SPECIAL CASE,
BUT TO GIVE THE GENERAL FORMULA
IS QUITE NONTRIVIAL, AND HAS FORCED
US TO UNDERSTAND MORE PRECISELY THE
MATHEMATICAL FOUNDATIONS OF ORIENT'S.

2 THERE IS A LOT OF LANGUAGE
AND MANY CONCEPTS
WHICH WILL BE UNFAMILIAR TO MANY
STRING-THEORISTS. SO THIS TALK WILL
BE A BROAD OVERVIEW, SKIPPING
MANY DETAILS.

A. TWO MOTIVATIONS

α : THERE ARE PROPOSED TYPE II COMPACTIFICATIONS ON $M_4 \times K$ WITH ALL MODULI STABILIZED.

AN ESSENTIAL ELEMENT IN MANY SUCH CONSTRUCTIONS ARE ORIENTIFOLDS (GKP), NEEDED TO EVADE NO-GO THEOREMS (G, MN)

EXTRAORDINARY CLAIMS MERIT EXTRAORDINARY PROOFS: THERE ARE MANY TECHNICAL POINTS WHICH OUGHT TO BE CLEARED UP.

EXAMPLE: SOLVABILITY OF

$$d_H G = (d+H)G = \int_{D\text{-brane}} + \int_{O\text{-plane}}$$

$\Rightarrow \int_D + \int_O$ IS TRIVIAL IN $(d+H)$ -COHOMOLOGY

* THIS IS NECESSARY, BUT NOT SUFFICIENT
FOR TADPOLE CANCELLATION

- THERE ARE EXAMPLES WHERE TADPOLE CONDITIONS ARE SATISFIED IN DE RHAM COHOMOLOGY, BUT NOT K-THEORETICALLY.

Moreover, ...

- THERE ARE CONSISTENCY CONDITIONS FOR D-BRANES IN ORIENTIFOLDS WHICH MUST BE STATED AT THE K-THEORY LEVEL.

B: THE MATHEMATICAL FORMULATION IS A VERY TIGHT AND NONTRIVIAL STRUCTURE FORCING US TO UNDERSTAND MORE DEEPLY TWO KEY THEMES

① HOW TO APPLY

TWISTED, EQUIVARIANT,
DIFFERENTIAL, GENERALIZED,
COHOMOLOGY THEORIES

COMBINED WITH

② FORMULATING

THE THEORY OF A
SELF-DUAL FIELD QUANTIZED
BY A PONTRYAGIN SELF-DUAL
COHOMOLOGY THEORY.

B. SUMMARY OF RESULTS

1. WORLDSHEET FORMULATION OF THE GENERAL ORIENTIFOLD σ -MODEL
2. ORIENTIFOLD SPACETIMES IN A NUTSHELL
3. DEFINITION OF THE RR CHARGE OF O-PLANES @ THE K-THEORETIC LEVEL.
4. COMPUTATION OF THE CHARGE ONCE WE INVERT 2.
5. CONSISTENCY CONDITIONS FOR D-BRANES IN ORIENTIFOLDS

2. THE WORLDSHEET

IT IS NOT TRIVIAL TO FORMULATE CAREFULLY THE ORIENTIFOLDED σ -MODEL (ALL GENUS, INCLUDE FERMIONS...)

FOCUS TODAY ON ONE QUESTION:
WHAT ARE THE TWISTED SECTORS?

REVIEW ORBIFOLDS:

σ MODEL: $\Sigma \xrightarrow{\phi} Y$, RIEMANNIAN

Γ : DISCRETE GROUP OF ISOMETRIES ON Y .

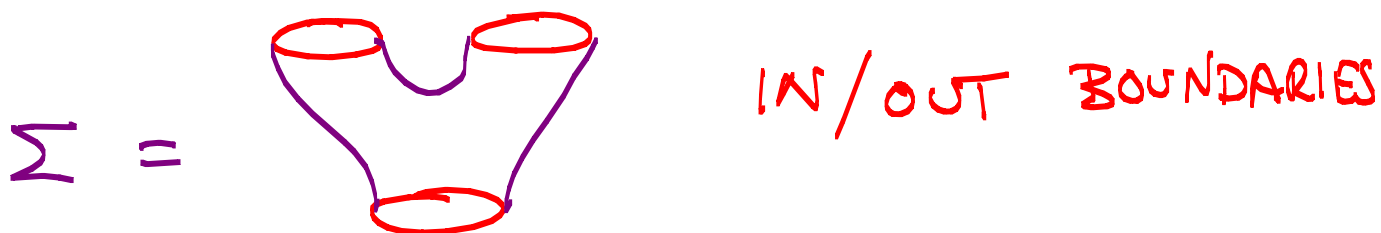
GAUGE Γ -SYMMETRY:

- $\Sigma \rightarrow$ PRINCIPAL Γ -BUNDLE $\tilde{\Sigma}$
- $\phi \rightarrow$ EQUIVARIANT MAP $\tilde{\phi}$

$$\begin{array}{ccc}
 \tilde{\Sigma} & \xrightarrow{\tilde{\phi}} & Y \\
 \gamma_{\tilde{\Sigma}} \downarrow & & \downarrow \gamma_Y \\
 \Sigma & \xrightarrow{\phi} & Y
 \end{array}$$

$\gamma_{\tilde{\Sigma}}$: Fixed point free: $\Sigma = \tilde{\Sigma} / \Gamma$

(THIS DEFINES A MAP $\phi: \Sigma \rightarrow \mathcal{X}$
 $\mathcal{X} = Y // \Gamma$ A "STACK" .)



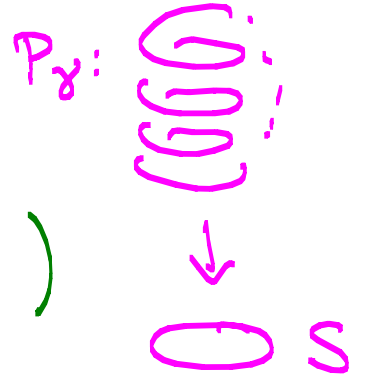
- RESTRICTING TO IN/OUT CIRCLE S
 GET A \mathbb{T}^2 -BUNDLE $S_{\mathbb{T}^2} \rightarrow S$.
- THESE ARE CLASSIFIED BY HOLONOMY

$$[\gamma] \in \text{Hom}(\pi_1(S), \Gamma) / \text{Conj.}$$

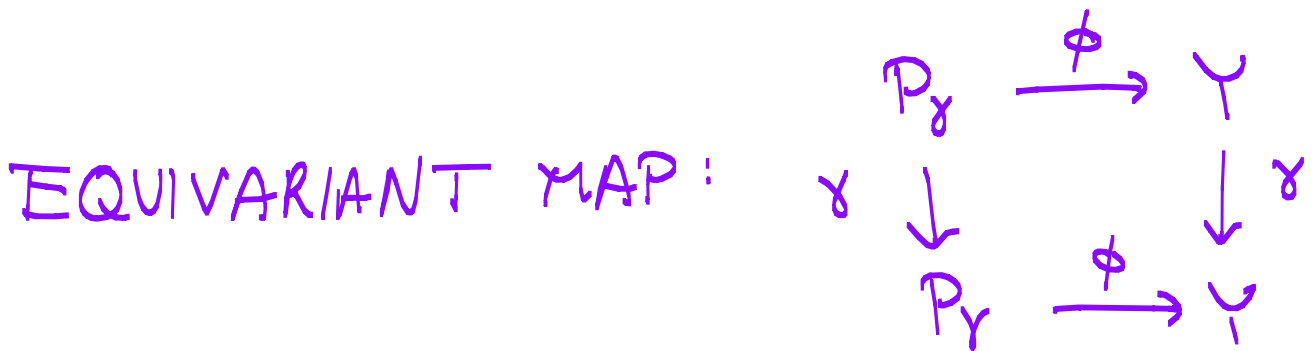
- WE CAN REPLACE S_T BY AN EQUIVALENT BUNDLE

$$P_\gamma = \mathbb{R} \times \mathbb{Z} \langle \gamma \rangle$$

$$(x, \gamma^n) \sim (x+1, \gamma^{n+1})$$



- P_γ IS A SINGLE CIRCLE \Rightarrow SINGLE HILBERT SPACE OF COVERING THEORY



$$\Rightarrow \boxed{\phi(x+1) = \gamma \cdot \phi(x)}$$

TWISTED SECTORS \equiv NONTRIVIAL BUNDLES $\begin{matrix} S_T \\ \downarrow \\ S \end{matrix}$

ORIENTIFOLDS: NEW DATA

$$1 \rightarrow \Gamma_0 \rightarrow \Gamma \xrightarrow{\omega} \mathbb{Z}_2 \rightarrow 1$$

$$\Gamma = \Gamma_0 \rtimes \Gamma_1$$

AGAIN GAUGE Γ : BUT $\gamma \in \Gamma_1$ IS
ACCOMPANIED BY PARITY ON W.S.

$$\begin{array}{ccc} \tilde{\Sigma} & \xrightarrow{\tilde{\phi}} & Y \\ \gamma_{\tilde{\Sigma}} \downarrow & & \downarrow \gamma_Y \\ \Sigma & \xrightarrow{\phi} & Y \end{array}$$

- $\gamma_{\tilde{\Sigma}}$ FIXED POINT FREE
- Σ ORIENTED SURFACE
- $\gamma_{\tilde{\Sigma}}$ ORIENTATION PRESERVING $\gamma \in \Gamma_0$
- " " REVERSING $\gamma \in \Gamma_1$

AGAIN RESTRICTING TO IN/OUT
CIRCLE GET Γ -BUNDLE $S^1 \rightarrow S^1$.

CLAIM: HOLONOMY MUST BE
 $[\gamma]$ WITH $\gamma \in \Gamma_0$

RECALL WE CAN REDUCE TO P_γ ,
BUT P_γ IS TOPOLOGICALLY S^1

THERE IS NO FIXED-POINT FREE
ORIENTATION-REVERSING MAP
 $S^1 \rightarrow S^1$.

THAT IS WHY THERE ARE
NO TWISTED SECTORS ASSOCIATED
TO $\gamma \in \Gamma_1$.

PUT DIFFERENTLY: THERE IS A \mathcal{L}
 MAP $\hat{\phi}$ OF DOUBLE COVERS:

$$\begin{array}{ccc}
 \tilde{\Sigma} & \xrightarrow{\tilde{\phi}} & Y \\
 \downarrow \Gamma_0 & & \downarrow \\
 \tilde{\Sigma} / \Gamma_0 & \xrightarrow{\hat{\phi}} & Y // \Gamma_0 \\
 \downarrow \mathbb{Z}_2 & & \downarrow \\
 \Sigma = \tilde{\Sigma} / \Gamma & \xrightarrow{\phi} & Y // \Gamma
 \end{array}$$

- (1) $\tilde{\Sigma} / \Gamma_0 = \text{OR}(\Sigma)$
 (2.) $\text{OR}(S) = S \amalg S$

RESTRICTING TO ONE COPY OF S GIVES
 AN EXPLICIT REDUCTION OF STRUCTURE
 GROUP TO T_0 .

A WORD ABOUT DOUBLE-COVERS...

RECALL: A DOUBLE-COVER

$$\tilde{M} \rightarrow M$$

IS A PRINCIPAL \mathbb{Z}_2 -BUNDLE OVER M .

DOUBLE-COVERS ARE CLASSIFIED BY $H^1(M, \mathbb{Z}_2)$

$$[\text{Hom}(\pi_1(M), \mathbb{Z}_2) \cong \text{Hom}(H_1(M), \mathbb{Z}_2) \cong H^1(M; \mathbb{Z}_2)]$$

$$\begin{array}{ccc}
 \left. \begin{array}{l} \text{CLASS} \\ \omega_1(\Sigma) \end{array} \right\} \begin{array}{c} \tilde{\Sigma} / \Gamma_0 \\ \downarrow \mathbb{Z}_2 \\ \Sigma = \tilde{\Sigma} / \Gamma \end{array} & \begin{array}{c} \xrightarrow{\hat{\phi}} \\ \\ \xrightarrow{\phi} \end{array} & \begin{array}{c} Y // \Gamma_0 \\ \downarrow \\ Y // \Gamma \end{array} \left. \begin{array}{l} \text{CLASS} \\ w \\ \text{IN} \\ H^1(Y // \Gamma, \mathbb{Z}_2) \end{array} \right\}
 \end{array}$$

$$\Rightarrow \phi^*(w) = \omega_1(\Sigma)$$

NEXT NEED TO MAKE SENSE OF
THE B-FIELD AMPLITUDE:

$$\exp\left[2\pi i \int_{\Sigma} \phi^* B\right].$$

THIS IS QUITE TRICKY: Σ IS
UNORIENTED AND IN GENERAL
UNORIENTABLE.

• B WILL LIVE IN SOME COHOMOLOGY
THEORY... \int_{Σ} WILL BE INTEGRATION
IN THAT COHOMOLOGY THEORY.

$$\bullet \int_{\Sigma} : H^{2+w_1(\Sigma)}(\Sigma; \mathbb{R}/\mathbb{Z}) \rightarrow \mathbb{R}/\mathbb{Z}$$

• $\phi^*(w) = w_1(\Sigma) \Rightarrow$ IF B WERE
A W-TWISTED FORM IT COULD BE
INTEGRATED...

3. SPACETIME AS A "STACK"

CATEGORY: $\mathcal{X}_1 \begin{matrix} \xrightarrow{p_0} \\ \xrightarrow{p_1} \end{matrix} \mathcal{X}_0$

MORPHISMS OBJECTS

PICTURE: $f \in \mathcal{X}_1$

COMPOSITION:

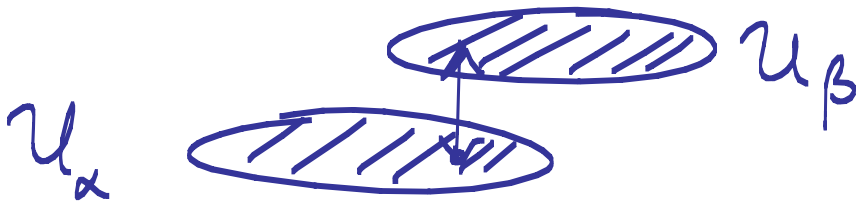
GROUPOID: ALL MORPHISMS INVERTIBLE

VIEW SPACETIME AS A GROUPOID:

$\mathcal{X}_0, \mathcal{X}_1$ MANIFOLDS

EXAMPLE A: $Y =$ MANIFOLD WITH OPEN COVER $\{U_\alpha\}$

$$\mathcal{X}_0 = \coprod_{\alpha} U_\alpha \quad \mathcal{X}_1 = \coprod_{\alpha \leq \beta} U_{\alpha\beta}$$



p_0, p_1 : INCLUSIONS $U_{\alpha\beta} \rightarrow U_\alpha$
 $U_{\alpha\beta} \rightarrow U_\beta$

ISOMORPHISM CLASSES OF OBJECTS IN \mathcal{X}
ARE

POINTS OF THE MANIFOLD Y .

BUT THE GROUPOID "REMEMBERS" THE
EXTRA DATA OF THE COVERING.

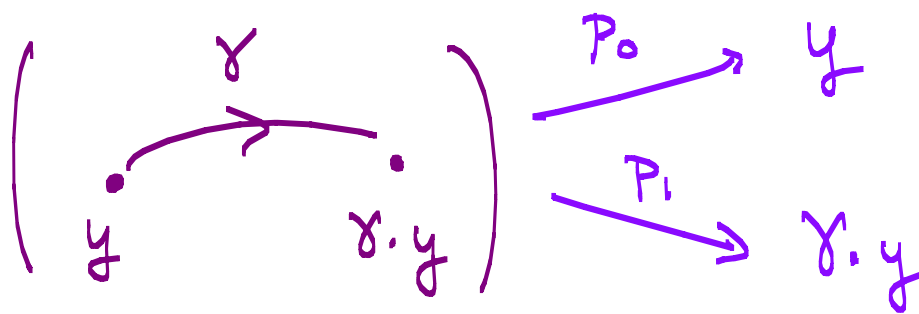
[\Rightarrow NOTION OF "EQUIVALENT GROUPOIDS"]

EXAMPLE B: $Y = \text{MANIFOLD WITH}$
 Γ -ACTION

DEFINES A GROUPOID \mathcal{X} DENOTED $Y//\Gamma$

OBJECTS: $\mathcal{X}_0 = Y$

MORPHISMS: $\mathcal{X}_1 = Y \times \Gamma$



ISOMORPHISM CLASSES OF OBJECTS IN $Y//\Gamma$
ARE

POINTS OF Y/Γ ,

BUT THE GROUPOID "REMEMBERS"
THE STABILIZER GROUP OF POINTS

PUTTING THESE NOTIONS TOGETHER WE
GET AN "ORBIFOLD" IN THE SENSE OF
SATAKE ("V-MANIFOLDS") AND THURSTON

BUT TO AVOID CONFUSION WE WILL CALL
IT A (DELIGNE-MUMFORD) "STACK."

(STACK = EQUIVALENCE CLASS OF GROUPOIDS.)

GIVEN A GROUPOID (OR GENERAL CATEGORY) ONE FORMS A "SIMPLICIAL SPACE"

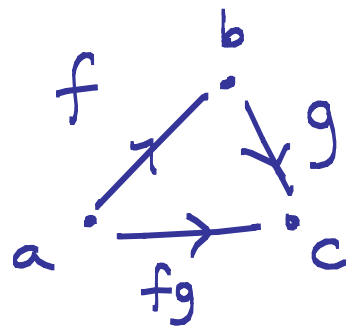
$$\dots \mathcal{X}_3 \rightrightarrows \mathcal{X}_2 \rightrightarrows \mathcal{X}_1 \rightrightarrows \mathcal{X}_0$$

$\mathcal{X}_n :=$ SET OF n -COMPOSABLE MORPHISMS

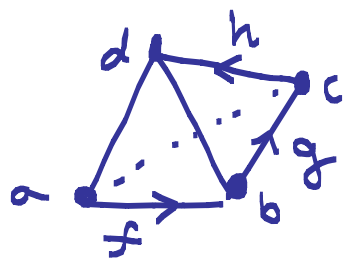
$$\mathcal{X}_2 = \{ a \xrightarrow{f} b \xrightarrow{g} c \}$$

$$\mathcal{X}_3 = \{ a \xrightarrow{f} b \xrightarrow{g} c \xrightarrow{h} d \}$$

Points of $\mathcal{X}_2 \Rightarrow$ 2-SIMPLICES:



POINT OF $\mathcal{X}_3 \Rightarrow$ 3-SIMPLICES:



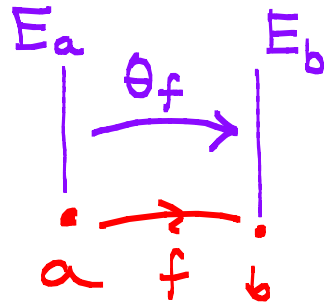
\Rightarrow CW COMPLEX $|\mathcal{X}|$

DEF: VECTOR BUNDLE ON A GROUPOID \mathcal{X}

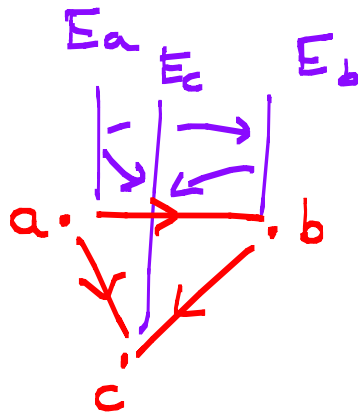
- BUNDLE E OVER \mathcal{X}_0



- ISOM. $\theta: p_0^*E \xrightarrow{\sim} p_1^*E$ OVER \mathcal{X}_1



- COMPATIBLE OVER \mathcal{X}_2



EXAMPLE A: ORDINARY V.B. ON Υ

CONDITION ON \mathcal{X}_2 : COCYCLE CONDITION

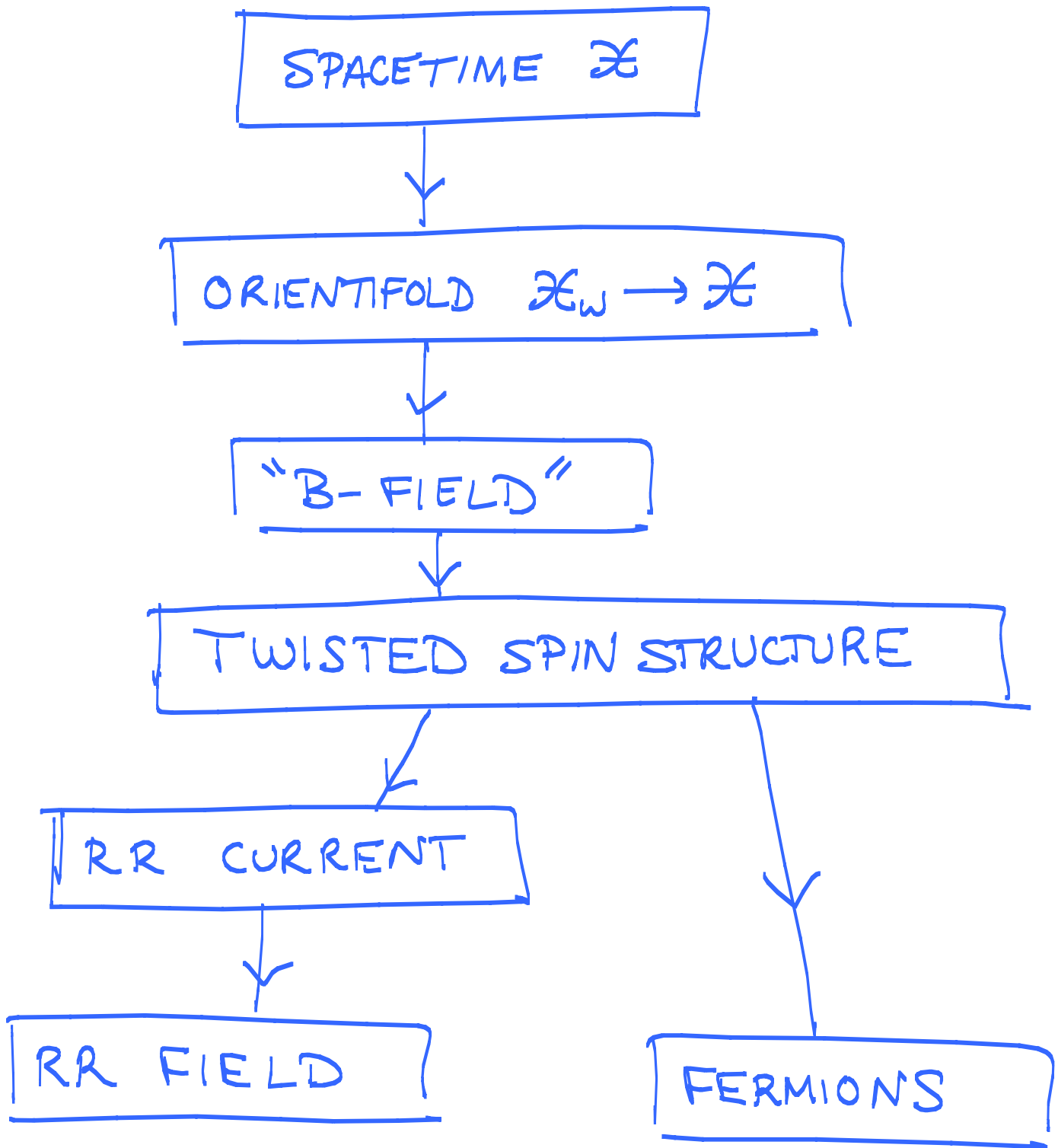
EXAMPLE B: Γ -EQUIVARIANT BUNDLE ON Υ

4. ORIENTIFOLD BACKGROUNDS IN A NUTSHELL

WE WANT TO DESCRIBE A TYPE II
ORIENTIFOLD SPACETIME IN A CONCISE
AND PRECISE WAY.

CLAIM: THE DATA NEEDED TO
SPECIFY A PERTURBATIVE, TYPE II,
ORIENTIFOLD SPACETIME IS:

⌋ WARNING: YOU MIGHT NOT
UNDERSTAND MANY WORDS;
I WILL SUBSEQUENTLY EXPLAIN
(A LITTLE BIT...)



1. SPACETIME: \mathcal{X} IS A SMOOTH D-M STACK EQUIPPED WITH RIEMANNIAN METRIC AND DILATON.

2. ORIENTIFOLD DATA: $\mathcal{X}_w \rightarrow \mathcal{X}$ is a
DOUBLE-COVER.

3. B-FIELD: \check{B} is a GEOMETRICAL
TWISTING OF $K\check{R}(\mathcal{X}_w)$.

TOPOLOGICALLY: $[\check{B}] = (t, a, h)$

$$\in H^0(\mathcal{X}, \mathbb{Z}_2) \times H^1(\mathcal{X}, \mathbb{Z}_2) \times H^{3+w}(\mathcal{X}, \mathbb{Z})$$

4. TWISTED SPIN-STRUCTURE

$$w_1(\mathcal{X}) = tw$$

$$w_2(\mathcal{X}) = tw^2 + aw$$

5. RR-CURRENT: ROUGHLY $\check{J}_D \in K\check{R}^{\check{B}}(\mathcal{X}_w)$

6. RR-FIELD: ROUGHLY $d\check{F} = \check{J}_D + \check{J}_0$

$$\Delta\check{F} \in K\check{R}^{-1+\check{B}}(\mathcal{X}_w)$$

5. GENERALIZATIONS OF COHOMOLOGY

SINGULAR COHOMOLOGY IS A FUNCTOR

TOP. SPACES \longrightarrow ABELIAN GROUPS

$$Y \longrightarrow H^j(Y)$$

SATISFYING NATURALNESS + LOCALITY

WE NEED FOUR GENERALIZATIONS:

α . GENERALIZED COHOMOLOGY

β . EQUIVARIANT COHOMOLOGY

γ . TWISTED COHOMOLOGY

δ . DIFFERENTIAL COHOMOLOGY

ϵ . PUTTING THEM TOGETHER

$$\epsilon = \alpha + \beta + \gamma + \delta$$

α . GENERALIZED COHOMOLOGY

BASIC PROPERTIES OF NATURALNESS AND MEYER-VIETORIS ALLOW OTHER THEORIES - NOTABLY K -THEORY.

COMMON FEATURE: "SPECTRUM,"
A SPACE \mathcal{S} SO THAT

$$h(Y) = [Y, \mathcal{S}]$$

EXAMPLE:

$$H^n(Y, \mathbb{Z}) = [Y, K(n, \mathbb{Z})]$$

\uparrow
Eilenberg-MacLane Spaces

FOR K-THEORY:

$\mathcal{H} = \mathbb{Z}_2$ -GRADED HILBERT SPACE

$\mathcal{F} =$ SKEW ADJOINT ODD FREDHOLMS

$$K(Y) = [Y, \mathcal{F}]$$

PHYSICS: NONCOMMUTATIVE TACHYON

$$T, Y \longrightarrow \mathcal{F}$$

HOMOTOPY CLASS OF THE "SOLITON"
IS THE K-THEORY CLASS.

$\mathcal{H} =$ "UNIVERSAL CHAN-PATON BUNDLE"

FOR BRANE - BRANE SYSTEMS.

B. EQUIVARIANT COHOMOLOGY

A. EQUIVARIANT COHOMOLOGY

$$\begin{aligned} H_{\Gamma}^j(Y) &:= H^j(Y//\Gamma) \\ &= \begin{cases} H^j(Y/\Gamma) & \text{FREE} \\ H^j(Y \times |P\mathcal{Y}/\Gamma|) & \text{FIXED} \end{cases} \end{aligned}$$

FOR STACKS: $H^j(Y//\Gamma) := H_{\Gamma}^j(Y)$

- $H_{\Gamma}^1(Y; \mathbb{Z}_2) =$ DOUBLE COVERS $\tilde{Y} \rightarrow Y$ TOGETHER WITH A LIFT OF Γ ACTION TO \tilde{Y}
- $\omega_1^{\text{eq}}(Y) :$ MEASURES WHETHER Γ PRESERVES AN ORIENTATION
- $\omega_2^{\text{eq}}(Y) :$ MEASURES WHETHER Y ADMITS A SPIN STRUCTURE WITH A SPIN ACTION OF Γ

B. EQUIVARIANT K-THEORY

\mathcal{H} NOW HAS Γ -ACTION

$\widehat{\mathcal{F}}_\Gamma \subset \mathcal{F}$: OPERATORS COMMUTING WITH Γ

$$K_\Gamma(Y) := [Y, \widehat{\mathcal{F}}_\Gamma]$$

i.e. EQUIVARIANT TACHYON FIELD

$$\gamma \cdot T(y) = T(\gamma \cdot y) \gamma \quad \forall \gamma \in \Gamma \\ y \in Y$$

FAMILIAR FROM D-BRANES ON ORBIFOLDS

C. ORIENTIFOLD K -THEORY

\mathcal{H} CARRIES A "REPRESENTATION" OF Γ

Γ_0 : LINEAR

Γ_1 : ANTI-LINEAR

$$\omega K_\Gamma(Y) := [Y, \mathbb{F}_\Gamma]$$

$$\gamma T(y) = T(\gamma y) \cdot \gamma$$

• FOR $\Gamma = \mathbb{Z}_2$ ε_1 $\omega = \text{Id}$

$$\omega K_\Gamma(Y) = KR(Y)$$

• FOR STACKS $\mathcal{X} = Y // \Gamma$

$$\omega K_\Gamma(Y) = KR(\mathcal{X}_\omega)$$

8. TWISTED COHOMOLOGY

EXAMPLE 1: ANY COHOMOLOGY THEORY \mathcal{E} CAN BE TWISTED BY A DOUBLE COVER

$$\tau: \tilde{Y} \rightarrow Y$$

$\mathcal{E}^\tau(Y) :=$ ISO. CLASSES OF COCYCLES ON \tilde{Y} ODD UNDER τ

EXAMPLE 2: K-THEORY CAN BE TWISTED BY A "GERBE"

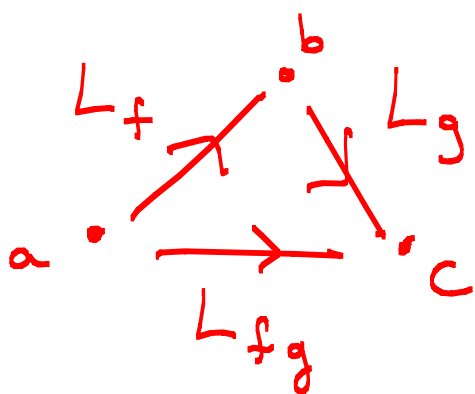
REPLACE: $Y \rightsquigarrow$ EQUIVALENT GROUPOID \mathcal{X}

EXAMPLE: IF
$$\begin{array}{ccc} G & \rightarrow & P \\ & & \downarrow \\ & & Y \end{array}$$
 IS PRINCIPAL

$$Y \rightsquigarrow P/G$$

A (GEOMETRICAL) TWISTING τ OF $K(X)$ IS:

- A (\mathbb{Z}_2 -GRADED) LINE BUNDLE $L \rightarrow \mathcal{X}_1$
- WITH A MULTIPLICATION ON \mathcal{X}_2



$$\mu: L_f \otimes L_g \cong L_{fg}$$

- AND A COMPATIBILITY ON \mathcal{X}_3

EXAMPLE: $\mathcal{X} = \text{pt} // \Gamma$

A TWISTING IS A CENTRAL EXTENSION

$$1 \rightarrow \mathbb{C}^* \rightarrow \hat{\Gamma} \rightarrow \Gamma \rightarrow 1$$

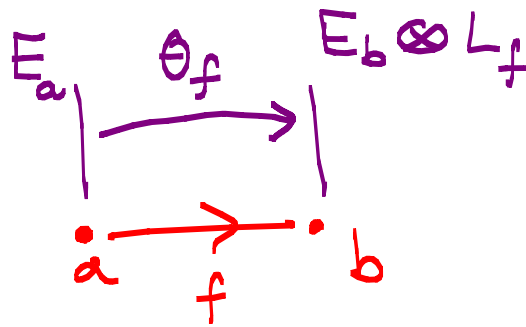
- MULTIPLICATION DEFINED BY COCYCLE
- COMPATIBILITY ON \mathcal{X}_3 : ASSOCIATIVE LAW FOR $\hat{\Gamma}$

NOW RECALL p.20: VECTOR BUNDLES ON GROUPOIDS ---

A TWISTED VECTOR BUNDLE E ON X
WITH TWISTING τ IS

• A VECTOR BUNDLE E ON \mathcal{X}_0 (NOT X !!)

• AN ISOMORPHISM $\theta: p_0^* E \otimes L \xrightarrow{\sim} p_1^* E$
ON \mathcal{X}_1



• COMPATIBILITY ON \mathcal{X}_2

EXAMPLE $\mathcal{X} = pt // \Gamma$

A TWISTED VB. ON A POINT IS A PROJECTIVE REP. OF Γ ;

IN PARTICULAR A REP. OF THE "TWISTING" $\hat{\Gamma}$.

TWISTINGS OF $KR(\mathcal{X}_w)$

NOW, IN THE MULTIPLICATION

$$L_f \otimes L_g \longrightarrow L_{fg}$$

WE SOMETIMES HAVE COMPLEX CONJ.

$$\overline{L}_f \otimes L_g \longrightarrow L_{fg}$$

FOR EXAMPLE: $\mathcal{X} = \text{pt} // \Gamma$

NOW WE GET A NONCENTRAL EXT.

$$1 \rightarrow \mathbb{C}^* \rightarrow \widetilde{\Gamma} \rightarrow \Gamma \rightarrow 1$$

\Rightarrow FORMULATION OF "DISCRETE TORSION"

FOR ORIENTIFOLDS:

$$H_{\Gamma}^{2+w}(\text{pt}; \mathbb{R}/\mathbb{Z})$$

8. Differential Cohomology

THE GAUGE INVARIANT INFORMATION IN AN ABELIAN GAUGE FIELD IS

- [MAXWELL] Fieldstrength $F \in \left\{ \begin{array}{l} \text{DIFF} \\ \text{FORMS} \end{array} \right\}$
- [DIRAC] INTEGRAL COHOMOLOGY CLASS IN SOME GENERALIZED COHOMOLOGY THEORY.
- [AHARONOV-BOHM - WILSON - 't HOOFT]
A FLAT FIELD IN SOME COHOMOLOGY WITH \mathbb{R}/\mathbb{Z} COEFF'S.

THESE DATA ARE CORRELATED.

CORRECT CONCEPT IS A DIFFERENTIAL COHOMOLOGY GROUP.

GIVE TWO EXAMPLES

EXAMPLE A: U(1) GAUGE FIELD

FLAT

FIELDSTRENGTH

$$0 \rightarrow H^1(X, \mathbb{R}/\mathbb{Z}) \rightarrow \check{H}^2(X) \rightarrow \Omega_{\mathbb{Z}}^2(X) \rightarrow 0$$

$$0 \rightarrow \Omega^1(X)/\Omega_{\mathbb{Z}}^1(X) \rightarrow \check{H}^2(X) \rightarrow H^2(X, \mathbb{Z}) \rightarrow 0$$

TOPOLOGICALLY
TRIVIAL GAUGE POTENTIALS
MOD GAUGE EQUIVALENCE

TOPOL. CLASS
 $C_1(L)$

EXAMPLE B: TYPE IIA RR FIELDS

IN THE ABSENCE OF RR CURRENT

FLAT RR

$$G = G_0 + \bar{u}^1 G_2 + \dots + \bar{u}^5 G_{10}$$

$$0 \rightarrow K^{-1}(X, \mathbb{R}/\mathbb{Z}) \rightarrow \check{K}^0(X) \rightarrow \left[\Omega_{\mathbb{Z}}^{\bullet}(X, \mathbb{R}[\bar{u}^i]) \right]_{\rightarrow 0}^0$$

$$0 \rightarrow \frac{(\Omega^{\bullet}(X, \mathbb{R}[\bar{u}^i]))^{-1}}{\Omega_{\mathbb{Z}}^{\bullet}(\dots)^{-1}} \rightarrow \check{K}^0(X) \rightarrow \underbrace{K^0(X)}_{\text{TOPOL. CLASS OF RR FIELD}} \rightarrow 0$$

$$C = \bar{u}^1 C_1 + \dots + \bar{u}^5 C_9$$

TOPOL. CLASS
OF RR FIELD

$$\underline{\varepsilon := \alpha + \beta + \gamma + \delta}$$

NOW ONE NEEDS TO COMBINE
THESE FOUR CONCEPTS.

AN IMPORTANT OUTCOME
IS THAT

THE GEOMETRICAL TWISTINGS \check{B} OF
 $\check{K}R(\mathcal{X}_\omega)$ ARE THEMSELVES

TWISTED, DIFFERENTIAL, CLASSES
IN A (SMALL) GENERALIZATION
OF $\check{H}^3(\mathcal{X})$.

THE TWISTING GENERALIZES

THE RULE $B \rightarrow -\gamma^*(B) \quad \gamma \in \Gamma,$

TOPOLOGICAL CLASSES OF TWISTINGS

TWISTED BY \mathcal{X}_w
 \downarrow
 \mathcal{X}

$$H^{3+w}(\mathcal{X}; \mathbb{Z}) \rightarrow \mathcal{P}^3(\mathcal{X}) \rightarrow H^0(\mathcal{X}, \mathbb{Z}_2) \times H^1(\mathcal{X}, \mathbb{Z}_2)$$

$$\text{Top. Class}([\check{B}]) = (t, a, h)$$

$$t = 0, 1 \iff \mathbb{I}A, \mathbb{I}B$$

$$a \in H^1(\mathcal{X}; \mathbb{Z}_2) \iff \text{WHEN } (-1)^{\mathbb{F}_L} \text{ ACCOMPANIES } \gamma \text{ IN } \omega_{\text{S. THEORY}}$$

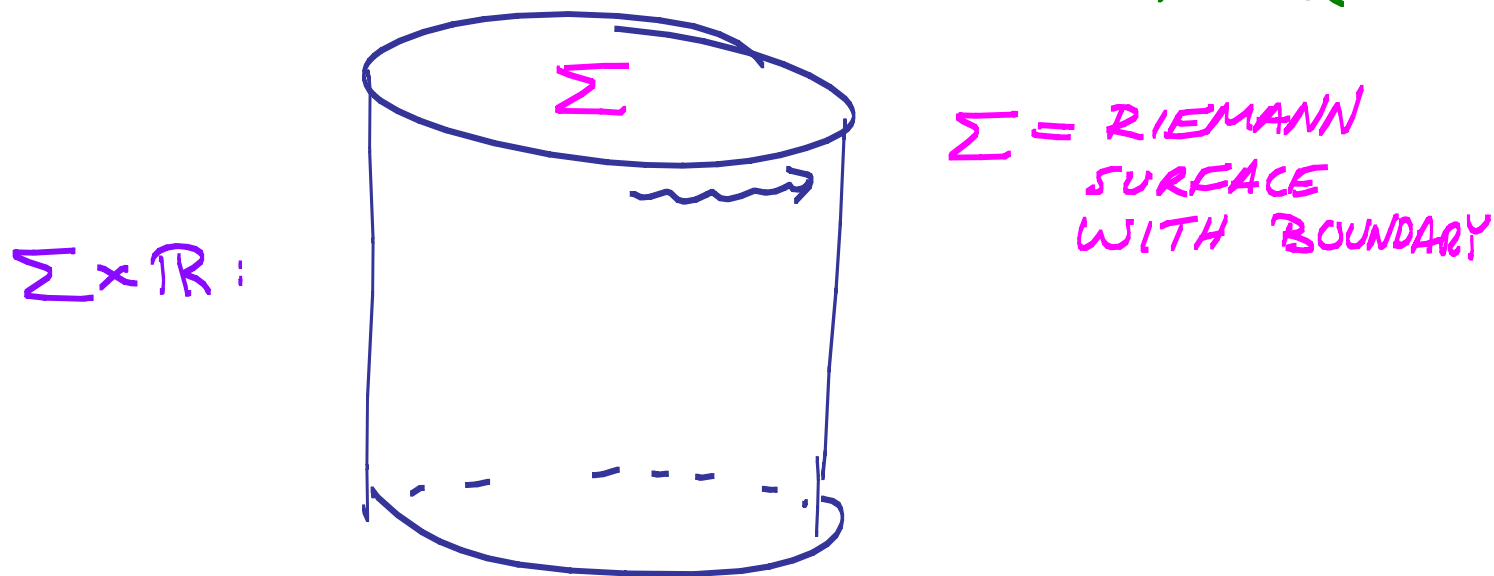
$$h \in H^{3+w}(\mathcal{X}; \mathbb{Z}) \iff \text{GENERALIZES FAMILIAR QUANT. LAW OF THE FIELD STRENGTH}$$

6. SELF-DUAL FIELDS

THE SECOND MAIN INGREDIENT IS THE THEORY OF SELF-DUAL FIELDS APPLIED TO $KR^{\check{B}}(\mathcal{X}_w)$.

RECALL THE FAMILIAR PARADIGM

3D ABELIAN CHERN-SIMONS \iff SELF-DUAL SCALAR



CS IN THE BULK IS HOLOGRAPHICALLY DUAL TO CHIRAL EDGE STATES ON THE BOUNDARY.

THIS CONSTRUCTION CAN BE GENERALIZED TO ANY PONTRYAGIN SELF-DUAL COHO. THEORY

FOR EXAMPLE, FOR THE SELF-DUAL

FIELD $[\check{F}] \in \check{H}^{2l+1}(M_{4l+2})$

USE CHERN-SIMONS IN $(4l+3)$ -DIM'S:

KEY OBSERVATION: $A \in \Omega^{2l+1}(M_{4l+3})$

$$S(A) = \frac{1}{2} \int_M A dA \quad \Rightarrow$$

$$\begin{aligned} S(A_1+A_2) - S(A_1) - S(A_2) &= \int_M A_1 dA_2 \\ &= \int^{\check{H}} A_1 * A_2 \end{aligned}$$

THE CHERN-SIMONS ACTION IS A QUADRATIC REFINEMENT OF THE \check{H} BILINEAR FORM ON THE DIFF L COHOMOLOGY!

THE KEY INGREDIENT IN DEFINING
A SELF-DUAL THEORY IS THE QUADRATIC
REFINEMENT OF A \wedge BILINEAR FORM.

WHEN APPLIED TO THE SELF-DUAL
RR FIELD IN ORIENTIFOLDS WE NEED
AN 11-DIMENSIONAL CHERN-SIMONS
THEORY WITH ACTION:

$$S(A) = \int_{\check{K}O} A * \bar{A} \quad A \in \check{K}R(\mathcal{X}_{11})$$

TO MAKE SENSE OF THE ACTION
WE NEED THE ISOMORPHISM OF
TWISTINGS:

$$\check{B} + \overline{\check{B}} \cong \mathcal{Z}(\check{K}O(\mathcal{X}_w))$$

THE EXISTENCE OF THIS ISOM
REQUIRES A "TWISTED SPIN STRUCTURE"

$$w_1(\mathcal{X}) = \pm w$$

$$w_2(\mathcal{X}) = \pm w^2 + aw$$

A NEW TOPOLOGICAL
CONSISTENCY CONDITION
ON ORIENTIFOLD BACKGROUNDS

- THE TWISTED SPIN STRUCTURE IS ALSO NEEDED TO FORM THE ACTION FOR GRAVITINOS/DILATINOS
- THE TWISTED SPIN STRUCTURE IS ALSO NEEDED TO CONSTRUCT THE WORLDSHEET THEORY.

7. A FORMULA FOR THE CHARGE OF O_p^\pm PLANES

PUTTING THESE IDEAS TOGETHER
WE GET

A. A DEFINITION OF THE K-THEORETIC CHARGE OF AN O-PLANE

B. A COMPUTATION OF THE CHARGE IN A SPECIAL CASE:

- $\Gamma = \mathbb{Z}_2$
- $B =$ "UNIVERSAL TWISTING" (OFTEN ASSUMED IN LIT.)
- $\text{Cod}(F) = 0 \pmod{4}$
- F Spin.

$F =$ FIXED POINT LOCUS

$$\mu = \mp \frac{1}{2} \tau_* \left(\frac{C(F)}{\psi_{1/2}(\text{EULER}(V))} \right) \in KR^0(\mathbb{R}_w) \left[\frac{1}{2} \right]$$

- $\tau: F \hookrightarrow Y$ INCLUSION OF FIXED POINT LOCUS, WITH NORMAL BUNDLE V

- $\psi_{1/2}$: MULT. INVERSE OF ADAMS OPERATION ψ_2

- $C(F)$: BOTT'S "CANNIBILISTIC CLASS"

A KO-THEORETIC VERSION OF THE

WU-CLASS:

$$\int_F^{KO} \psi_2(y) = \int_F^{KO} C(F) y \quad y \in KO(F)$$

PROOF INVOLVES ATIYAH-SEGAL
LOCALIZATION THEOREM.

TAKING CHERN CHARACTERS WE
REPRODUCE

$$[\mu(\mathcal{O}_P^\pm)]_{DR} = \pm 2^{p-4} \zeta_* \left(\sqrt{\frac{L(\mathcal{R}_T/4)}{L(\mathcal{R}_V/4)}} \right)$$
$$= \pm 2^{p-4} \left(1 + \frac{1}{96} (p_1(\pi) - p_1(\nu)) + \dots \right) \delta(F)$$

- REPRODUCES MORALES-SCRUCCA-SERONE,
SCRUCCA-SERONE
- CONTRADICTS MUKHI-SURYANARAYANA,
JULIA-LABORDERE, AND

$$\mu(\mathcal{O}_P^+) = + 2^{p-4} \left(1 - \frac{5}{96} (p_1(\pi) - p_1(\nu)) + \dots \right) \delta(F)$$

POSSIBLY WRITTEN [COMP'T W/OUT
VECTOR STRUCTURE]

8. CONSISTENCY CONDITIONS FOR D-BRANES

THIS FORMALISM ALSO GIVES
CONSISTENCY CONDITIONS ON D-BRANES.

D-BRANE WORLDVOLUME

$$\dot{z}: W \hookrightarrow Y$$

INVARIANT UNDER Γ

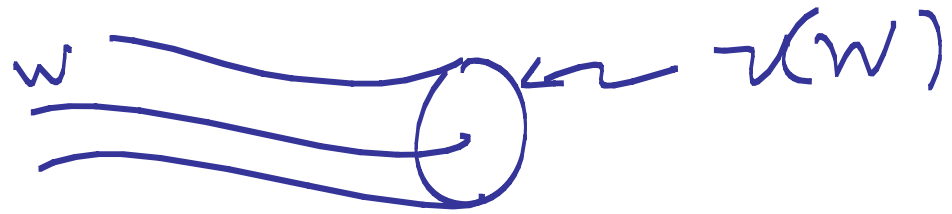
GAUGE BUNDLE ON W DEFINES
AN ELEMENT $E \in {}^\omega K_{\Gamma}^{\tau_W}(W)$.

TO COMPUTE RR CURRENT $z_*(E)$
NEED

$$z_*: {}^\omega K_{\Gamma}^{\tau_W}(W) \longrightarrow {}^\omega K_{\Gamma}^B(Y)$$

THOM ISOMORPHISM \Rightarrow CONSISTENCY CONDITION

LET $\pi: \mathcal{V}(W) \rightarrow W$ BE
PROJECTION OF THE NORMAL BUNDLE



THOM ISOMORPHISM \Rightarrow ON $\mathcal{V}(W)$

$$\pi^*(\tau_W + \tau_{\mathcal{V}}) = \mathcal{B}$$

$$\Rightarrow a(\tau_W) + w_1^{eq}(\mathcal{V}) = a(\mathcal{B})$$

$$h(\tau_W) + \tilde{\beta} \left(a(\tau_W) w_1^{eq}(\mathcal{V}) + (w_1^{eq}(\mathcal{V}))^2 + w_2^{eq}(\mathcal{V}) \right) = h(\mathcal{B})$$

\Rightarrow FAMILIAR RULES FOR WHEN W
CARRIES SO OR Sp GAUGE GROUP
AND $W \subset$ SPIN COMPONENT OF F ,
AND MORE...

9. CONCLUSIONS

-] LOTS MORE TO SAY ABOUT D-BRANES IN ORIENTIFOLDS & THEIR TWISTED GAUGE THEORIES
- MUCH WORK REMAINS TO BE DONE TO COMPLETE OUR PICTURE
- SOME PUZZLES NEED TO BE RESOLVED
-] FURTHER CONSISTENCY CONDITIONS FOR D-BRANE ANOMALY CANCELLATION
- IT WOULD BE GOOD TO LOOK AT EXAMPLES OF STRING BACKGROUNDS AND EXAMINE THE CONSEQUENCES OF OUR CONSISTENCY CONDITIONS.
- INCLUDE T-DUALITY
- S-DUALITY?
- RELATION TO M & F THEORY?