LIFE AFTER RCFT

— OR —

WHERE DO WE GO FROM HERE?

I. RCFT: GO THE DISTANCE

II. GENERALIZE

III. WHAT ABOUT EXPERIMENT?

MODULAR FUNCTOR

Friedan & Shenker

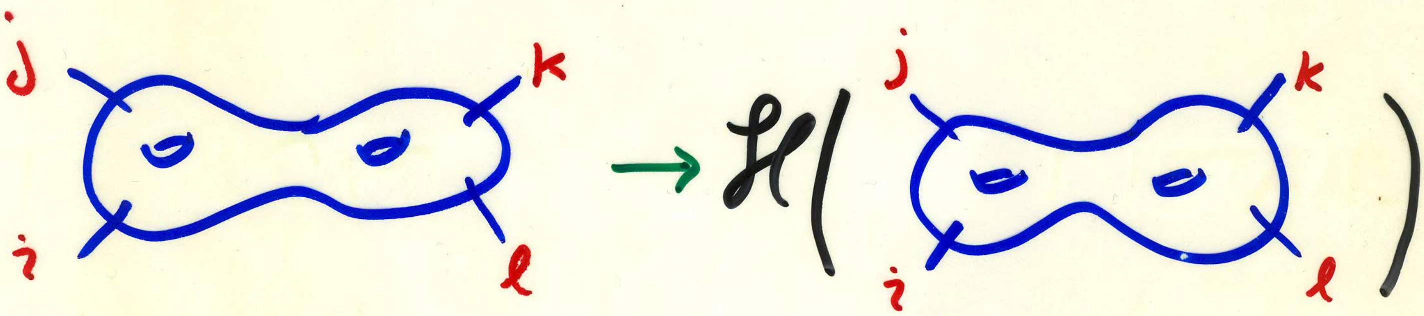
Segal

DATA:

1. I – FINITE SET OF LABELS (REPS)

$i \to i^\vee \to i \; ; \; 0^\vee = 0$

2. $(\Sigma ; (i_k, P_k)) \to \mathcal{H}(\Sigma ; (i_k\, P_k))$



DEPENDS ON TOPOLOGY & ORIENTATION

3. GEOMETRIC AUTOMORPHISMS → LINEAR TMNS "DUALITY TMNS"

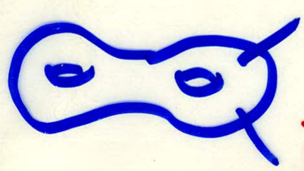
$f \to \mathcal{H}(f)$

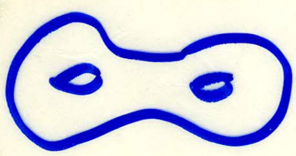
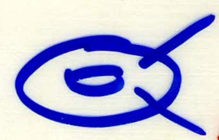
CONDITIONS:

1. $f \mapsto \mathcal{H}(f)$ REPRESENTS M.C.G.

2. $\mathcal{H}(\overline{\Sigma}) \cong \mathcal{H}(\Sigma)^{\vee}$

3. $\mathcal{H}(\Sigma_1 \amalg \Sigma_2) \cong \mathcal{H}(\Sigma_1) \otimes \mathcal{H}(\Sigma_2)$

4. $\mathcal{H}($  $^{i}_{j}) \cong \bigoplus_{k \in I} \mathcal{H}($  $k) \otimes \mathcal{H}(k^{\vee}$  $^{i}_{j})$

$\mathcal{H}($  $) \cong \bigoplus_{k \in I} \mathcal{H}($  $^{k}_{k^{\vee}})$

5. $\mathcal{H}($  $^{j_1}_{j_2}) \cong \delta_{j_1, j_2^{\vee}}\, \mathbb{C}$

CONJECTURE 1: A MF DETERMINES THE C.A. OF A UNITARY RCFT –

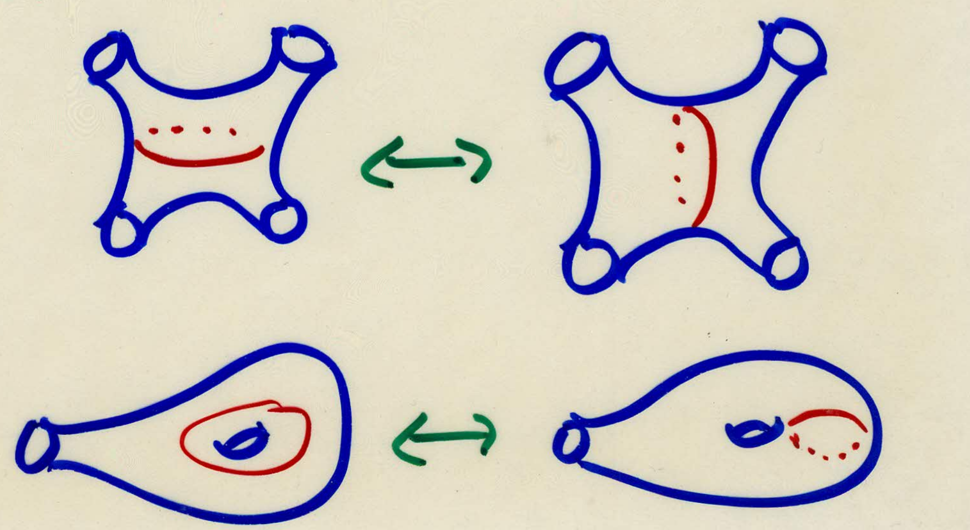
– UP TO ⊗ PRODUCT WITH "HOLOMORPHIC $c = 0 \bmod 24$ THEORIES ..."

(N.B. PRECISE EQUIVALENCE RELATION IS UNKNOWN)

THUS: WE SHOULD CLASSIFY MODULAR FUNCTORS

GLUEING ⟹ DECOMPOSE $\mathcal{H}$ INTO V^{i}_{jk}

MANY SEWING RELATIONS:

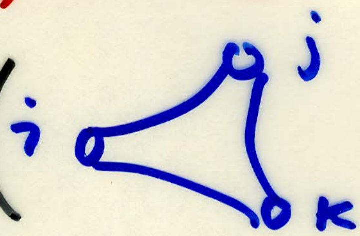


MODULAR TENSOR CTGRY

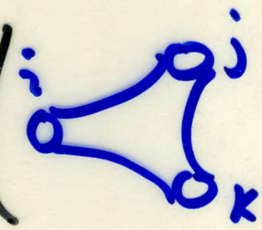
DATA:

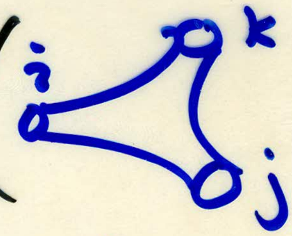
1. I = LABELS, REP'S, S-SEL. SECTORS

   $i \to i^\vee \to i \,;\; 0^\vee = 0$

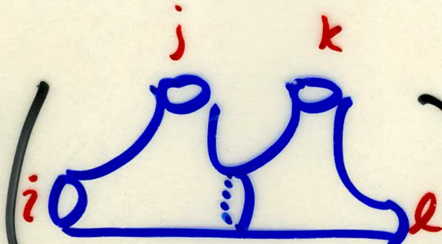
2. $V^i_{jk} = \mathcal{H}\Big($ 

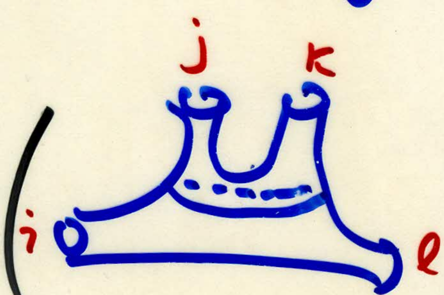
$\Big)$ "3-POINT COUPLINGS"

3. $\Omega^i_{jk} : \mathcal{H}\Big($ 

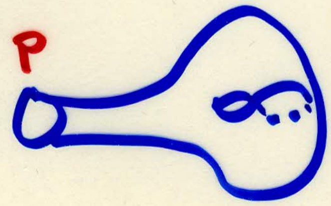
$\Big) \cong \mathcal{H}\Big($ 

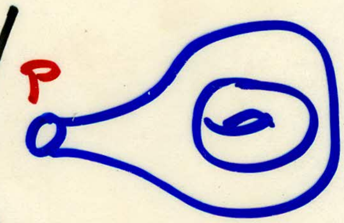
$\Big)$

4. $F : \mathcal{H}\Big($ 

$\Big) \cong \mathcal{H}\Big($ 

$\Big)$

5. $S(p) : \mathcal{H}\Big($ 

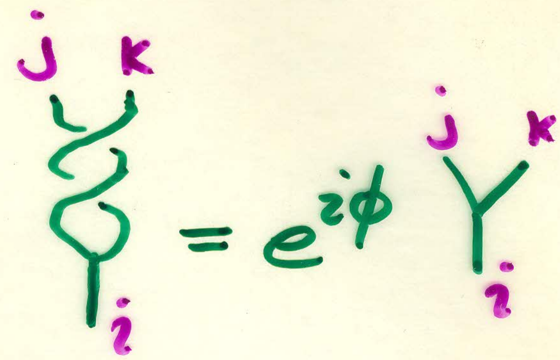
$\Big) \cong \mathcal{H}\Big($ 

$\Big)$

6. $e^{2\pi i c/8}$

CONDITIONS:

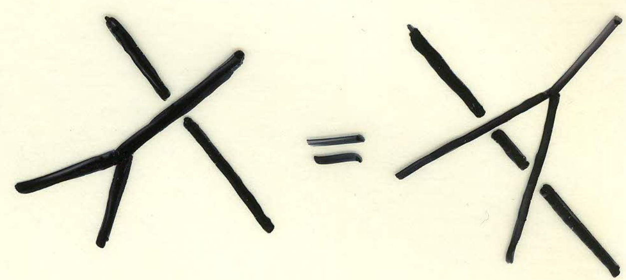
1. $V_{0\,j}^{\,i} = \delta_i^{\,j}\,\mathbb{C}$ ETC



2. $\Omega^2 = e^{i\phi}\cdot \mathbb{1}$

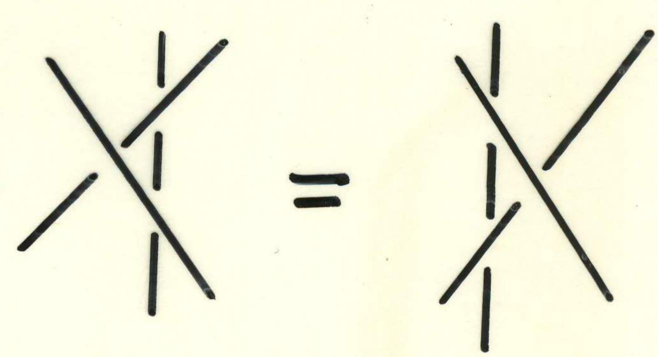
3. PENTAGON:

$$FFF = FF$$



4. 2 HEXAGONS:

$$F\Omega^{\pm 1}F = \Omega^{\pm 1}F\Omega^{\pm 1}$$



5. 3 TORUS EQS.

$$S^2(p) = \pm e^{-i\pi\Delta_p} C$$

$$STS = T^{-1}ST^{-1}$$

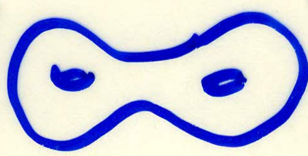
$$SaS^{-1} = b$$

COMPLETENESS THM:

w/ NATHAN SEIBERG

AN MTC DEFINES A PROJECTIVE M.F.

PF: DEF: $\mathcal{H}(\Sigma) \equiv \bigoplus V \otimes V \otimes \cdots \otimes V$

e.g. $\mathcal{H}\Big($  $\Big) \equiv \bigoplus_{ijk} V_{ij}^{i} \otimes V_{kk^\vee}^{j}$

DEFINE $\mathcal{H}(f)$ VIA Ω, F, S

CHECK: 1. INDEPENDENCE OF SEWING

2. RELATIONS OF DUALITY GROUPOID

$\therefore$ MODULAR FUNCTORS ARE DETERMINED BY A <u>FINITE</u> SET OF DATA + CONDITIONS

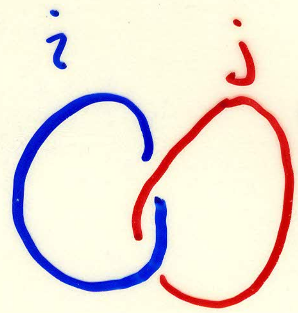
REMARK FOR MODULAR MAVENS

w/ N. SEIBERG

CAN SIMPLIFY $SaS^{-1} = b$:

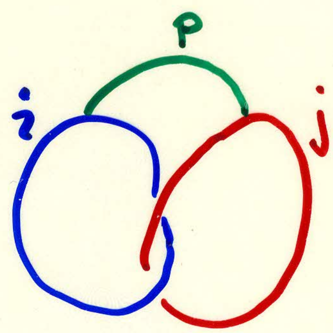
TWO CONSEQUENCES OF THIS EQUATION ARE

$$\frac{S_{ij}}{S_{00}} \overset{\text{CSW}}{=}$$



$$\underset{\text{Turaev}}{\overset{\text{Resh.}}{=}} \frac{\left(B^2\begin{bmatrix} i & j \\ i & j \end{bmatrix}\right)_{00}}{F_i\, F_j}$$

$$\frac{S_{ij}(p)}{S_{00}} \overset{\text{CSW}}{=}$$



$$= \sqrt{F_p}\, \frac{\left(B^2\begin{bmatrix} i & j \\ i & j \end{bmatrix}\right)_{p0}}{F_i\, F_j}$$

RESULT: NO FURTHER CONSEQUENCES

∴ <u>DEFINE</u> S IN TERMS OF Ω, F

ANALOGY WITH GROUP THRY

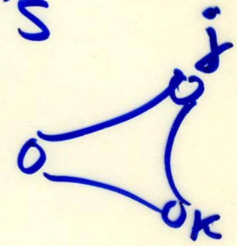
I: REP'S,
$i \to i^\vee$ CONJ.

I : LABELS,
$i \to i^\vee$, $0^\vee = 0$

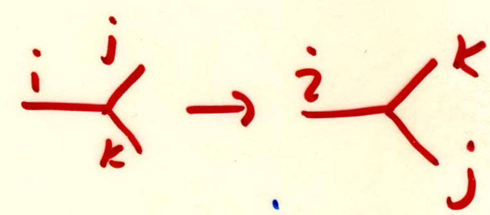
SPACE OF 3J COUPLINGS:

$V_{jk}^{i} = \mathrm{Hom}(V_j \otimes V_k, V_i)$

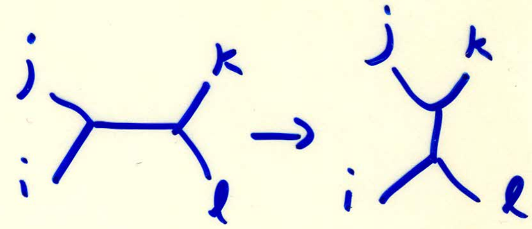
SPACE OF CVO'S

$V_{jk}^{i} = \mathcal{H}($  $)$

SYMMETRY OF CPLNGS (COMMUTATIVITY CNSTRT)

Ω: 

6J SYMBOLS (ASSOC. CONSTRT)

F: 

KEY POINT: $g=0$ AXIOMS, $\Omega^2 = 1$ PLUS INTEGRALITY CONDITION: $\frac{1}{F_i} \in \mathbb{Z}_+$ $\Rightarrow$ CLASSICAL RECONSTRUCTION OF AN (ALGEBRAIC) GROUP

DELIGNE

DOPLICHER-ROBERTS

ASCENT TO QUANTUM RECONSTRCTN!

HOW TO GENERALIZE GROUP THEORY?

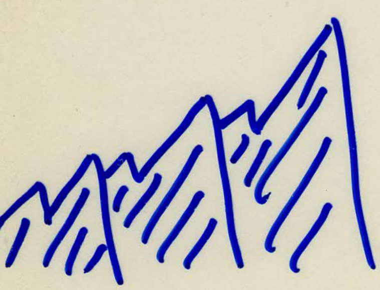
MEANING OF $g=0$ IS CLEAR:

$\Omega: V\otimes W \approx W\otimes V$

$F: V\otimes(W\otimes Z) \approx (V\otimes W)\otimes Z$

$g=0$: COMPATIBILITY AXIOMS FOR REP'S OF QUASITRIANGULAR HOPF ALG'S.

WHAT DO $g=1$ EQS. MEAN??



WHAT IS THE ANALOG OF DELIGNE'S INTEGRALITY CONDITION???

MY FAVORITE FUNCTORS

G : ANY COMPACT GROUP

λ : INVARIANT INTEGRAL FORM ON LIE(G)

(Technically : $\lambda \in H^4(BG;\mathbb{Z})$ Dijkgraaf & Witten)

$(G,\lambda) \Longrightarrow$ GET A M.F. (2 WAYS)

$\boxed{\begin{array}{c}\text{CSW}(G,\lambda)\text{ : CSW GAUGE THEORY} \\ S = \frac{1}{4\pi}\int \lambda(A, dA + A^2) \bmod 2\pi\mathbb{Z}\end{array}}$

QG(G,λ) : LET U = RATIONALLY DEFORMED QUANTUM GROUP

DEFINE A M.T.C. VIA REP(U)

e.g. G = SIMPLE $\lambda = k \cdot \mathbb{I}$

$U = U_q(\mathfrak{g}) \quad q = e^{2\pi i/k+h}$

PHENOMENOLOGY

A: $CSW(G,\lambda) = Q.G.(G,\lambda)$

WHEN BOTH SIDES ARE KNOWN

B: THE M.F. OF **EVERY** KNOWN UNITARY RCFT IS IN THIS LIST!

WITTEN
G.M. & N.S.

$\boxed{\text{CONJECTURE 2: THE M.F. OF ANY UNITARY RCFT} = CSW(G,\lambda) \text{ FOR SOME PAIR } G,\lambda;\ G = \text{CMPT GRP},\ \lambda \in H^4(BG;\mathbb{Z})}$

1. $\hat{G}_k \longrightarrow (G, k\cdot\mathbb{1})$ G - simple, $\pi_1 = 0$

2. EXTENDED ALGEBRAS $\longrightarrow$ $G \rightarrow G/Z$ $Z \subset$ CENTER

3. $G/H \longrightarrow \frac{G \times H}{Z}$; $\lambda_{G/H} = \begin{pmatrix} \lambda_G & \\ & -\lambda_H \end{pmatrix}$

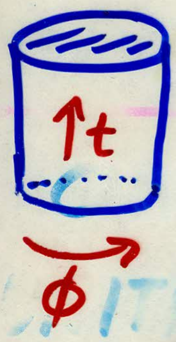
4. ORBFLDS $\longrightarrow$ $G \rightarrow P \ltimes G$

STRONG CONJECTURE

EXTEND M.F. TO $\mathcal{H}(\Sigma)$, $\partial\Sigma \neq \emptyset$

$\mathcal{H}$(DISK) = FULL CHIRAL ALG.

WITTEN

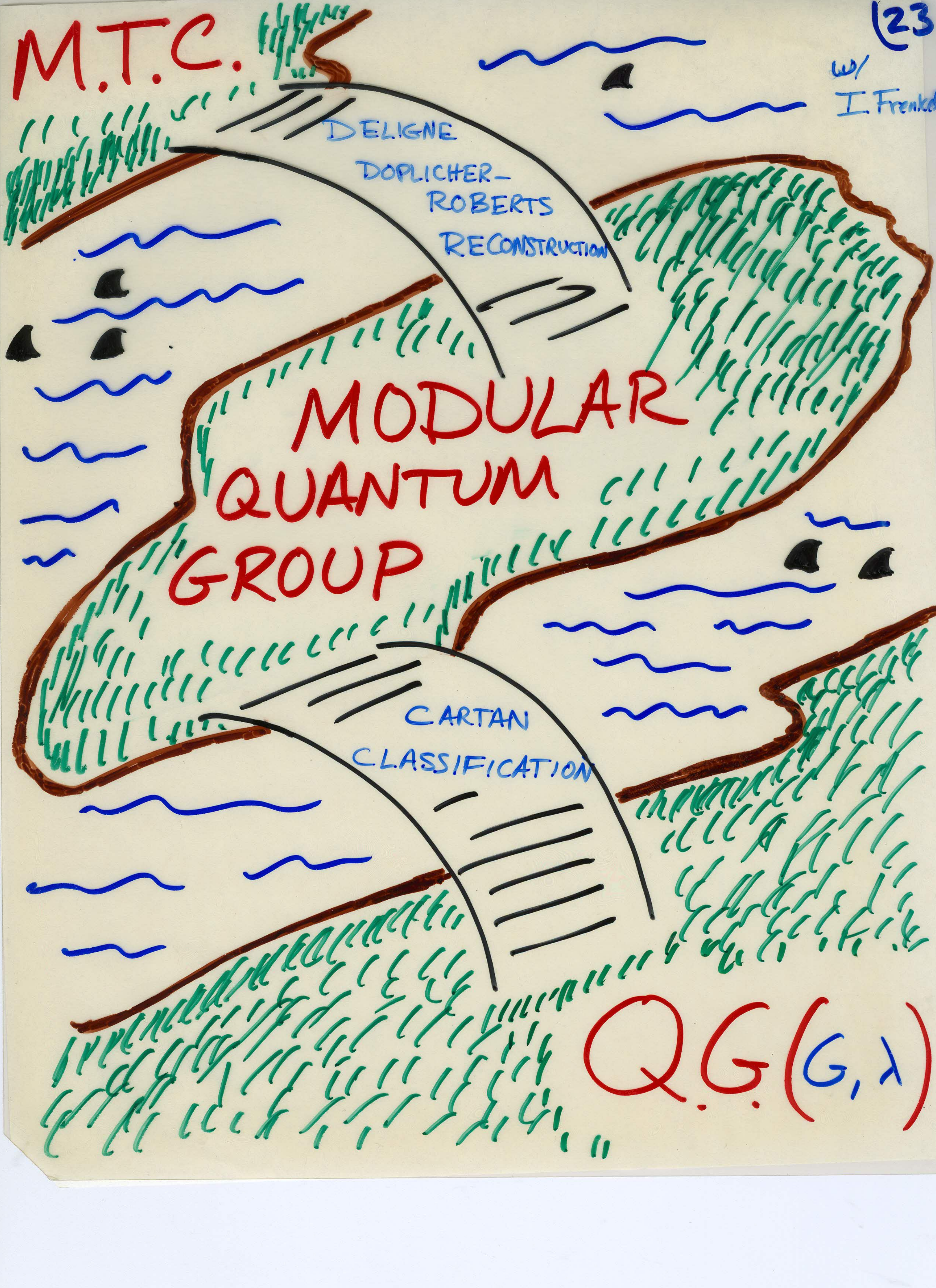


$$\int \frac{DA}{\mathrm{vol}\,\mathcal{G}} e^{ikCS(A)} = \int Dg(\phi,t)\, e^{ikS_{WZW}(g)}$$

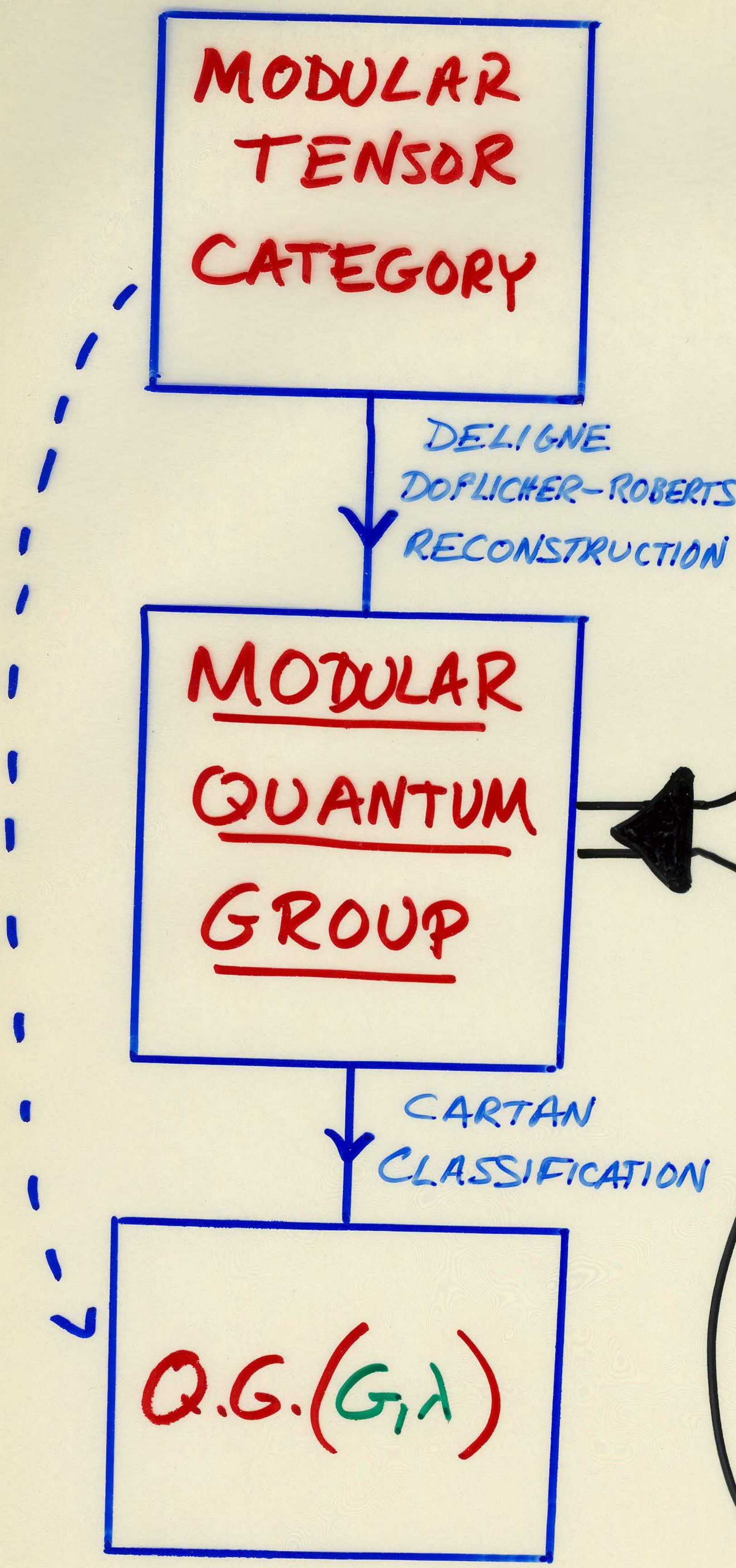
NONTRIVIAL REPS:

CONJECTURE 2': ALL C.A.'s ARE OBTAINED FROM QUANTIZATION OF CSW(G, λ) ON $D \times \mathbb{R}$

N.B. NOT ALL C.A.'S HAVE BEEN OBTAINED THIS WAY (YET)


M.T.C.
23
w/ I. Frenkel
DELIGNE
DOPLICHER-
ROBERTS
RECONSTRUCTION
MODULAR
QUANTUM
GROUP
CARTAN
CLASSIFICATION
Q.G.(G, λ)

w/ I. Frenkel



EXPLAINING THE MODULAR PROPERTIES OF RATIONALLY DEFORMED QUANTUM GROUPS SHOULD LEAD TO A SUITABLE DEFINITION OF AN ABSTRACT "MODULAR QUANTUM GROUP"

w/ I. Frenkel

3 TORUS EQS. $\Longleftrightarrow$ HARMONIC ANALYSIS ON SELF-DUAL QUANTUM GROUPS

PETER-WEYL: $L^2(G) \cong \bigoplus_\lambda V_\lambda \otimes V_\lambda^\vee$

FOURIER TMN: $L^2(G) \to L^2(\hat{G}) \equiv \bigoplus_\lambda \mathrm{End}(V_\lambda)$

SO $G = \hat{G}$ (e.g. $G = \mathbb{R}, \mathbb{Z}/N$) $L^2(G) \circlearrowleft$

$$Sf(k) = \int e^{ikx} f(x)\, dx$$

1. $S^2 = C$ $\Longleftrightarrow$ $S^2 f(x) = f(-x)$

2. $STS = T^{-1}ST^{-1}$ $\Longleftrightarrow$ $\begin{cases} \text{GAUSSIAN} \longrightarrow \text{GAUSSIAN} \\ \int e^{ikx} e^{-\frac{i}{2}x^2} e^{ixk'} dx = e^{\frac{i}{2}k^2} e^{ikk'} e^{\frac{i}{2}(k')^2} \end{cases}$

3. $SaS^{-1} = b$ $\Longleftrightarrow$ $S(f * g) = S(f) \cdot S(g)$

MODULAR QUANTUM GROUPS $(sl(2))$

PETER WEYL: $U_q(sl(2))^* \cong \bigoplus_{j=0}^{\infty} V_j \otimes V_j^{\vee} \qquad |q| \neq 1$

TRUNCATE: $\mathfrak{A} \equiv \bigoplus_{j=0}^{k/2} V_j \otimes V_j^{\vee} / \sim \qquad q = e^{\frac{2\pi i}{k+2}}$

$\mathfrak{A}$ SPANNED BY: $\chi_j^{p,\mu} \equiv \mathrm{Tr}_j \left[q^{J^3} \binom{j}{p\,j} (\mu \otimes \cdot) \right]$

"Q.G. TORUS" 1-POINT FNS $= \chi_j^{p,\mu} = j \bigcirc\!\!\overset{p}{\text{———}}\, \mu$

$\exists$ HAAR MEASURE $\Longrightarrow$ CONVOLUTION:

e.g. $\chi_j * \chi_{j'} = \dfrac{\delta_{jj'}}{\dim_q V_j} \chi_j$ etc.

ALSO PRODUCT STRUCTURE:

e.g. $\chi_j \cdot \chi_{j'} = \sum N_{jj'}^{j''} \chi_{j''}$ etc.

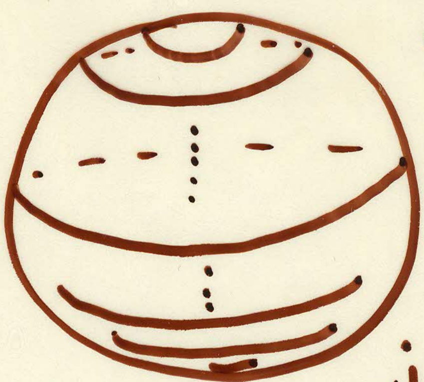
4 MAIN RESULTS

w/ I. Frenkel

1. $\mathfrak{a}$ IS A SEMISIMPLE ALG.

$$\mathfrak{a} \cong 1\,\overset{1}{\square} \oplus 2\,\overset{2}{\square} \oplus \cdots \oplus \overset{\frac{k}{2}+1}{\boxed{}} \oplus \cdots \oplus 2\,\overset{2}{\square} \oplus 1\,\overset{1}{\square}$$

VERLINDE'S CONJUGACY CLASSES: $e^{2\pi i \cdot \frac{2j+1}{k+2} T^3}$



2. $\exists$ INVERTIBLE F.T. $S : \mathfrak{a} \to \mathfrak{a}$

$$S(f * g) = S(f) \cdot S(g)$$

$$\Rightarrow \quad S^1 \sim B^2/F \cdot F$$

RECALL CLASSICAL HEAT KERNEL ON A COMPACT GROUP:

$$\rho(g,t) = \sum_{\lambda} (\dim V_\lambda)\, \chi_\lambda(g)\, e^{-\frac{t}{2} c_\lambda}$$

3. DEFINE Q.G. HEAT KERNELS

$$\rho^{\pm 1} \equiv \sum_{j} (\dim_q V_j)\, \chi_j\, q^{\pm c_j}$$

THEN: $S(\rho^{\pm 1}) = e^{\pm 2\pi i \frac{c}{8}} \rho^{\mp 1}$

— PLANE WAVES FREELY PROPAGATE TO PLANE WAVES —

4. (2) & (3) ABOVE, VIEWED AS CONDITIONS ON S, ARE EQUIVALENT TO 3 TORUS EQS.

EXAMPLE OF #4

$$S(\chi_j)\cdot S(\chi_{j'}) = S(\chi_j * \chi_{j'}) = \frac{\delta_{jj'}}{\dim_q V_j}\, S(\chi_j)$$

SO

$$S N_j S^T = \lambda_j = \begin{pmatrix} \lambda_j^{(0)} & & \\ & \ddots & \\ & & \lambda_j^{(k)} \end{pmatrix}$$

SO

VERLINDE'S THEOREM $\Longleftrightarrow$ INTERTWINING PROPERTY ON CHARACTERS

QUANTUM GROUP HARMONIC ANALYSIS IS THE SPACETIME MANIFESTATION OF WORLD-SHEET MODULAR INVARIANCE

WHERE'S THE ALGEBRA?

DELIGNE: $\frac{1}{F_i} = \dim V_i \in \mathbb{Z}_+$ (*)

OCNEANU PATH MODEL FOR FUSION GRAPH

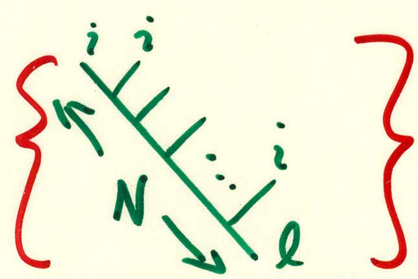
$\Rightarrow \frac{1}{F_i} = \frac{\dim \mathcal{H}_i}{\dim \mathcal{H}_0} = [A:B] =$ JONES INDEX

PASQUIER
FREDENHAGEN et. al.
Alvarez-Gaumé et. al.
....

SO $\Leftrightarrow$ GNLZTION OF (*) AUTOMATIC

$\therefore$ HOPE: MTC NEEDS NO FURTHER AXIOMS!

REVERSE BRAUER-WEYL RECIPROCITY

$V_i^N \equiv \bigoplus_{\ell=0}^{|I|}$  $\otimes V_\ell$

$\left\{\begin{matrix}\text{B-MATRIX}\\ \text{ALGEBRA}\end{matrix}\right\}' \overset{?}{=}$ ALGEBRA OF A M.Q.G.

BUT... V_ℓ NOT KNOWN...

1. $N \to \infty$ ERROR "WASHESOUT"?

2. $\dim_q V_\ell \longrightarrow$ MINIMAL CHOICE FOR V_ℓ?

"STANDARD MODEL"

– FOR SPACE OF UNITARY RCFT's –

ANY CHIRAL ALGEBRA

$\sim X \otimes Y$

X : MINIMAL OBJECT FOR (G,λ)

Y : "$c = 0 \bmod 24$ THEORY"

ANY GLUING OF LEFT-MOVERS WITH RIGHT-MOVERS

Dijkgraaf-Verlinde
G.M. &
N. Seiberg

$\sim \omega$: AUTOMORPHISM OF THE F.R.A.

$\omega : I \rightarrow I \quad N_{ijk} = N_{\omega(i)\omega(j)\omega(k)}$

CHOICE OF CHALLENGE

MODEL LOVERS : PROVE IT'S RIGHT: RECONSTRUCT!

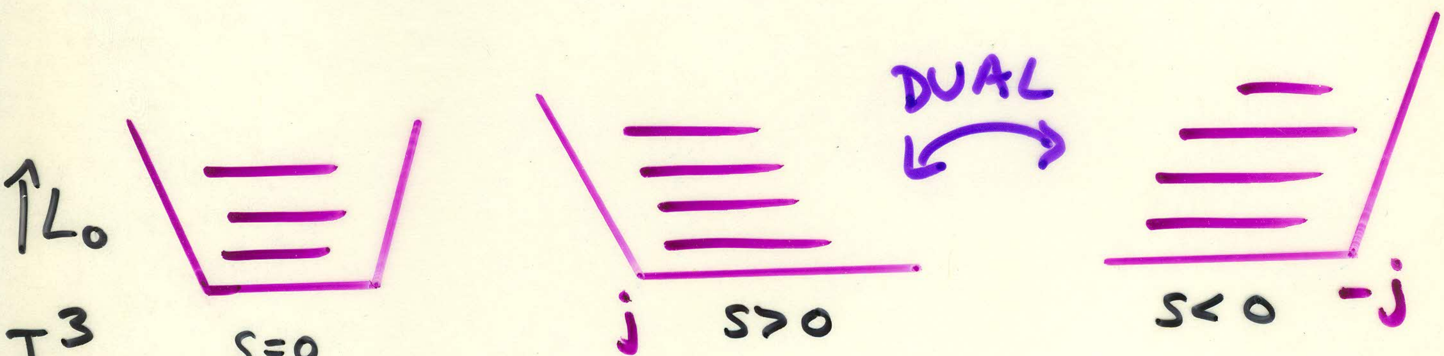
MODEL HATERS : PROVE IT'S WRONG
FIND SPORADIC RCFT's!

1. FUN WITH FRACTIONAL LEVEL $sl(2)$

KAC-WAKIMOTO / KPZ

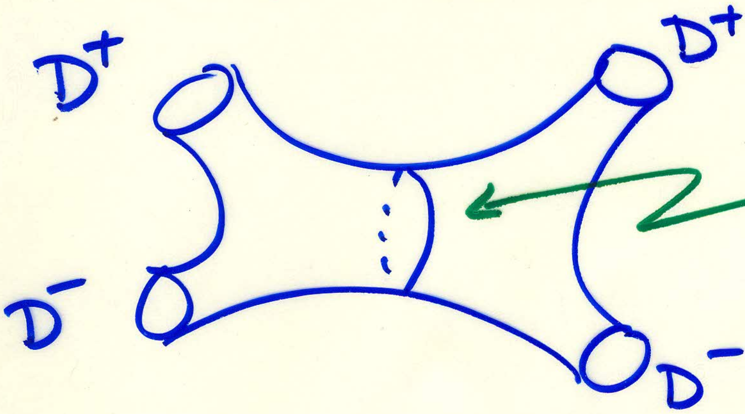
$k+2 = p/q \quad J_{r,s} = \frac{1}{2}\left[(r-1) - s(k+2)\right] \quad \begin{array}{l} 1 \le r \le p-1 \\ 0 \le s \le q-1 \end{array}$

MODULAR INVT SET OF REPS:



$F(j = \frac{r-1}{2}) \qquad D^-(j) \qquad D^+(-j)$

... MANY CONCEPTUAL PROBLEMS ...



CONTINUOUS SPECTRUM OF 2-SIDED REP'S?

DIXON
LYKKEN
PESKIN

$N_{ijk} < 0$!?

KOH-SORBA
BERNARD-FELDER

FREE FIELD EXAMPLE

C. Crnkovic
G.M.
N. Read
G. Zuckerman

β, γ BOSONIC GHOSTS, $\lambda = 1/2$

$J^+ = -\frac{1}{2}\gamma^2 \quad J^3 = \frac{1}{2}\beta\gamma \quad J^- = \frac{1}{2}\beta^2 \qquad k+2 = 3/2$

$$\mathcal{H}_{NS} = \mathcal{H}_{NS}^+ \oplus \mathcal{H}_{NS}^- = F(0) \oplus F(1/2)$$

$$\mathcal{H}_R = \mathcal{H}_R^+ \oplus \mathcal{H}_R^- = D^+(-\tfrac{1}{4}) \oplus D^-(+\tfrac{1}{4})$$

$e^{\phi/2} \times e^{-\phi/2} = 1 \Longrightarrow$ $\widehat{sl(2)}$ Fusion Rules Truncate All 2-Sided Reps!

GENERAL SPECTRUM : $\displaystyle\bigoplus_{j=0}^{\frac{p-2}{2}} F(j) \oplus' \left(D^+(J_{r,s}) \oplus D^-(-J_{r,s}) \right)$

$\therefore$ HOPE

1. FRACTIONAL LEVEL $sl(2)$ DOES MAKE SENSE
2. NO OBSTRUCTION TO 2D GRAVITY CORRELATORS IN L.C.G.
3. $s \neq 0$ FIELDS GENERALIZE FMS FERMION EMISSION VERTEX!

2. DUAL NONUNITARY THEORIES?

C. Crnković
G. M.
N. Read

AIM

UNITARY RCFT's : GROUPS = NONUNITARY RCFT's : SUPER-GROUPS

"ARGUMENT"

DUALITY $\Rightarrow$ MODULAR FUNCTOR $\Rightarrow$ 3D TOPOLOGICAL FIELD THEORY

CONJECT 3: ALL 3D T.F.T.'s ARE CSW FOR SOME "GROUP" WITTEN

ORDINARY GROUPS $\Rightarrow$ UNITARY

"SO" IT MUST BE A SUPERGROUP!

N.B. $[\text{super}, \text{affine}] \neq 0$

- CHANGE OF VARIABLES ON $D \times \mathbb{R}$
- KAC-CHARACTER FORMULA
- FUSION RULES

NEW KNOT POLYNOMIALS? N. Reshetikhin L. Crane

$\Rightarrow$ TIME-DEPENDENT STRING BCKGNDS?

3. IRRATIONAL CFT?

RECENT WORK OF VERLINDE'S STRONGLY SUGGESTS THAT MODULAR GEOMETRY MAKES SENSE IN IRRATIONAL CASE

.... MANY SUBTLETIES ...

NAIVELY: $$\chi_h(\tau) \equiv \frac{q^{h-c/24}}{\prod(1-q^n)}$$

$$\chi_h(-1/\tau) = \int_0^\infty dh' \, S(h,h') \, \chi_{h'}(\tau)$$

$$\boxed{S(h_1,h_2) = \int_{i\mathbb{R}+\epsilon} dz \, z^{-1/2} \, e^{2\pi\left(-\frac{h_1}{z} + h_2 z\right)}}$$

EXISTS... BUT LEADS TO INCONSISTENCIES...

WHEN S IS KNOWN TO MAKE SENSE IT IS A FOURIER TMN.

4. q- DEFORMATIONS

DRINFELD: $U_q(\hat{\mathfrak{g}})$: QUANTIZE Q.F.T.! (CURRENT ALGEBRA)

FRENKEL-JING VERTEX OPERATOR REP

$$[\alpha_n, \alpha_m] = \delta_{n+m,0} \frac{1}{n\hbar^2}(q^{2n} - q^{-2n})(q^n - q^{-n})$$

CURRENT ALGEBRA : $J^+(z) J^-(w) \sim \frac{1}{(z-w)^2} + \cdots$

q- CURRENT ALGEBRA : $J^+(z) J^-(w) \sim \frac{1}{(z-qw)(z-q^{-1}w)} + \cdots$

1. WHAT IS THE GEOMETRICAL MEANING OF THIS SPLIT SINGULARITY?

q - Related to "Spectral Parameter"

Reshetikhin-Semionov-Tian-Shansky

2. IS THERE A q-DEFORMED $\hat{\mathfrak{g}}$ SYMMETRY IN INTEGRABLE FIELD THEORIES?

$q-1 \sim$ mass

8. NEW PERSPECTIVES

1. UNIFICATION VIA 4-GEOMETRY — ATIYAH, WITTEN

2. REPRESENTATIONS OF GEOMETRIC CATEGORIES — SEGAL, FRENKEL, ⋮

ANY QUANTUM THEORY

SPACE $\longrightarrow$ HILBERT SPACE OF STATES

COBORDISM + ... $\longrightarrow$ LINEAR TRANSFORMATION e.g. PROPAGATION

SO QUANTUM MECHANICS IS A FUNCTOR!

IS IT USEFUL?

PUT DIFFERENTLY: MTC'S ARE USEFUL FOR RCFT BECAUSE OF $\approx$ T.F.T.

WHAT ABOUT OTHER THEORIES?

- INTEGRABLE 1+1
- 2D GRAVITY
- STRING THEORY
- NONTOPOLOGICAL F.T.'s!

PART III: EXPERIMENT ??

"REAL WORLD" APPLICATIONS IN CONDENSED MATTER PHYSICS?

2D 2nd ORDER PHASE TRNS – OF COURSE

CSW $\Longrightarrow$ APPL'S TO F.Q.H.E. & ANYONS

BASIC PRINCIPLES OF (NON)RELTVSTIC 2+1 QFT $\Longrightarrow$ DATA FOR MTC + g=0 AXIOMS

Fröhlich
Gabbiani
Marchetti

– NONABELIAN ANYONS NOT RULED OUT –

F.Q.H. SYSTEM $\Longrightarrow$ L.G. THEORY

Girvin-MacDonald
N. Read

$$A(z) \sim \int \frac{\langle \alpha | \psi^{+}\psi(z') | \alpha \rangle}{z - z'} \, d^2_z{}'$$

w/ CS TERM

LOW ENERGY, LONG RANGE: PURE CSW

$\Longrightarrow$ FULL MTC !?

w/ N. Read

EXAMPLES? GO IN REVERSE! USE RCFT TO PRODUCE CORRELATED ELECTRON GROUND STATES

$$\Psi_{LAUGHLIN} = \Big\langle \underbrace{\prod_{1}^{N} e^{i\sqrt{q}\,\phi(z_i)}}_{\text{Rational Torus } U(1)_{q/2}} \Big\rangle$$

q ODD

$\nu = \frac{1}{q}$

ELECTRONS WITH SPIN:

$$\Psi_{HALPERIN}(z_1^{\uparrow} \cdots z_N^{\uparrow} ; z_1^{\downarrow} \cdots z_N^{\downarrow}) =$$

$$\underbrace{\langle V^{+}(z_1^{\uparrow}) \cdots V^{+}(z_N^{\uparrow})\, V^{-}(z_1^{\downarrow}) \cdots V^{-}(z_N^{\downarrow}) \rangle}_{\widehat{SU}(2)\ k=1} \Big\langle \prod_{1}^{2N} e^{i\sqrt{q+\frac{1}{2}}\,\phi(z_i)} \Big\rangle$$

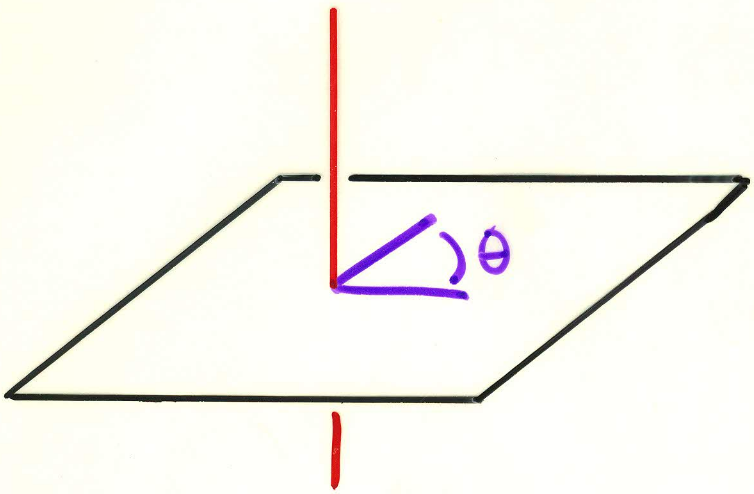
q EVEN

$\nu = \frac{2}{2q+1}$

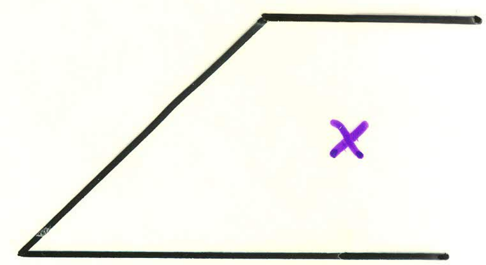
FROM CS-DESCRIPTION

SINGULAR GAUGE TMN $\Longleftrightarrow$ INSERTION OF VERTEX OP'S

w/ N. SEIBERG



$\approx$



$g(\theta) \sim e^{i\theta}$ $\qquad$ $V(z_0) \sim e^{i\phi(z_0)}$

SO WE HAVE

QUASIHOLE EXCITATIONS $\Longleftrightarrow$ INSERTION OF VERTEX OP'S

LAUGHLIN: $e^{\frac{i}{\sqrt{q}}\phi(z_0)}$ $\qquad$ HALPERIN: $V^{\pm}(z_0)\, e^{\frac{i}{2\sqrt{q+1/2}}\phi(z_0)}$

CHRGE: $\frac{1}{q}$ $\qquad$ $\frac{1}{2q+1}$

NEW STATES

w/ N. Read

EXAMPLE: $\mathrm{Pf}\,\frac{1}{z_{ij}} \cdot \prod_{i<j}(z_i - z_j)^q$

$$\Psi_{Pf} = \underbrace{\langle \psi(z_1) \cdots \psi(z_{2N}) \rangle}_{\text{ISING}} \langle \prod_{1}^{2N} e^{i\sqrt{q}\,\phi(z_i)} \rangle$$

1. $\nu = \frac{1}{q}$ q even !

2. $\exists$ HAMILTONIAN SUCH THAT Ψ_{Pf} IS NONDEGENERATE, INCOMPRESSIBLE, GROUNDSTATE

$$\mathcal{H} \sim \sum (-i\nabla_i - A)^2 + V \sum_{i<j} P^{ij}_{q+1} + \text{MANY BODY INT'S}$$

3. ORDER PARAMETER : $U^q(z)\psi^\dagger(z) \frac{1}{z-w} U^q(w)\psi^\dagger(w)$

N. Read

$$\Big\langle \underbrace{U^{q}(z)\,\psi^{\dagger}(z)}\ \underbrace{U^{q}(w)\,\psi^{\dagger}(w)} \Big\rangle_{Pf}$$

HAS VEV IN THE PFAFFIAN STATE

LAUGHLIN ORDER PARMETER: **PAIRED**

$\Rightarrow \exists$ EXCITATIONS OF FLUX $= \frac{1}{2}$ AND CHARGE $= 1/2q$

– AND ALWAYS COME IN PAIRS –

ADIABATIC FLUX ARGUMENT $\Rightarrow$

EXPLICIT FORMULA FOR PAIR EXCITATION WAVEFUNCTION:

$$\Psi^{PAIR}(z_1, \cdots z_{2N}; z_0, \tilde{z}_0) =$$

$$\left\{ \sum_{\sigma} \frac{\mathrm{sgn}\,\sigma}{z_{\sigma(1)\sigma(2)} \cdots z_{\sigma(2N-1)\sigma(2N)}} \prod_{k=1}^{2N} \Big[(z_{\sigma(2k-1)} - z_0)(z_{\sigma(2k)} - \tilde{z}_0) + z_0 \leftrightarrow \tilde{z}_0 \Big] \right\} \cdot$$

$$\cdot \prod_{i<j} (z_i - z_j)^{q}$$

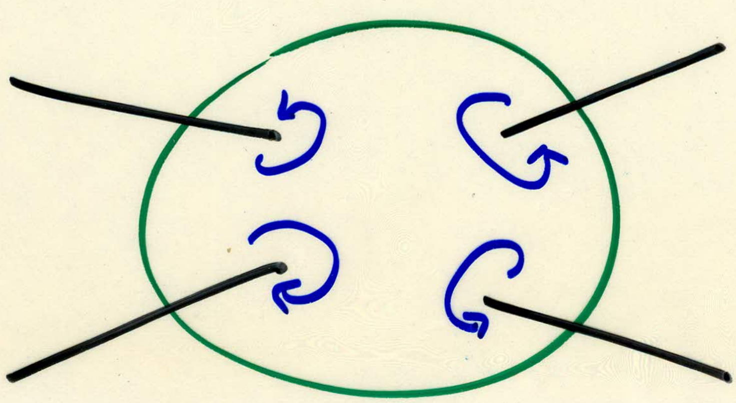
!! EXACTLY REPRODUCED BY !!

$$\left\langle \prod_{1}^{2N} \psi(z_i)\, e^{i\sqrt{q}\,\phi(z_i)}\, \sigma(z_0)\, e^{\frac{i}{2\sqrt{q}}\phi(z_0)} \cdot \sigma(\tilde{z}_0)\, e^{\frac{i}{2\sqrt{q}}\phi(\tilde{z}_0)} \right\rangle$$

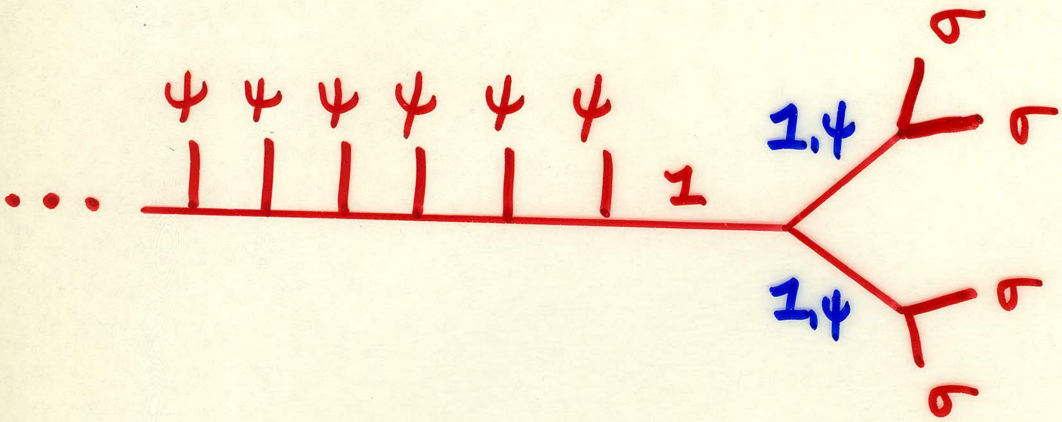
ASSUME EXISTENCE OF L.G. DESCRIPTION W/ CS-TERM

4 QUASIPARTICLE STATES:

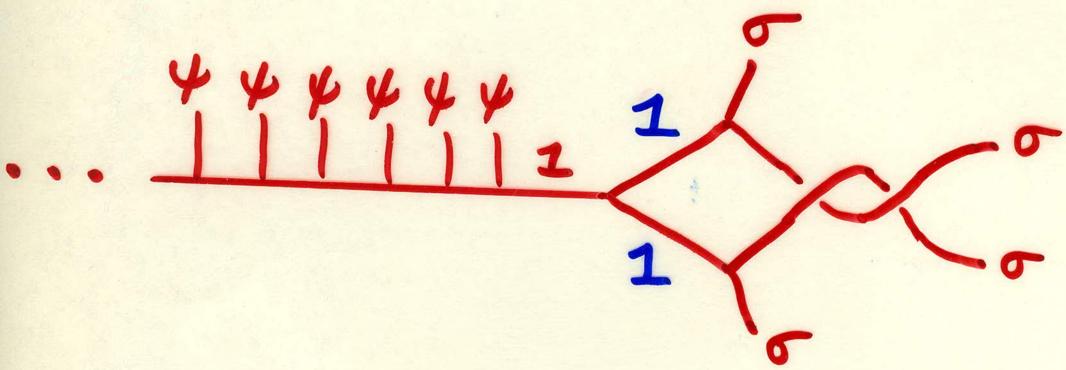
$$\left\langle \prod_{1}^{2N} \psi(z_i)\, e^{i\sqrt{q}\,\phi(z_i)} \prod_{1}^{4} \sigma(w_i)\, e^{\frac{i}{2\sqrt{q}}\phi(w_i)} \right\rangle$$



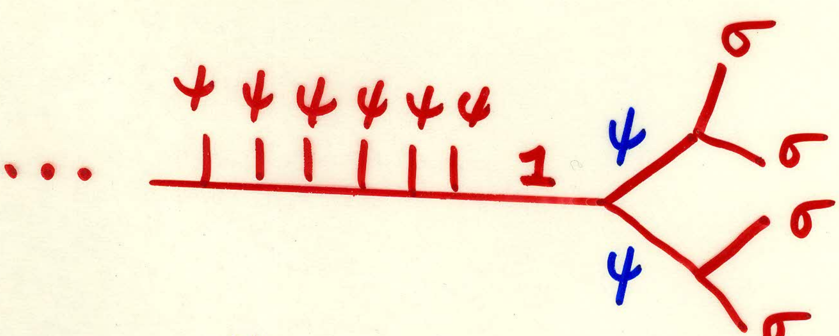
DOUBLY DEGENERATE:



ANALYTIC CONTINUATION:



$=$



$\times\ e^{2\pi i/8}$

"PHYSICAL" REALIZATION OF

<u>NONABELIAN</u> <u>ANYONS</u>

CONCLUSIONS

1. ORGANIZED APPROACH TO PROOF OF THE MAIN CONJECTURE

2. MEANING OF $g=1$ EQS. : SUGGESTS NEW INTERPRETATION OF SPACETIME MEANING OF MOD INVCE

3. PARADIGM FOR MANY FUTURE DIRECTIONS

4. POSSIBLE APPLICATIONS TO FQHE