Physics 619 **HOMEWORK # 3**

DUE NOVEMBER 7, 2016

Problem I (5 pt)

Let χ be a free boson field with the propagator $\mathcal{D}(x)$,

$$\langle\, \chi(x)\, \chi(x')\, \rangle = \mathcal{D}(x - x')\ .$$

Show that

$$\langle\, \mathrm{e}^{\mathrm{i}\chi(x)\ -\mathrm{i}\chi(0)} \rangle = \mathrm{e}^{\mathcal{D}(\mathrm{x})-\mathcal{D}(0)}$$

Problem II (5 pt)

Show that in D dimensions

$$\gamma_b \gamma_a \gamma_b = (D-2)\ \gamma_a\ , \quad \gamma_b \gamma_{a_1} \gamma_{a_2} \gamma_b = 4\delta_{a_1 a_2} + (4-D)\ \gamma_{a_1}\gamma_{a_2}\ ,$$

$$\gamma_b \gamma_{a_1} \gamma_{a_2} \gamma_{a_3} \gamma_b = 2\, \gamma_{a_3}\gamma_{a_2}\gamma_{a_1} + (D-4)\ \gamma_{a_1}\gamma_{a_2}\gamma_{a_3}$$

Problem III (20 pt)

(a) Calculate the electron-self energy $\Sigma_2(p)$ at one-loop order in the generalized Lorentz gauge[1] and with dimensional regularization, i.e., reduce it to a linear combination of scalar "bubble" integrals of the form

$$J(p, m_1, m_2) = \int \frac{\mathrm{d}^d k}{(2\pi)^d}\ \frac{1}{(k^2+m_1^2)((k-p)^2+m_2^2)}\ .$$

It is convenient also introduce

$$I(m) = \int \frac{\mathrm{d}^d k}{(2\pi)^d}\ \frac{1}{k^2+m^2} \qquad \Longrightarrow \qquad J(0, m_1, m_2) = \frac{I(m_2) - I(m_1)}{m_1^2 - m_2^2}\ .$$

(b) Using the result of (a) calculate in the bare theory the one-loop correction to the fermion mass. What can be said of the gauge dependence of the result? Explain.

(c) Using the result of (a) calculate the divergent part of the counterterm $\delta_2^{(2)}$. Analyze two cases: (a) $\xi = 0$ (Landau gauge) and (b) $\xi = 3$ (Yennie gauge).

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[1] $\tilde{D}_{ab} = \frac{1}{k^2+\mu^2}\left(\delta_{ab} - (1-\xi)\ \frac{k_a k_b}{k^2+\xi\mu^2}\right)$. Keep μ finite.

Problem IV (5 pt)

Show that the following change of variables $\psi,\ \bar\psi,\ A$ ("C-conjugation")

$$\psi \to (\bar\psi C)^t\ ,\quad \bar\psi \to (C\psi)^t\ ,\quad A \to -A$$

where $C = \gamma_4\gamma_2$, are the symmetry of the action $\mathcal{A}_{QED}[\psi, \bar\psi, A]$.

Problem V (10 pt)

Using dimensional regularization and the renormalization condition $\Pi(\mu^2) = 0$ show that

$$\delta_3 = \frac{e^2}{12\pi^2}\Big(-\frac{2}{\epsilon} + \log(\mu^2) + const\Big) + O(e^4)$$