Physics 619 **HOMEWORK # 2**

DUE OCTOBER 9, 2016

Problem I (10 pt) :

The Dirac Lagrangian in a presence of external electromagnetic field A_μ is given by

$$\mathcal{L} = \bar{\psi}\big(i\gamma^\mu D_\mu - m\big)\,\psi \qquad \text{with} \qquad D_\mu = \partial_\mu + ie\,A_\mu\ .$$

(a) Derive the Euler-Lagrange equation for ψ.

(b) Show that

$$\Big(\,D_\mu D^\mu + m^2 + e\,S^{\mu\nu}\,F_{\mu\nu}\,\Big)\,\Psi = 0 \tag{1}$$

with $S^{\mu\nu} = \frac{i}{4}\,[\gamma^\mu, \gamma^\nu]$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

(c) Chose the gauge $A_0 = 0$ and turn off the electric field by assuming $\partial_t \vec{A} = 0$. Show that (1) reduces to

$$\left[\partial_t^2 + m^2 - (\vec{\nabla} - ie\vec{A})^2 - e\,\begin{pmatrix} \vec{\sigma}\cdot\vec{B} & 0 \\ 0 & \vec{\sigma}\cdot\vec{B} \end{pmatrix}\right]\,\psi = 0\ .$$

Show that in the non-relativistic limit this equation simplifies to the Schrödinger equation for an electron in a magnetic field (Pauli equation).

(d) Consider the relativistic Dirac equation in a constant magnetic field. Determine the spectrum of energies of one-particle excitation.

Problem II (10 pt) :

Let us modify the Dirac Lagrangian by adding a term

$$\Delta\mathcal{L} = \frac{ae}{2m}\,\bar{\psi}\,S^{\mu\nu}\,\psi\,F_{\mu\nu}$$

where a is a constant. Consider the nonrelativistic limit of the modified Dirac equation and show that the resulting modification of the Pauli equation amounts of changing the magnetic moment of the electron. Express the modified gyromagnetic ratio in terms of a.

Problem III (10 pt)

Compute the spin-averaged differential cross section for the Compton scattering (scattering of a photon by a charged particle) to a leading order in perturbation theory.