Physics 619 **HOMEWORK # 1**

DUE SEPTEMBER 26, 2016

Problem I (Peskin and Schroeder, Problem 2.1) (10 pt)

Classical electromagnetism (with no sources) follows from the action

$$S = \int d^4x \left(-\tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} \right), \qquad \text{where} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ .$$

(a) Derive Maxwell's equations as the Euler-Lagrange equations of this action, treating the components $A_\mu(x)$ as the dynamical variables. Write the equations in standard form identifying $E^i = -F^{0i}$ and $\epsilon^{ijk}B^k = -F^{ij}$.

(b) Construct the energy-momentum tensor for this theory. Note that usual procedure does not result in a symmetric tensor. To remedy that, we can add to $T^{\mu\nu}$ a term of the form $\partial_\lambda K^{\lambda\mu\nu}$, where $K^{\lambda\mu\nu}$ is antisymmetric in its first two indices. Such an object is automatically divergenceless, so

$$\hat{T}^{\mu\nu} = T^{\mu\nu} + \partial_\lambda K^{\lambda\mu\nu}$$

is an equally good energy-momentum tensor with the same globally conserved energy and momentum. Show that this construction with

$$K^{\lambda\mu\nu} = F^{\mu\lambda}A^\nu \ ,$$

leads to an energy-momentum tensor $\hat{T}^{\mu\nu}$ that is symmetric and yields the standard formulae for the electromagnetic energy and momentum densities:

$$\mathcal{E} = \tfrac{1}{2}\left(\vec{E}^2 + \vec{B}^2\right); \quad \vec{S} = \vec{E} \times \vec{B} \ .$$

Problem II (10 pt)

Consider a photon that is linearly polarized along x-axis and moves in z-direction with respect to an inertial frame S. What is the polarization measured by an observer S' that moves in x-direction with a velocity v?

Problem III (10 pt)

Calculate explicitly the photon propagator in the Feynman gauge in the coordinate (Minkowski space) representation.

Problem IV (10 pt)

Consider a family of gauge conditions,

$$G(A) = \mathrm{div}\vec{A} - \omega(x) = 0$$

Integrating over ω with

$$\rho[\omega] = \mathrm{e}^{-\frac{1}{2\xi} \int \mathrm{d}^4x \, \omega^2(x)}$$

find the action $\mathcal{A}_\xi$ and the propagator $\tilde{D}_{ab}(k)$.