

FINAL EXAM

**Room:** ARC-107

**Time:** Thursday, December 17, 10am-1pm

GROUND RULES

- There are four problems based on the above-listed material.
- Closed book
- Closed notes
- Partial credit will be given. Do as many parts of a problem as possible.

PROGRAM

I. NEWTONIAN MECHANICS

• **Space (Survey of undergraduate level vector algebra)** : Euclidean space. Vectors. Orthogonal transformation. Kronecker and Levi-Civita symbols. Euler theorem. Cross product.

**Literature:** 1) R.A. Sharipov, Quick Introduction to Tensor Analysis (Chapters I-III);  
2) H. Goldstein: Classical Mechanics (Chapters 4.1-3, 4.5-4.7)  
3) P. Lampert. Course Notes (Chapters 7.1-7.3)

• **Newton's laws (Survey of undergraduate level mechanics):** Time. Reference frame. Material point (particle). Velocity. Galilean transformations and principle of relativity. Newton's laws. Mass and force. Examples of forces.

**Literature:** 1) P. Lammert, Course Notes (Chapters 1.1, 1.2, 7.1)

• **Systems of particles (Survey of undergraduate level mechanics):** Internal and external forces. Linear and angular momentum. Energy. Conservative and nonconservative forces. Virial theorem.

**Literature:** 1). Lammert, Course Notes (Chapters 1.3-1.8)

- 2). H. Goldstein: Classical Mechanics (Chapters 1.1, 1.2, 3.4);
- 3). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 8, 10).

• **Motion in one dimension:** Local solution of the Newton equation. Phase curves.

**Literature:** 1). L.D. Landau and E.M. Lifshitz: Mechanics (Chapter 11).

- 2). Lammert, Course Notes (Chapters 4.1-1.2)

• **Two-body problem:** Reduced mass. Motion in a central field. Second Kepler law. Binet's equation.

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 3.1-3.5);

- 2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 13, 14).
- 3). Lammert, Course Notes (Chapter 4.3)

• **Kepler problem:** Closed orbits. The Laplace-Runge-Lenz vector.

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 3.7-3.9);

- 2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 15).
- 3). Lammert, Course Notes (Chapter 4.4)

• **Scattering:** Differential scattering cross section. Rutherford's formula.

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 3.10, 3.11);

- 2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 18, 19).

## II. LAGRANGIAN MECHANICS

• **Configuration space:** Generalized coordinates. Examples (cylindrical, spherical, ellipsoidal coordinates). Generalized velocities. Kinetic energy. Holonomic constraints. Degrees of freedom.

**Literature:** 1). Lammert, Course Notes (Chapters 2.1-2.2);

- 2). H. Goldstein: Classical Mechanics (Chapters 1.3);
- 3). L.D. Landau and E.M. Lifshitz: Mechanics (Chapter 1).

• **Lagrange's equation:** D'Alembert's principle. Lagrange's equation.

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 1.4, 1.6);

- 2). Lammert, Course Notes (Chapters 2.3-2.4);

3). Lammert, Course Notes (Supplemental, Chapter 1)

• **Hamilton's principle:** Calculus of Variations. Hamilton's principle. Properties of the Lagrange function (Lagrangian). Lagrange's equations for systems with constraints. Lagrange multipliers.

- Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 2.1-2.3);  
2). Lammert, Course Notes (Chapters 2.5-2.8);  
3). Lammert, Course Notes (Supplemental, Chapters 2, 3)  
4). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 2-5).

• **Symmetries and conservation laws:** Cyclic coordinates. Routh function (Routhian). Integrals of motion. Symmetries. Noether's theorem. Energy. Momentum. Angular momentum.

- Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 2.6-2.7, 8.3);  
2). Lammert, Course Notes (Chapters 2.9);  
3). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 6-10, 41).

### III. RIGID BODY MOTION

• **Kinematics of rigid body motion:** Configurational space of a rigid body . Euler angles. Angular velocity.

- Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 4.1-4.8);  
2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 31).

• **The Lagrangian for a rigid body:** Inertia tensor. Principal axis. Angular momentum and kinetic energy of a rigid body. Heavy symmetrical top. Rigid body in contact. Non-holonomic constraints.

- Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 5.1-5.4, 5.7);  
2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 32-34, 38).

• **The equation of motion of a rigid body (self-study):** Euler's equations. Free asymmetrical top.

- Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 5.5, 5.6);

2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 34, 36, 37).

• **Motion in a non-inertial frame of reference (self-study):** Motion in a non-inertial frame of reference. Coriolis force.

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapter 4.10);

2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapter 39).

3) P. Lampert. Course Notes (Chapters 7.4-7.6)

#### IV. SMALL OSCILLATIONS

• **Oscillations of systems with more than one degree of freedom:** Formulation of the problem. Pair of forms. Characteristic frequencies. Normal coordinates (modes).

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 6.1-6.4);

2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 21, 23, 24).

3) P. Lampert. course Notes (Chapters 3.1-3.3, 5.1-5.7)

#### V. HAMILTONIAN MECHANICS

• **Canonical equations:** Legendre transformation. Phase space. Hamiltonian. Canonical equations of motion. Modified Hamilton's principle. Hamiltonian and energy.

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 8.1, 8.3);

2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 40, 41)

• **Variational principle and Liouville's theorem:** Modified Hamilton's principle. Liouville's theorem.

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 8.6, 9.9);

2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapter 46)

• **Poisson bracket:** Poisson bracket. Integrals of motions. Liouville-Arnold theorem (weak form).

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 9.5-9.7);

2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapter 42)

- **Canonical transformations:** Canonical transformations. Generating functions.

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 9.1-9.3);

2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapter 45)

## VI. HAMILTON-JACOBI THEORY (self-study)

- **Hamilton-Jacobi equation:** Hamilton-Jacobi equation. Separation of the variables.

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 10.1-10.5);

2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 47, 48)

- **Angle-action variables:** Angle-action variables. Adiabatic invariants. Canonical perturbation theory.

**Literature:** 1). H. Goldstein: Classical Mechanics (Chapters 10.6-10.8, 12.5);

2). L.D. Landau and E.M. Lifshitz: Mechanics (Chapters 49, 50, 52)