PHYSICS 504 Spring 2025

ELECTRICITY & MAGNETISM II
(Tentative Syllabus)

This is a tentative schedule of what we will cover in the course. It is subject to change, often without notice. These will occur in response to the speed with which we cover material, individual class interests, and possible changes in the topics covered. Use this plan to read ahead from the lecture notes and following textbooks, so you are better equipped to ask questions in class.

[1] L D Landau and E.M. Lifshitz "The Classical Theory of Fields", Volume 2

[2] J.D. Jackson "Classical Electrodynamics" 3rd edition

[3] A. Zangwill, "Modern Electrodynamics", 1st edition

[4] H.Goldstein, C. Poole and J. Safko, "Classical Mechanics", 3rd edition

Special Relativity with basics of relativistic Field Theory

- **PRELIMINARIES**

  Space in Classical Physics: Cartesian space. Euclidean structure. Curvilinear coordinates. Metric-preserving coordinate transformations. Translations of the origin. Proper and improper orthogonal transformations. Euler's theorem. Isometries of $\mathbb{E}^3$. Active and passive points of view.

  Suggested literature: Lecture notes
  Secs.1.2.1-1.2.4,1.7 in [3]
  Secs. 4.1-4.6 in [4]

  Euclidean tensors: Euclidean vectors and pseudovectors. Levi-Cevita symbol. Cross product. Matrix of finite rotations. Kronecker product. Tensors. Invariant tensors and pseudotensors. Tensors of rank two. Irreducible representations of $SO(3)$. Angular momentum addition. Parity transformations. Irreducible representations of $O(3)$.

  Suggested literature: Lecture notes
  Secs. 1.2.5,1.8 in [3]
  Secs. 4.7,4.8 in [4]

  Spacetime in Classical Physics: Event. Causal structure in classical spacetime. Inertial frames. Galilean transformations. Galilean principle of relativity. Newton's first law.

  Suggested literature: Lecture notes
  Sec.22.2 in [3]

- **KINEMATICS OF SPECIAL RELATIVITY**

  Spacetime in Special Relativity: Causal structure in Special Relativity. Light cone. Spacetime interval. Proper time. Pseudo-Euclidean (Minkowski) space $\mathbb{M}^{1,3}$. Einstein principle of relativity.

  Suggested literature: Lecture notes
  §§1-3 in [1]
  Sec. 11.1 in [2]
  Sec.22.3 in [3]
  Sec.7.1 in [4]

  Lorentz group: Definition. Parity and time reversal transformations. Proper, improper, orthochronous, non-orthochronous Lorentz transformations. General structure of the Lorentz group. Lorentz boosts. Group of proper, orthochronous Lorentz transformations $SO^+(1,3)$.

  Suggested literature: Lecture notes
  §§4,5 in [1]
  Sec.11.2 in [2]
  Sec.22.4 in [3]

  Tensors in the Minkowski space: 4-velocity. Covariant and contravariant vectors. Tensors of rank 2. Metric tensor. Inner product in the Minkowski space. Tensors of higher rank in $\mathbb{M}^{1,3}$. Levi-Cevita symbol in $\mathbb{M}^{1,3}$. Pseudotensors.

  Suggested literature: Lecture notes
  §§6,7 in [1]
  Secs.11.3,11.4,11.6 in [2]
  Secs.22.5.1,22.5.2 in [3]

  Matrix representations of the Lorentz group: Rank 2 antisymmetric tensor. Quadratic invariants. Finite dimensional irreducible representations of $SO^+(1,3)$, $O^+(1,3)$ and $O(1,3)$.

  Suggested literature: Lecture notes

- **COVARIANT FORM OF MAXWELL'S EQUATIONS**

  First pair of Maxwell's eqs.: Fields. Field-strength tensor. Covariant form(s) of the first pair of Maxwell's eqs.

  Suggested literature: Lecture notes
  §§23-26 in [1]
  Secs.11.9,11.10 in [2]

  Simple physics behind Maxwell's eqs: Stokes's theorem. Faraday's law of induction. Monopoles. Gauss-Ostrogradsky theorem. Gauss's law. Ampère's law. Displacement current.

  Suggested literature: Lecture notes
  §§23-25 in [1]
  Secs.1.3,1.4,5.1-5.3,5.15,6.1,6.11,6.12 in [2]
  Secs.1.4,2.1,2.2 in [3]

  Second pair of Maxwell's eqs.: Covariant form. 4-current. The continuity equation.

  Suggested literature: Lecture notes
  §§28-30 in [1]
  Sec.1.5 in [3]

  Differential p-forms: Helmholtz's decomposition theorem. Definition of differential p-forms. Exterior derivative. Closed and exact forms. Poincaré lemma.

  Suggested literature: Lecture notes
  Sec.1.9 in [3]

4-potential: Definition. Bianchi identity. Maxwell's equation in terms of the 4-potential. Gauge invariance. Gauge fixing condition. Lorenz gauge.

Suggested literature: Lecture notes
§18 in [1]
Secs.6.2,6.3 in [2]
Secs.15.3 in [3]

- **VARIATIONAL PRINCIPLE**

Poisson's equation in curvilinear coordinates: Variational principle for Poisson's equation. Laplacian in curvilinear coordinates. Orthogonal coordinates.

Suggested literature: Lecture notes
Secs.1.7-1.12 in [2]

Variational principle for Maxwell's equations: The principle of least action in relativistic Field Theory. Lagrangian density. Euler-Lagrange equations. The action functional of the electromagnetic field.

Suggested literature: Lecture notes
§§27,30,32 in [1]
Sec.12.7 in [2]
Secs.13.1,13.2 in [4]

Maxwell's equations in curvilinear coordinates: Tensor fields in curvilinear coordinates. Differentiation. Exterior derivative. Divergency of a vector field. First pair of Maxwell's equations in curvilinear coordinates. The action functional of the electromagnetic field in curvilinear coordinates. Lorenz gauge fixing condition in curvilinear coordinates.

Suggested literature: Lecture notes
§§81-83,90 in [1]

Functional action for particles in electromagnetic field: The principle of least action for a free moving particle. Point-like charge in an external field. Covariant form of the equation of motions. Energy conservation law for a charge in a stationary external field. Energy density and energy flux. Poynting vector. Poynting's theorem.

Suggested literature: Lecture notes
§§$8, 9, 15-17$ in [1]
Secs.6.7,12.1 in [2]
Secs.7.9,7.10 in [4]

- **CONSERVATION LAWS**

Symmetries: Continuous and discrete symmetries of Classical Electrodynamics. Noether's theorem.

Suggested literature: Lecture notes
Sec.6.10 in [2]
Secs.15.1,15.2,24.4.2 in [3]
Sec.13.7 in [4]

Energy-momentum tensor: Canonical energy-momentum tensor in Field Theory. Symmetric energy-momentum tensor. Energy-momentum tensor for the electromagnetic field. Conservations of energy and momentum in a local, relativistic invariant field theory. Stress tensor. Energy-momentum tensor of a system of particles.

Suggested literature: Lecture notes
§§32,33,94 in [1]
Secs.6.7,12.10 in [2]
Secs.13.3,13.5,13.6 in [4]

Rotational invariance and angular momentum: 4-tensor of angular momentum. The center-of-energy theorem. Pauli-Lubanski 4-vector.

Suggested literature: Lecture notes
§§14, 32 in [1]
Secs.15.6,15.7 in [3]

Applications of Classical Electrodynamics

- **MAGNETOSTATICS**

Magnetic moment: Static magnetic field. Vector potential in the Coulomb gauge. Magnetic fields of a localized current distribution. Relation between magnetic and mechanical moments.

Suggested literature: Lecture notes
§§43,44 in [1]
Sec.5.3-5.6 in [2]
Secs.11.1,11.2 in [3]

Macroscopic equations: Magnetization. The magnetic field (intensity). Boundary conditions. Relation between magnetic (field) induction and magnetic field (intensity). Methods of solving boundary value problems in magnetostatic.

Suggested literature: Lecture notes
Sec.5.8-5.13 in [2]

Simple magnetic matter: Magnetic moment in an external magnetic field (torque, force, potential energy). Larmor's theorem. Diamagnetism. Paramagnetism. Curie's law. Exchange interaction.

Suggested literature: Lecture notes
§§45 in [1]
Sec.5.7 in [2]

- **QUASI-STATIC FIELDS**

Energy in a magnetic field: Energy of magnetic matter. Total free energy of a magnetic. Energy of a system of currents. Self- and mutual inductance. Estimation of self-induction for simple circuits.

Suggested literature: Lecture notes
Sec.5.16,5.17 in [2]

Quasi-static EM fields in conductors: Coulomb gauge. Quasi-static approximation. Skin effect.

Suggested literature: Lecture notes
Sec.6.3,5.18 in [2]
Secs.14.5-14.7, 14.10 in [3]

- **ELECTROMAGNETIC WAVES**

Waves in vacuum: Wave equation. Plane EM waves. Monochromatic waves. Helmholtz equation. Doppler effect. Elliptical, linear and circular polarization.

Suggested literature: Lecture notes
§§46-48 in [1]
Sec.7.1,7.2 in [2]
Secs.16.1-16.4.4, 16.6 in [3]

Waves in simple matter: Waves in nondispersive media. Wave impedance. Index of refraction. Reflection and refraction: Snell's law, Fresnel equations, reflection and transmission coefficients, polarization by reflection, Brewster's angle, total internal reflection.

Suggested literature: Lecture notes
Sec.7.1,7.3,7.4 in [2]
Secs.17.1-17.3 in [3]

Waves in dispersive matter I: Constitutive relations in a dispersive medium. Kramers-Kronig relations. Lorentz model for dispersion.

Suggested literature: Lecture notes
Secs.6.10,7.5, 7.10 in [2]
Secs.18,1, 18.2, 18.51,18.54, 18.7 in [3]

Waves in dispersive matter II: Plane waves in dispersive media. Phase velocity and group velocity. Conservation of energy in dispersive media: Poynting vector, effective energy density.

Suggested literature: Lecture notes
Secs.6.7,6.8, 7.8 in [2]
Secs.18.3,18.4, 18.6 in [3]

- **RETARDATION AND RADIATION**

Fields from moving charges: Green's functions for the wave equation. Lienard-Wiechert potentials and fields for a point charge. Point charge in uniform motion. Spectral decomposition of the retarded potentials.

Suggested literature: Lecture notes
§§62-64, in [1]
Secs.6.4, 6.5, 12.11, 14.1 in [2]
Secs.20.1-20.3, 23.1, 23.2 in [3]

Multipole fields and radiation: Fields of a system of charges at large distances. Dipole radiation. Quadrupole and magnetic dipole radiation.

Suggested literature: Lecture notes
§§ 66,67,71 in [1]
Secs.9.1-9.3 in [2]
Secs.20.5, 20.7 in [3]