PHYSICS 504   Spring 2024

ELECTRICITY & MAGNETISM II

Midterm Exam

Room: SEC-204

Time: February 29, (Thursday), 12:10-1:30 pm

Ground rules

- There are four problems based on the material listed below.
- This is a closed book, closed notes exam.
- Partial credit will be given. Do as many parts of a problem as possible.

Program

- **PRELIMINARIES**

  Space in Classical Physics: Cartesian space. Euclidean structure. Curvilinear coordinates. Metric-preserving coordinate transformations. Translations of the origin. Proper and improper orthogonal transformations. Euler's theorem. Isometries of $\mathbb{E}^3$. Active and passive points of view.

  Suggested literature: Lecture notes
  Secs.1.2.1-1.2.4,1.7 in [3]
  Secs. 4.1-4.6 in [4]

  Euclidean tensors: Euclidean vectors and pseudovectors. Levi-Cevita symbol. Cross product. Matrix of finite rotations. Kronecker product. Tensors. Invariant tensors and pseudotensors. Tensors of rank two. Irreducible representations of $SO(3)$. Angular momentum addition. Parity transformations. Irreducible representations of $O(3)$.

  Suggested literature: Lecture notes
  Secs. 1.2.5,1.8 in [3]
  Secs. 4.7,4.8 in [4]

  Spacetime in Classical Physics: Event. Causal structure in classical spacetime. Inertial frames. Galilean transformations. Galilean principle of relativity. Newton's first law.

  Suggested literature: Lecture notes
  Sec.22.2 in [3]

- **KINEMATICS OF SPECIAL RELATIVITY**

  Spacetime in Special Relativity: Causal structure in Special Relativity. Light cone. Spacetime interval. Proper time. Pseudo-Euclidean (Minkowski) space $\mathbb{M}^{1,3}$. Einstein principle of relativity.

  Suggested literature: Lecture notes
  §§1-3 in [1]
  Sec. 11.1 in [2]
  Sec.22.3 in [3]
  Sec.7.1 in [4]

  Lorentz group: Definition. Parity and time reversal transformations. Proper, improper, orthochronous, non-orthochronous Lorentz transformations. General structure of the Lorentz group. Lorentz boosts. Group of proper, orthochronous Lorentz transformations $SO^+(1,3)$.

  Suggested literature: Lecture notes
  §§4,5 in [1]
  Sec.11.2 in [2]
  Sec.22.4 in [3]

  Tensors in the Minkowski space: 4-velocity. Covariant and contravariant vectors. Tensors of rank 2. Metric tensor. Inner product in the Minkowski space. Tensors of higher rank in $\mathbb{M}^{1,3}$. Levi-Cevita symbol in $\mathbb{M}^{1,3}$. Pseudotensors.

  Suggested literature: Lecture notes
  §§6,7 in [1]
  Secs.11.3,11.4,11.6 in [2]
  Secs.22.5.1,22.5.2 in [3]

  Matrix representations of the Lorentz group: Rank 2 antisymmetric tensor. Quadratic invariants. Finite dimensional irreducible representations of $SO^+(1,3)$, $O^+(1,3)$ and $O(1,3)$.

  Suggested literature: Lecture notes

- **COVARIANT FORM OF MAXWELL'S EQUATIONS**

  First pair of Maxwell's eqs.: Fields. Field-strength tensor. Covariant form(s) of the first pair of Maxwell's eqs.

  Suggested literature: Lecture notes
  §§23-26 in [1]
  Secs.11.9,11.10 in [2]

  Simple physics behind Maxwell's eqs: Stokes's theorem. Faraday's law of induction. Monopoles. Gauss-Ostrogradsky theorem. Gauss's law. Ampère's law. Displacement current.

  Suggested literature: Lecture notes
  §§23-25 in [1]
  Secs.1.3,1.4,5.1-5.3,5.15,6.1,6.11,6.12 in [2]
  Secs.1.4,2.1,2.2 in [3]

  Second pair of Maxwell's eqs.: Covariant form. 4-current. The continuity equation.

  Suggested literature: Lecture notes
  §§28-30 in [1]
  Sec.1.5 in [3]

  Differential p-forms: Helmholtz's decomposition theorem. Definition of differential p-forms. Exterior derivative. Closed and exact forms. Poincaré lemma.

  Suggested literature: Lecture notes
  Sec.1.9 in [3]

4-potential: Definition. Bianchi identity. Maxwell's equation in terms of the 4-potential. Gauge invariance. Gauge fixing condition. Lorenz gauge.

Suggested literature: Lecture notes
§18 in [1]
Secs.6.2,6.3 in [2]
Secs.15.3 in [3]

- **VARIATIONAL PRINCIPLE**

Poisson's equation in curvilinear coordinates: Variational principle for Poisson's equation. Laplacian in curvilinear coordinates. Orthogonal coordinates.

Suggested literature: Lecture notes
Secs.1.7-1.12 in [2]

Variational principle for Maxwell's equations: The principle of least action in relativistic Field Theory. Lagrangian density. Euler-Lagrange equations. The action functional of the electromagnetic field.

Suggested literature: Lecture notes
§§27,30,32 in [1]
Sec.12.7 in [2]
Secs.13.1,13.2 in [4]

Functional action for particles in electromagnetic field: The principle of least action for a free moving particle. Point-like charge in an external field. Covariant form of the equation of motions. Energy conservation law for a charge in an external field. Energy density and energy flux. Pointing vector.

Suggested literature: Lecture notes
§§$8, 9, 15 - 17$ in [1]
Secs.6.7,12.1 in [2]
Secs.7.9,7.10 in [4]

Literature

[1] L D Landau and E.M. Lifshitz "The Classical Theory of Fields", Volume 2

[2] J.D. Jackson "Classical Electrodynamics" 3rd edition

[3] A. Zangwill, "Modern Electrodynamics", 1st edition

[4] H.Goldstein, C. Poole and J. Safko, "Classical Mechanics", 3rd edition