# **ELECTRICITY & MAGNETISM II**

# Midterm Exam

Room: ARC-204

Time: March 3, (Monday), 12:10-1:30 pm

### Ground rules

- There are four problems based on the material listed below.
- This is a closed book, closed notes exam.
- Partial credit will be given. Do as many parts of a problem as possible.

### Program

#### • PRELIMINARIES

**Space in Classical Physics:** Cartesian space. Euclidean structure. Curvilinear coordinates. Metric-preserving coordinate transformations. Translations of the origin. Proper and improper orthogonal transformations. Euler's theorem. Isometries of  $\mathbb{E}^3$ . Active and passive points of view.

Suggested literature: Lecture notes

Secs. 1.2.1-1.2.4, 1.7 in [3] Secs. 4.1-4.6 in [4]

**Euclidean tensors:** Euclidean vectors and pseudovectors. Levi-Cevita symbol. Cross product. Matrix of finite rotations. Kronecker product. Tensors. Invariant tensors and pseudotensors. Tensors of rank two. Irreducible representations of SO(3). Angular momentum addition. Parity transformations. Irreducible representations of O(3).

Suggested literature: Lecture notes

Secs. 1.2.5, 1.8 in [3]Secs. 4.7, 4.8 in [4]

**Spacetime in Classical Physics:** Event. Causal structure in classical spacetime. Inertial frames. Galilean transformations. Galilean principle of relativity. Newton's first law.

Suggested literature: Lecture notes Sec.22.2 in [3]

#### • KINEMATICS OF SPECIAL RELATIVITY

**Spacetime in Special Relativity:** Causal structure in Special Relativity. Light cone. Spacetime interval. Proper time. Pseudo-Euclidean (Minkowski) space  $\mathbb{M}^{1,3}$ . Einstein principle of relativity.

Suggested literature: Lecture notes

 $\S$ 1-3 in [1] Sec. 11.1 in [2] Sec. 22.3 in [3] Sec. 7.1 in [4]

**Lorentz group:** Definition. Parity and time reversal transformations. Proper, improper, orthochronous, non-orthochronous Lorentz transformations. General structure of the Lorentz group. Lorentz boosts. Group of proper, orthochronous Lorentz transformations  $SO^+(1,3)$ .

Suggested literature: Lecture notes

 $\S$ 4,5 in [1] Sec.11.2 in [2] Sec.22.4 in [3]

**Tensors in the Minkowski space:** 4-velocity. Covariant and contravariant vectors. Tensors of rank 2. Metric tensor. Inner product in the Minkowski space. Tensors of higher rank in  $\mathbb{M}^{1,3}$ . Levi-Cevita symbol in  $\mathbb{M}^{1,3}$ . Pseudotensors.

Matrix representations of the Lorentz group: Rank 2 antisymmetric tensor. Quadratic invariants. Finite dimensional irreducible representations of  $SO^+(1,3)$ ,  $O^+(1,3)$  and O(1,3). Suggested literature: Lecture notes

#### • COVARIANT FORM OF MAXWELL'S EQUATIONS

**First pair of Maxwell's eqs.:** Fields. Field-strength tensor. Covariant form(s) of the first pair of Maxwell's eqs.

Suggested literature: Lecture notes  $\S$ 23-26 in [1] Secs.11.9,11.10 in [2]

Simple physics behind Maxwell's eqs: Stokes's theorem. Faraday's law of induction. Monopoles. Gauss-Ostrogradsky theorem. Gauss's law. Ampère's law. Displacement current.

Suggested literature: Lecture notes

 $\begin{array}{l} \S 23\text{-}25 \text{ in } [1] \\ \text{Secs.}1.3, 1.4, 5.1\text{-}5.3, 5.15, 6.1, 6.11, 6.12 \text{ in } [2] \\ \text{Secs.}1.4, 2.1, 2.2 \text{ in } [3] \end{array}$ 

Second pair of Maxwell's eqs.: Covariant form. 4-current. The continuity equation.

Suggested literature: Lecture notes  $\S$ 28-30 in [1] Sec. 1.5 in [3]

**Differential** *p***-forms:** Helmholtz's decomposition theorem. Definition of differential *p*-forms. Exterior derivative. Closed and exact forms. Poincaré lemma.

Suggested literature: Lecture notes Sec.1.9 in [3] **4-potential:** Definition. Bianchi identity. Maxwell's equation in terms of the 4-potential. Gauge invariance. Gauge fixing condition. Lorenz gauge.

# Literature

- [1] L D Landau and E.M. Lifshitz "The Classical Theory of Fields", Volume 2
- [2] J.D. Jackson "Classical Electrodynamics" 3rd edition
- [3] A. Zangwill, "Modern Electrodynamics", 1st edition
- [4] H.Goldstein, C. Poole and J. Safko, "Classical Mechanics", 3rd edition