Problem set "Quasi-static fields"

Due April 10, 2025

Problem I

(a) Let $\tilde{\mathcal{F}}$ be the total free energy of a magnetic (for the definition of $\tilde{\mathcal{F}}$, see the lecture notes). Without making any assumptions about the relation between $\vec{B}$ and $\vec{H}$ show that

$$\delta\tilde{\mathcal{F}} = \delta F_0 - \int_V d^3x\ \vec{M}\cdot\delta\vec{B}_0\ .$$

(b) A piece of a simple magnetic material of magnetic susceptibility χ_m is placed in a homogeneous external magnetic field $\vec{B}_0$. Determine the heat that is generated as a result of the magnetization of the body. Assume that $|\chi_m| \ll 1$ and ignore any change of the specific volume of the magnetic.

(c) Apply the result (b) to a paramagnetic ideal gas of magnetic dipoles.

Problem II

A circular wire loop of radius a and a long straight wire lie in the same plane such that the center of the loop is at a distance h from the wire, where $h > a$. Calculate the coefficient of mutual induction of the wires.

Problem III

(a) Consider a cylindrical surface of radius R, whose axis is along the z direction. Suppose there is a surface current with density $K(\phi) = \frac{I}{2R}\ \cos(\phi)$ flowing along the z direction. Find the magnetic induction produced by the current inside and outside the cylinder.

(b) Calculate the total magnetostatic field energy per unit length. How is it divided inside and outside the cylinder?

(c) Let the cylinder be divided lengthwise into two equal parts. Suppose that current flows up one half in the positive z-direction and down the other half in the negative z-direction. What is the inductance per unit length of the system?

Problem IV

Calculate the magnetic moment per unit length of a long cylindrical conductor (of radius a, conductivity σ and magnetic permeability μ) in a homogeneous periodic magnetic field

$$\vec{H}(t) = \vec{H}_0 \ \cos(\omega t) \ .$$

The constant vector $\vec{H}_0$ is parallel to the axis of the cylinder. Examine the limiting cases of large and small ω.