Problem set "Magnetostatics"

Due April 3, 2025

Problem I

A magnetic field is defined by the vector potential, whose components in the cylindrical coordinates (ρ, ϕ, z) read as

$$A_\phi = A(\rho, z)\,, \qquad A_\rho = A_z = 0\ .$$

Here $A = A(\rho, z)$ is a given function of ρ and z.

(a) Find the equations that describe the lines of magnetic induction.

(b) Apply the result of (a) to the case of an elementary magnetic dipole.

Problem II

The half spaces $x > 0$ and $x < 0$ are filled with a material of permeability μ_1 and μ_2, respectively. Consider an infinitely long wire at $x = a > 0,\ y = 0$, carrying current I.

(a) Find the magnetic field everywhere.

(b) Find the force per unit length acting on the current. Be explicit about the direction of the force.

Problem III

A ball of radius a carries a charge q. The ball is rotated about a diameter with constant angular velocity ω. Find the vector potential and magnetic induction both inside and outside the ball. Consider two cases

(a) The charge is uniformly distributed along the surface of the ball.

(b) The charge is uniformly distributed over the volume.

Problem IV

(a) The magnetic susceptibility of copper (in the solid phase) is $\chi_m = -8.8 \times 10^{-8}$. Estimate the mean distance of the electrons from the nucleus in an atom of copper.

(b) The magnetic moment of an oxygen molecule is m $= 2.6 \times 10^{-23}$ *amp* m^2. Estimate the magnetic susceptibility of oxygen under normal conditions.