# Problem set "Lorentz transformations in $1+1$ dimensions" 

Due February 12, 2024

## Problem I

Consider the Minkowski space $\mathbb{M}^{1,1}$. By definition, a Poincaré transformation is a coordinate transformation,

$$
x \mapsto \tilde{x}: \quad \tilde{x}^{0}=\tilde{x}^{0}\left(x^{0}, x^{1}\right), \quad \tilde{x}^{1}=\tilde{x}^{1}\left(x^{0}, x^{1}\right),
$$

which preserves the form of the pseudometric:

$$
d s^{2}=\left(d x^{0}\right)^{2}-\left(d x^{1}\right)^{2}=\left(d \tilde{x}^{0}\right)^{2}-\left(d \tilde{x}^{1}\right)^{2}
$$

(i) Show that any Poincaré transformation can be expressed as a composition of a translation, Lorentz boost along the $x \equiv x^{1}$ direction, parity $(x \rightarrow-x)$ and time reversal transformations $\left(t \rightarrow-t\right.$ with $\left.t \equiv x^{0} / c\right)$.
(ii) Show that the set of Poincaré transformations form a Lie group.
(iii) Find the commutation relations for the generators of the Poincaré Lie algebra.

## Problem II

A cart rolls on a long table with velocity $v$. A smaller cart rolls on the first cart in the same direction with velocity $v$ relative to the first cart. A third cart rolls on the second cart in the same direction with velocity $v$ relative to the second cart, and so on up to $n$ carts. What is the velocity $v_{n}$ of the $n^{\text {th }}$ cart in the frame of the table? What does $v_{n}$ tend to as $n \rightarrow \infty$ ?

