Problem set "Lorentz transformations in 1+1 dimensions"

Due February 17, 2025

Problem I

Consider the Minkowski space $\mathbb{M}^{1,1}$. By definition, a Poincaré transformation is a coordinate transformation,

$$x \mapsto \tilde{x} \ : \qquad \tilde{x}^0 = \tilde{x}^0(x^0, x^1) \ , \quad \tilde{x}^1 = \tilde{x}^1(x^0, x^1) \ ,$$

which preserves the form of the pseudometric:

$$ds^2 = (dx^0)^2 - (dx^1)^2 = (d\tilde{x}^0)^2 - (d\tilde{x}^1)^2 \ .$$

(i) Show that any Poincaré transformation can be expressed as a composition of a translation, Lorentz boost along the $x \equiv x^1$ direction, parity ($x \to -x$) and time reversal transformations ($t \to -t$ with $t \equiv x^0/c$).

(ii) Show that the set of Poincaré transformations form a Lie group.

(iii) Find the commutation relations for the generators of the Poincaré Lie algebra.

Problem II

A cart rolls on a long table with velocity v. A smaller cart rolls on the first cart in the same direction with velocity v relative to the first cart. A third cart rolls on the second cart in the same direction with velocity v relative to the second cart, and so on up to n carts. What is the velocity v_n of the n^{th} cart in the frame of the table? What does v_n tend to as $n \to \infty$?