Problem set "EM fields in dispersive media"

Due April 28, 2025

Problem I

Consider a conductor or semiconductor which has current flowing along it because of an applied electric field. Suppose a transverse magnetic field is also applied. Then, there develops a component of the electric field in the direction orthogonal to both the applied electric field (direction of current flow) and the magnetic field, resulting in a voltage difference between the sides of the conductor. This phenomenon is known as the *Hall effect*.

(a) Using the known transformation properties of electromagnetic fields under rotations, spatial reflections and assuming there exists a Taylor series expansion around zero magnetic field strength; find the generalization of Ohm's law for an isotropic, homogeneous medium which is valid to second order in the magnetic field.

(b) What extra restrictions would be imposed if time reversal invariance was in addition assumed?

Problem II

Consider an electromagnetic wave of angular frequency ω in a medium containing free electrons of density n_e.

(a) Find the current density induced by $\vec{E}$ (neglect the interactions between the electrons).

(b) Derive the differential equations for the spatial dependence of a wave of frequency ω in such a medium.

(c) Under what condition would electromagnetic waves propagate in this medium indefinitely?

Problem III

An electromagnetic wave of the form $\vec{E}_{\text{inc}} = E_0 \; \vec{e}_x \, \mathrm{e}^{\mathrm{i}(kz-\omega t)}$ is incident from free space onto a flat surface located at $z = 0$ of a non-permeable material ($\mu = \mu_0$) of conductivity σ and dielectric permittivity ϵ.

(a) Find the amplitude E_0'' of the reflected wave $\vec{E}_{\text{ref}} = E_0'' \, \vec{e}_x \, \mathrm{e}^{\mathrm{i}(-kz-\omega t)}$.

(b) What is the reflectivity (reflection coefficient)? You should express the answer in terms of real numbers only. What is the reflectivity for the case of a good conductor when $\sigma/(\omega\epsilon) \gg 1$?

Problem IV

Suppose you are given a medium that can be described by the Lorentz model of harmonically bound electrons (number density N) with the bound frequency ω_0 and without damping. The medium is placed in a uniform magnetic field with magnetic induction $\vec{B}_0 = B_0\,\vec{e}_z$.

(a) Consider a circularly polarized wave propagating in the direction parallel to $\vec{B}$:

$$\vec{E} = \Re e\Big(E_0\,\vec{e}_\pm\;\mathrm{e}^{\mathrm{i}(kz-\omega t)}\Big) \qquad\qquad (\vec{e}_\pm = \vec{e}_x \pm \mathrm{i}\vec{e}_y)\ .$$

Here the subscripts + and - correspond to the left and right polarized waves, respectively. Calculate the electric susceptibility $\chi_\pm$ of the medium for both polarizations of the wave.

In finding the polarization assume that the magnetic field of the wave can be neglected.

(b) A plane, monochromatic, linearly polarized EM wave of frequency ω propagates in the medium in the direction along $\vec{B}_0$. Find the angle of rotation of the polarization plane, if the wave has passed a distance L in the medium.