PHYSICS 504 Spring 2025

ELECTRICITY & MAGNETISM II

Final Exam

Room: SEC-220

Time: May 14, (Wednesday), 10am-1pm

Ground rules

- There are four problems based on the material listed below.
- This is an open-book, open-notes exam.
- IPhones and laptops are not allowed.
- Partial credit will be given. Do as many parts of a problem as possible.

PROGRAM

Special Relativity with basics of relativistic Field Theory

- **PRELIMINARIES**

  Space in Classical Physics: Cartesian space. Euclidean structure. Curvilinear coordinates. Metric-preserving coordinate transformations. Translations of the origin. Proper and improper orthogonal transformations. Euler's theorem. Isometries of $\mathbb{E}^3$. Active and passive points of view.

  Suggested literature: Lecture notes
  Secs.1.2.1-1.2.4,1.7 in [3]
  Secs. 4.1-4.6 in [4]

  Euclidean tensors: Euclidean vectors and pseudovectors. Levi-Cevita symbol. Cross product. Matrix of finite rotations. Kronecker product. Tensors. Invariant tensors and pseudotensors. Tensors of rank two. Irreducible representations of $SO(3)$. Angular momentum addition. Parity transformations. Irreducible representations of $O(3)$.

  Suggested literature: Lecture notes
  Secs. 1.2.5,1.8 in [3]
  Secs. 4.7,4.8 in [4]

  Spacetime in Classical Physics: Event. Causal structure in classical spacetime. Inertial frames. Galilean transformations. Galilean principle of relativity. Newton's first law.

  Suggested literature: Lecture notes
  Sec.22.2 in [3]

- **KINEMATICS OF SPECIAL RELATIVITY**

  Spacetime in Special Relativity: Causal structure in Special Relativity. Light cone. Spacetime interval. Proper time. Pseudo-Euclidean (Minkowski) space $\mathbb{M}^{1,3}$. Einstein principle of relativity.

  Suggested literature: Lecture notes
  §§1-3 in [1]
  Sec. 11.1 in [2]
  Sec.22.3 in [3]
  Sec.7.1 in [4]

  Lorentz group: Definition. Parity and time reversal transformations. Proper, improper, orthochronous, non-orthochronous Lorentz transformations. General structure of the Lorentz group. Lorentz boosts. Group of proper, orthochronous Lorentz transformations $SO^+(1,3)$.

  Suggested literature: Lecture notes
  §§4,5 in [1]
  Sec.11.2 in [2]
  Sec.22.4 in [3]

  Tensors in the Minkowski space: 4-velocity. Covariant and contravariant vectors. Tensors of rank 2. Metric tensor. Inner product in the Minkowski space. Tensors of higher rank in $\mathbb{M}^{1,3}$. Levi-Cevita symbol in $\mathbb{M}^{1,3}$. Pseudotensors.

  Suggested literature: Lecture notes
  §§6,7 in [1]
  Secs.11.3,11.4,11.6 in [2]
  Secs.22.5.1,22.5.2 in [3]

  Matrix representations of the Lorentz group: Rank 2 antisymmetric tensor. Quadratic invariants. Finite dimensional irreducible representations of $SO^+(1,3)$, $O^+(1,3)$ and $O(1,3)$.

  Suggested literature: Lecture notes

- **COVARIANT FORM OF MAXWELL'S EQUATIONS**

  First pair of Maxwell's eqs.: Fields. Field-strength tensor. Covariant form(s) of the first pair of Maxwell's eqs.

  Suggested literature: Lecture notes
  §§23-26 in [1]
  Secs.11.9,11.10 in [2]

  Simple physics behind Maxwell's eqs: Stokes's theorem. Faraday's law of induction. Monopoles. Gauss-Ostrogradsky theorem. Gauss's law. Ampère's law. Displacement current.

  Suggested literature: Lecture notes
  §§23-25 in [1]
  Secs.1.3,1.4,5.1-5.3,5.15,6.1,6.11,6.12 in [2]
  Secs.1.4,2.1,2.2 in [3]

  Second pair of Maxwell's eqs.: Covariant form. 4-current. The continuity equation.

  Suggested literature: Lecture notes
  §§28-30 in [1]
  Sec.1.5 in [3]

  Differential p-forms: Helmholtz's decomposition theorem. Definition of differential p-forms. Exterior derivative. Closed and exact forms. Poincaré lemma.

  Suggested literature: Lecture notes
  Sec.1.9 in [3]

4-potential: Definition. Bianchi identity. Maxwell's equation in terms of the 4-potential. Gauge invariance. Gauge fixing condition. Lorenz gauge.

Suggested literature: Lecture notes
§18 in [1]
Secs.6.2,6.3 in [2]
Secs.15.3 in [3]

- **VARIATIONAL PRINCIPLE**

Poisson's equation in curvilinear coordinates: Variational principle for Poisson's equation. Laplacian in curvilinear coordinates. Orthogonal coordinates.

Suggested literature: Lecture notes
Secs.1.7-1.12 in [2]

Variational principle for Maxwell's equations: The principle of least action in relativistic Field Theory. Lagrangian density. Euler-Lagrange equations. The action functional of the electromagnetic field.

Suggested literature: Lecture notes
§§27,30,32 in [1]
Sec.12.7 in [2]
Secs.13.1,13.2 in [4]

Functional action for particles in electromagnetic field: The principle of least action for a free moving particle. Point-like charge in an external field. Covariant form of the equation of motions. Energy conservation law for a charge in a stationary external field. Energy density and energy flux. Poynting vector. Poynting's theorem.

Suggested literature: Lecture notes
§§$8, 9, 15-17$ in [1]
Secs.6.7,12.1 in [2]
Secs.7.9,7.10 in [4]

Symmetries: Continuous and discrete symmetries of Classical Electrodynamics.

Suggested literature: Lecture notes
Sec.6.10 in [2]
Secs.15.1,15.2,24.4.2 in [3]
Sec.13.7 in [4]

Applications of Classical Electrodynamics

- **MAGNETOSTATICS**

Magnetic moment: Static magnetic field. Vector potential in the Coulomb gauge. Magnetic fields of a localized current distribution. Relation between magnetic and mechanical moments.

Suggested literature: Lecture notes
§§43,44 in [1]
Sec.5.3-5.6 in [2]
Secs.11.1,11.2 in [3]

Macroscopic equations: Magnetization. The magnetic field (intensity). Boundary conditions. Relation between magnetic (field) induction and magnetic field (intensity). Methods of solving boundary value problems in magnetostatic.

Suggested literature: Lecture notes
Sec.5.8-5.13 in [2]

Simple magnetic matter: Magnetic moment in an external magnetic field (torque, force, potential energy). Larmor's theorem. Diamagnetism. Paramagnetism. Curie's law. Exchange interaction.

Suggested literature: Lecture notes
§§45 in [1]
Sec.5.7 in [2]

- **QUASI-STATIC FIELDS**

Energy in a magnetic field: Energy of magnetic matter. Total free energy of a magnetic. Energy of a system of currents. Self- and mutual inductance. Estimation of self-induction for simple circuits.

Suggested literature: Lecture notes
Sec.5.16,5.17 in [2]

Quasi-static EM fields in conductors: Coulomb gauge. Quasi-static approximation. Skin effect.

Suggested literature: Lecture notes
Sec.6.3,5.18 in [2]
Secs.14.5-14.7, 14.10 in [3]

- **ELECTROMAGNETIC WAVES**

Waves in vacuum: Wave equation. Plane EM waves. Monochromatic waves. Helmholtz equation. Doppler effect. Elliptical, linear and circular polarization.

Suggested literature: Lecture notes
§§46-48 in [1]
Sec.7.1,7.2 in [2]
Secs.16.1-16.4.4, 16.6 in [3]

Waves in simple matter: Waves in nondispersive media. Wave impedance. Index of refraction. Reflection and refraction: Snell's law, Fresnel equations, reflection and transmission coefficients, polarization by reflection, Brewster's angle, total internal reflection.

Suggested literature: Lecture notes
Sec.7.1,7.3,7.4 in [2]
Secs.17.1-17.3 in [3]

Waves in dispersive matter I: Constitutive relations in a dispersive medium. Kramers-Kronig relations. Lorentz model for dispersion.

Suggested literature: Lecture notes
Secs.6.10,7.5, 7.10 in [2]
Secs.18,1, 18.2, 18.51,18.54, 18.7 in [3]

Waves in dispersive matter II: Plane waves in dispersive media. Phase velocity and group velocity. Conservation of energy in dispersive media: Poynting vector, effective energy density.

Suggested literature: Lecture notes
Secs.6.7,6.8, 7.8 in [2]
Secs.18.3,18.4, 18.6 in [3]

- **RETARDATION AND RADIATION**

Fields from moving charges: Green's functions for the wave equation. Lienard-Wiechert potentials and fields for a point charge. Point charge in uniform motion. Spectral decomposition of the retarded potentials.

Suggested literature: Lecture notes
§§62-64, in [1]
Secs.6.4, 6.5, 12.11, 14.1 in [2]
Secs.20.1-20.3, 23.1, 23.2 in [3]

Multipole fields and radiation: Fields of a system of charges at large distances. Dipole radiation. Quadrupole and magnetic dipole radiation.

Suggested literature: Lecture notes
§§ 66,67,71 in [1]
Secs.9.1-9.3 in [2]
Secs.20.5, 20.7 in [3]

Literature

[1] L D Landau and E.M. Lifshitz "The Classical Theory of Fields", Volume 2

[2] J.D. Jackson "Classical Electrodynamics" 3rd edition

[3] A. Zangwill, "Modern Electrodynamics", 1st edition

[4] H.Goldstein, C. Poole and J. Safko, "Classical Mechanics", 3rd edition