Physics 503: Electromagnetism I
Problem Set #1, due Friday, September 11, 2015

Zangwill 3.5

3.5 Gauss' Law Practice Use Gauss' law to find the electric field when the charge density is:

(a) $\rho(x) = \rho_0 \exp\left\{-\kappa\sqrt{x^2}\right\}$. Expresses the answer in Cartesian coordinates.

(b) $\rho(x, y) = \rho_0 \exp\left\{-\kappa\sqrt{x^2 + y^2}\right\}$. Express the answer in cylindrical coordinates.

(c) $\rho(x, y, z) = \rho_0 \exp\left\{-\kappa\sqrt{x^2 + y^2 + z^2}\right\}$. Express the answer in spherical coordinates.

Zangwill 3.10

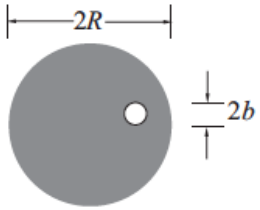
3.10 Two Electrostatic Theorems

(a) Use Green's second identity, $\int_V d^3r\,(f\nabla^2 g - g\nabla^2 f) = \int_S d\mathbf{S}\cdot(f\nabla g - g\nabla f)$, to prove that the potential $\varphi(0)$ at the center of a charge-free spherical volume V is equal to the average of $\varphi(\mathbf{r})$ over the surface S of the sphere. We proved this theorem in the text using Green's reciprocity relation.

(b) Use the result of part (a) to provide an alternative to the derivation of Earnshaw's theorem given in the text.

Zangwill 3.12

3.12 A Charged Spherical Shell with a Hole The figure below shows a circular hole of radius b (white) bored through a spherical shell (gray) with radius R and uniform charge per unit Area σ.



(a) Show that $\mathbf{E}(P) = (\sigma/2\epsilon_0)[1 - \sin(\theta_0/2)]\hat{\mathbf{r}}$, where P is the point at center of the hole and θ_0 is the opening angle of a cone whose apex is at the center of the sphere and whose open end coincides with the edge of the hole. Perform the calculation by summing the vector electric fields produced at P by all the other points of the shell.

(b) Use an entirely different argument to explain why $\mathbf{E}(P) \approx (\sigma/2\epsilon_0)\hat{\mathbf{r}}$ when $\theta_0 \ll 1$.