Significant figures help keep track of imprecision

It is important to record the precision of your measurements so that other people can understand and interpret your results. A common convention used in science to indicate precision is known as **significant figures.**

In the case of the measurement of the pencil as about 18.2 cm, the measurement has three significant figures. The significant figures of a measurement include all the digits that are actually measured (18 cm), plus one *estimated* digit. Note that the number of significant figures is determined by the precision of the markings on the measuring scale.

The last digit is reported as a 0.2 (for the estimated 0.2 cm past the 18 cm mark). Because this digit is an estimate, the true value for the measurement is actually somewhere between 18.15 cm and 18.25 cm.

When the last digit in a recorded measurement is a zero, it is difficult to tell whether the zero is there as a place holder or as a significant digit. For example, if a length is recorded as 230 mm, it is impossible to tell whether this number has two or three significant digits. In other words, it can be difficult to know whether the measurement of 230 mm means the measurement is known to be between 225 mm and 235 mm or is known more precisely to be between 229.5 mm and 230.5 mm.

One way to solve such problems is to report all values using scientific notation. In scientific notation, the measurement is recorded to a power of 10, and all of the figures given are significant. For example, if the length of 230 cm has two significant figures, it would be recorded in scientific notation as 2.3×10^2 cm. If it has three significant figures, it would be recorded as 2.30×10^2 cm.

Scientific notation is also helpful when the zero in a recorded measurement appears in front of the measured digits. For example, a measurement such as 0.000 15 cm should be expressed in scientific notation as 1.5×10^{-4} cm if it has two significant figures. The three zeros between the decimal point and the digit 1 are not counted as significant figures because they are present only to locate the decimal point and to indicate the order of magnitude. The rules for determining how many significant figures are in a measurement that includes zeros are shown in **Table 1-4.**

Significant figures in calculations require special rules

In calculations, the number of significant figures in your result depends on the number of significant figures in each measurement. For example, if someone reports that the height of a mountaintop, like the one shown in **Figure 1-11,** is 1710 m, that implies that its actual height is between 1705 and 1715 m. If another person builds a pile of rocks 0.20 m high on top of the mountain, that would not suddenly make the mountain's new height known accurately enough to be measured as 1710.20 m. The final answer cannot be more precise than the least precise measurement used to find the answer. Therefore, the answer should be rounded off to 1710 m even if the pile of rocks is included.
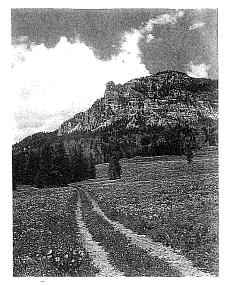

Figure 1-11
If a mountain's height is known with an uncertainty of 5 m, the addition of 0.20 m of rocks will not appreciably change the height.

Table 1-4   Rules for determining whether zeros are significant figures

Rule	Examples
1. Zeros between other nonzero digits are significant.	**a.** 50.3 m has three significant figures. **b.** 3.0025 s has five significant figures.
2. Zeros in front of nonzero digits are not significant.	**a.** 0.892 kg has three significant figures. **b.** 0.0008 ms has one significant figure.
3. Zeros that are at the end of a number and also to the right of the decimal are significant.	**a.** 57.00 g has four significant figures. **b.** 2.000 000 kg has seven significant figures.
4. Zeros at the end of a number but to the left of a decimal are significant if they have been measured or are the first estimated digit; otherwise, they are *not* significant. In this book, they will be treated as *not* significant.	**a.** 1000 m may contain from one to four significant figures, depending on the precision of the measurement, but in this book it will be assumed that measurements like this have one significant figure. **b.** 20 m may contain one or two significant figures, but in this book it will be assumed to have one significant figure.

Similar rules apply to multiplication. Suppose that you calculate the area of a room by multiplying the width and length. If the room's dimensions are 4.6 m by 6.7 m, the product of these values would be 30.82 m^2. However, this answer contains four significant figures, which implies that it is more precise than the measurements of the length and width. Because the room could be as small as 4.55 m by 6.65 m or as large as 4.65 m by 6.75 m, the area of the room is known only to be between 30.26 m^2 and 31.39 m^2. The area of the room can have only two significant figures because each measurement has only two. So it must be rounded off to 31 m^2. **Table 1-5** summarizes the two basic rules for determining significant figures when you are performing calculations.

Table 1-5   Rules for calculating with significant figures

Type of calculation	Rule	Example
addition or subtraction	The final answer should have the same number of digits to the right of the decimal as the measurement with the *smallest* number of digits *to the right of the decimal.*	97.3 + 5.85 103.15 $\xrightarrow{\text{round off}}$ 103.2
multiplication or division	The final answer has the same number of significant figures as the measurement having the *smallest* number of *significant figures.*	123 × 5.35 658.05 $\xrightarrow{\text{round off}}$ 658

Calculators do not pay attention to significant figures

When you use a calculator to analyze problems or measurements, you may be able to save time because the calculator can do the math more quickly than you. However, the calculator does not keep track of the significant figures in your measurements.

Calculators often exaggerate the precision of your final results by returning answers with as many digits as the display can show. To reinforce the correct approach, the answers to the sample problems in this book will always show only the number of significant figures that the measurements justify.

In order to provide answers with the correct number of significant figures, it will sometimes be necessary to round the results of a calculation. The rules described in **Table 1-6** will be used. In this book, the results of a calculation will be rounded after each type of mathematical operation. For example, the result of a series of multiplications should be rounded using the multiplication/division rule before it is added to another number. Similarly, the sum of several numbers should be rounded according to the addition/subtraction rule before the sum is multiplied by another number. You should consult your teacher to find out whether to round this way or to delay rounding until the end of all calculations.

Table 1-6   **Rules for rounding**

What to do	When to do it	Examples
round down	• whenever the digit following the last significant figure is a 0, 1, 2, 3, or 4	30.24 becomes 30.2
	• if the last significant figure is an even number and the next digit is a 5, with no other nonzero digits	32.25 becomes 32.2 32.65000 becomes 32.6
round up	• whenever the digit following the last significant figure is a 6, 7, 8, or 9	22.49 becomes 22.5
	• if the digit following the last significant figure is a 5 followed by a nonzero digit	54.7511 becomes 54.8
	• if the last significant figure is an odd number and the next digit is a 5, with no other nonzero digits	54.75 becomes 54.8 79.3500 becomes 79.4

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