GAUSS' LAW RECAP $\oint_S \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0$

- Relates charge distribution and electric field; equivalent to Coulomb's law

- New concept: electric flux through a closed surface

- You can find the electric flux through a closed surface if you know the charge distribution (just integrate the charge inside)

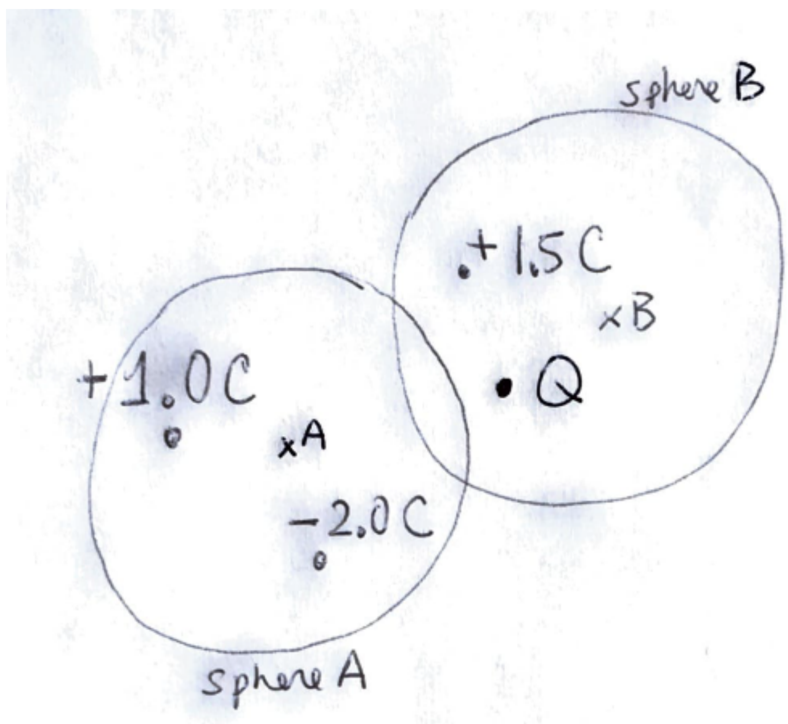
- You can find the charge inside a closed surface if you know the electric flux through the surface

PRELECTURE QUESTION: LECTURE 5

A set of charges is arranged in the plane of the page as shown. The two spheres A and B are centered in the plane of the page.

(a) What is the electric flux through sphere A?

(b) If the electric flux through sphere B is $(5.0/(8.854 \times 10^{-12}))\,\mathrm{Nm^2/C}$, what is the charge Q? (remember $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/(Nm^2)}$)

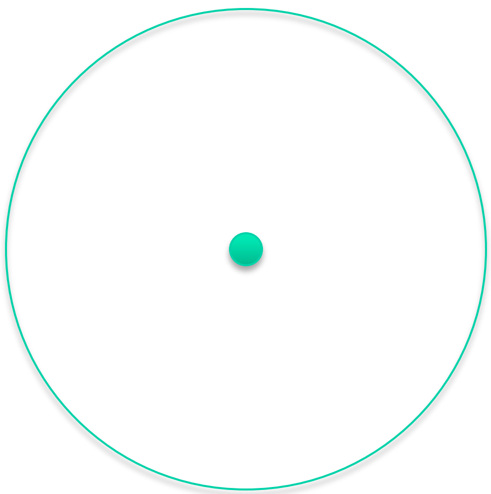


GAUSS' LAW RECAP

- Relates charge distribution and electric field; equivalent to Coulomb's law
- New concept: electric flux through a closed surface
- You can find the electric flux through a closed surface if you know the charge distribution (just integrate the charge inside)
- You can find the charge inside a closed surface if you know the electric flux through the surface
- Very simple way to find the electric field of a symmetrical charge distribution: spherical shell, cylindrical shell, plane

$$\oint_S \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0$$

BUT if we know the flux through a sphere of radius r centered on a point charge, we CAN find the field at all points on the sphere!

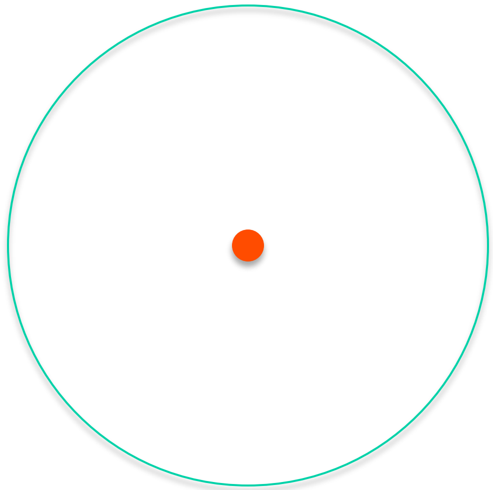


Get the necessary additional information about $\vec{E}(\vec{r})$ from SYMMETRY

$$\vec{E}(\vec{r}) = E(r)\hat{r}$$

Evaluate the flux integral
on sphere of radius r

$$\oint_S \vec{E} \cdot d\vec{A} = E(r)\pi r^2$$

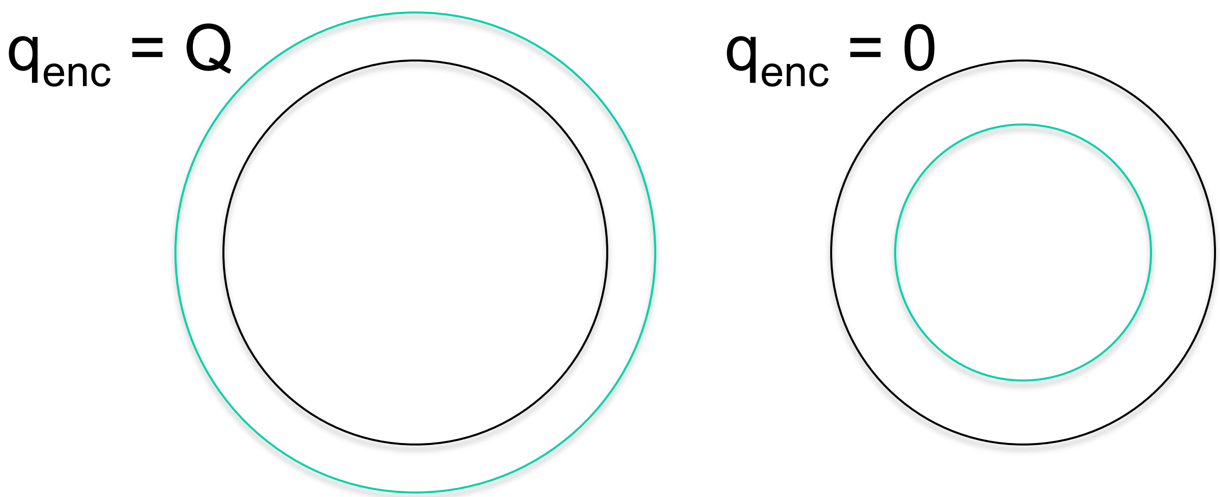


Gauss Law:
$E(r)\, 4\pi r^2 = q/\varepsilon_0$

$E(r) = q/(4\pi\varepsilon_0 r^2) = kq/r^2$
direction is radial
= Coulomb's law!

same argument works for any spherically symmetric charge distribution $\vec{E}(\vec{r}) = E(r)\hat{r}$

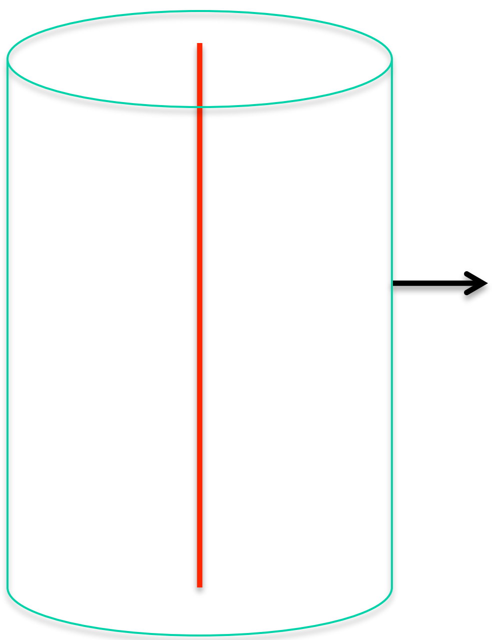
Use Gauss' law to prove the shell theorems
-field outside a uniformly charged spherical shell is the same as if the charge were concentrated at a point at the center
-field inside a uniformly charged spherical shell is zero



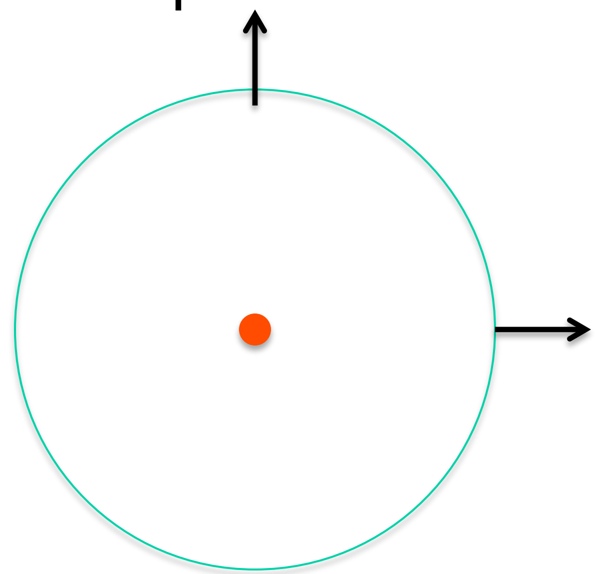
Symmetry of a uniformly charged infinite wire

180 degree flip around any point on the wire
rotation by any angle around the wire axis
translation by any distance along the wire

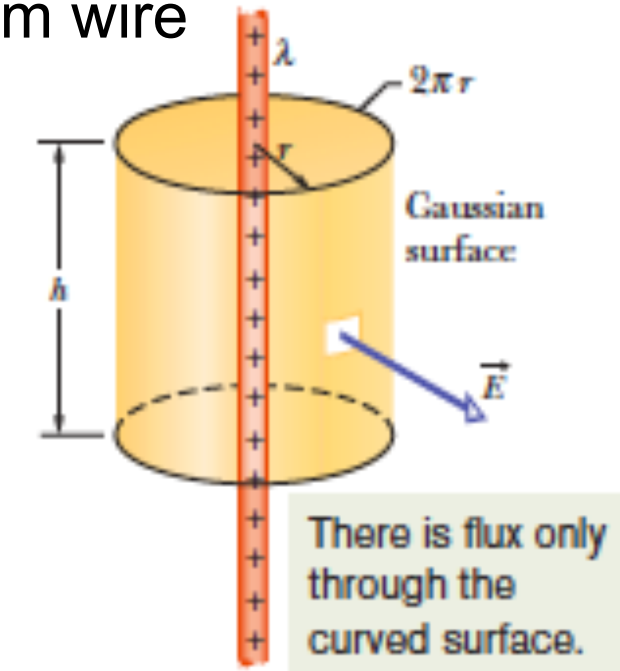
Side view



top view



Field of uniformly charged wire with linear charge density λ points directly away from the wire
magnitude depends only on distance r from wire



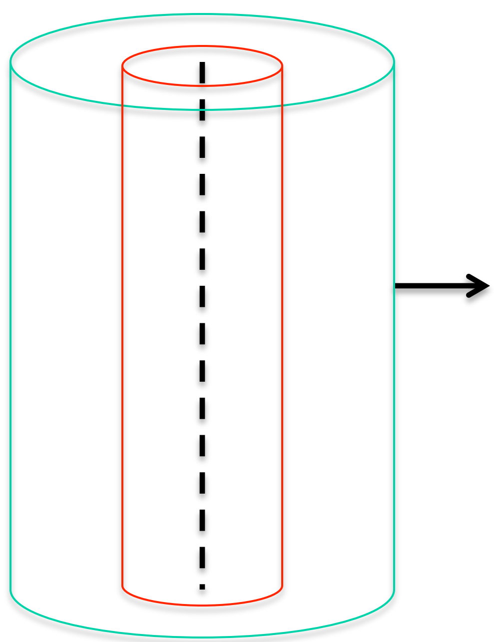
Let's choose our surface S to be
a cylinder of radius r and height h
centered on the wire

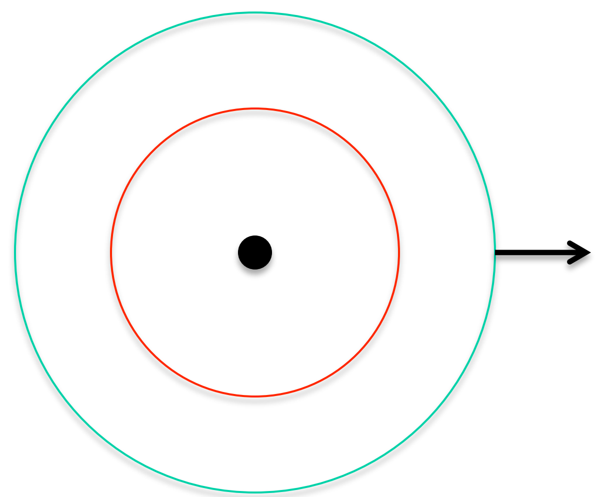
Flux through cylinder
$E(r)\ 2\ \pi\ r\ h$

Gauss' law: flux through S = (charge enclosed)/ε_0
$E(r)\ 2\ \pi\ r\ h = (\lambda\ h)\ /\ \varepsilon_0$
so
$E(r) = \lambda/(2\pi\varepsilon_0 r)$

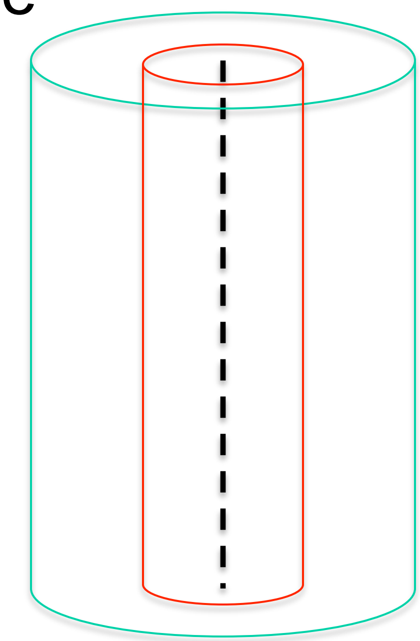
Symmetry of a uniformly charged infinite cylindrical shell

180 degree flip around any point on the wire
rotation by any angle around the wire axis
translation by any distance along the wire





Field of uniformly charged **shell** with linear charge density λ
points directly away from the wire
magnitude depends only on distance r from wire



Let's choose our surface S to be
a cylinder of radius r and height h
centered on the wire

Flux through cylinder
$E(r)\ 2\ \pi\ r\ h$

Gauss' law: flux through S = (charge enclosed)/ε_0
$E(r)\ 2\ \pi\ r\ h = (\lambda\ h)\ /\ \varepsilon_0$
so
$E(r) = \lambda/(2\pi\varepsilon_0 r)$

Simple way of finding the field – works for spherical symmetry and other very symmetrical charge distributions

- Wires and symmetric infinite cylinders (HRW 10 23-4)
-

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \text{(line of charge).}$$

pointing directly away from or towards the axis

- Infinite flat uniform sheets of charge (HRW 10 23-5)

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(sheet of charge).}$$

Pointing directly away from or towards the plane
DOES NOT DEPEND ON THE DISTANCE FROM THE PLANE!!

DEMO – the electric field inside a Faraday cage is zero no matter what is going on outside the cage!

DEMO – the electric field inside a Faraday cage is zero no matter what is going on outside the cage!

Hollow conductor

Electric field inside the cavity is ZERO

Conductors: (metal, tap water, the human body)

Some fraction of the electrons are free to move anywhere in the material

ADDED CHARGE IS FREE TO MOVE ANYWHERE IN THE MATERIAL

Rules for conductors in electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero

Field outside $= \dfrac{kQ}{r^2}\hat{r}$
$\sigma = Q/(4\pi R^2)$
Magnitude of field at r = R is $k(4\pi R^2 \sigma)/R^2 = \sigma / \varepsilon_0$

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero
Add a concentric spherical cavity

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero
Add a concentric spherical cavity
Electric field is zero inside

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero
Add a concentric spherical cavity
Electric field is zero inside
True for any cavity

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero
Add a concentric spherical cavity with charge q at the center

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the interior of the conductor**

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero
Add a concentric spherical cavity with charge q at the center
Q redistributes so that –q is spread uniformly on the inner surface and Q+q is spread uniformly on the outer surface

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the interior of the conductor**

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero
Add a concentric spherical cavity with charge q at the center
Q redistributes so that –q is spread uniformly on the inner surface and Q+q is spread uniformly on the outer surface

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the interior of the conductor**

Isolated spherical conductor with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Spread it uniformly on the surface of the sphere!
The shell theorem tells us that the field inside is zero
Add a concentric spherical cavity with charge q at the center
Q redistributes so that –q is spread uniformly on the inner surface and Q+q is spread uniformly on the outer surface
True even if spherical cavity is not concentric

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated conductor **of any shape** with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated conductor of any shape with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Depends on the shape

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated conductor of any shape with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Depends on the shape
Add a cavity of any shape

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

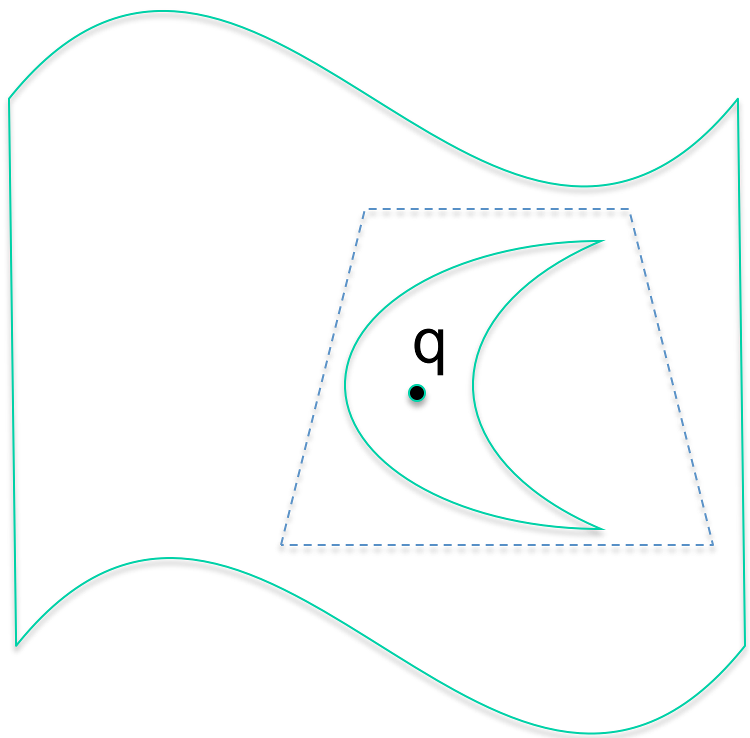
Isolated conductor of any shape with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Depends on the shape
Add a cavity of any shape
Electric field is zero inside

Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated conductor of any shape with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Depends on the shape
Add a cavity of any shape with charge q inside

All points on dashed surface are in the interior of the conductor
E=0 so electric flux through dashed surface is zero
By Gauss' law, net charge inside dashed surface is zero
So there must be a total of −q on the surface of the crescent moon shaped cavity



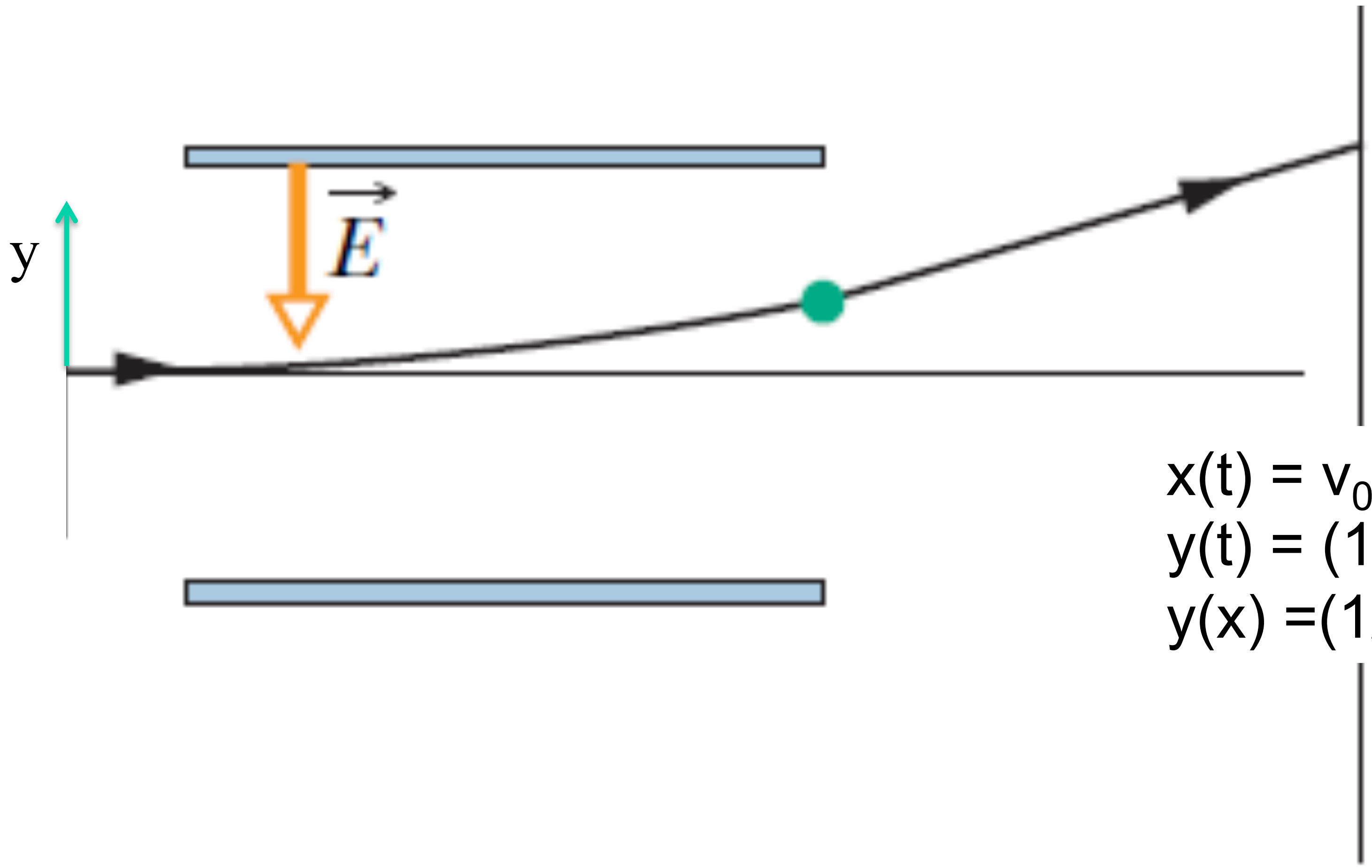
Electrostatics:

- Charge density is zero everywhere in the interior of a conductor – all added charge is on the surface(s)
- the charges on the surface rearrange themselves until **the electric field is zero everywhere in the conducting material**

Isolated conductor of any shape with excess charge Q
All the charge is on the surface
How to arrange the charge to get zero field in the interior?
Depends on the shape
Add a cavity of any shape with charge q inside
Q redistributes so that –q is spread on the inner surface and Q+q is spread on the outer surface

A Point Charge in an Electric Field : Motion in a uniform electric field

Given a uniform electric field **E**, how will charge q move?
Charge will feel force **F**=q**E** (same force for all positions)
F=m**a**, so **a** =**F**/m=q**E**/m (constant acceleration)



$x(t) = v_{0x}t$
$y(t) = (1/2)at^2$
$y(x) = (1/2)a(x/v_{0x})^2$

Motion of charged particle in electric field $\vec{F} = q\vec{E}$

$$\vec{a} = \vec{F} / m$$

•43 SSM An electron is released from rest in a uniform electric field of magnitude 2.00×10^4 N/C. Calculate the acceleration of the electron. (Ignore gravitation.)

•46 An electron is accelerated eastward at 1.80×10^9 m/s^2 by an electric field. Determine the field (a) magnitude and (b) direction.

•40 GO An electron with a speed of 5.00×10^8 cm/s enters an electric field of magnitude 1.00×10^3 N/C, traveling along a field line in the direction that retards its motion. (a) How far will the electron travel in the field before stopping momentarily, and (b) how much time will have elapsed? (c) If the region containing the electric field is 8.00 mm long (too short for the electron to stop within it), what fraction of the electron's initial kinetic energy will be lost in that region?

$$v(t) = v_0 + at$$
$$x(t) = v_0 t + at^2/2$$
$$v^2 - v_0^2 = 2a(x - x_0)$$
$$(x - x_0) = (v + v_0)t/2$$

•47 SSM Beams of high-speed protons can be produced in "guns" using electric fields to accelerate the protons. (a) What acceleration would a proton experience if the gun's electric field were 2.00×10^4 N/C? (b) What speed would the proton attain if the field accelerated the proton through a distance of 1.00 cm?

$$v(t) = v_0 + at$$
$$x(t) = v_0 t + at^2/2$$
$$v^2 - v_0^2 = 2a(x - x_0)$$
$$(x - x_0) = (v + v_0)t/2$$