2nd homework is due TODAY 11:59PM – Sept 15^{th}

Purchasing WebAssign access

Course web site
http://www.physics.rutgers.edu/~karin/227H

Outline

Electric flux through a closed surface
Gauss law – relates information about the electric field on surface to charge INSIDE

Start from Coulomb law and show Gauss law is true

Use Gauss law to find field of symmetric charge distributions – Point charge, uniform spherical shell, uniform spherical ball

Charge distribution determines electric field
sum, line integral, surface integral, volume integral

$$\vec{E}(\vec{r}) = \sum_i \frac{kQ_i}{|\vec{r}-\vec{r}_i|^2} \frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|}$$

--shouldn't there be an easier way to prove the shell theorem?
--does the electric field determine the charge distribution?
--how do you find the charge distribution from the electric field?

REFORMULATION OF COULOMB'S LAW

$$\oint_S \vec{E}\cdot d\vec{A} = q_{enc}/\varepsilon_0$$

GAUSS' LAW

"electric flux through the closed surface S"

What is a surface?

A surface is a set of points in three-dimensional space satisfying one equation (and optionally some inequalities)

$\{(x,y,z)$ such that $z = 1\}$ (plane)

$\{(x,y,z)$ such that $x^2+y^2+z^2 = 100\}$ (sphere)

$\{(x,y,z)$ such that $z = 1$ and $x^2+y^2 \leq 100\}$ (disk)

Open surface: has a boundary

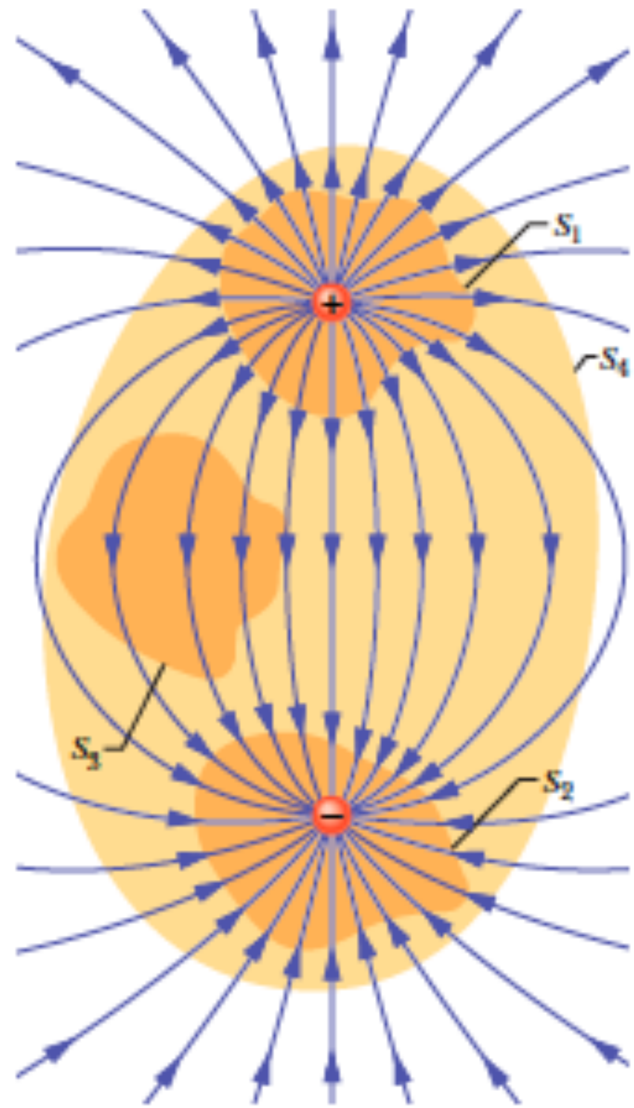
Closed surface: has no boundary, has an inside and an outside

$$\oint_S \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0$$

What is the right hand side?

q_{enc} is the NET charge inside the closed surface S

$\varepsilon_0 = 1/(4\,\pi\,k)$ or $k = 1/(4\,\pi\,\varepsilon_0)$
“permittivity of free space”



$$\oint_S \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0$$

What is the left hand side?

"Electric flux" through closed surface S

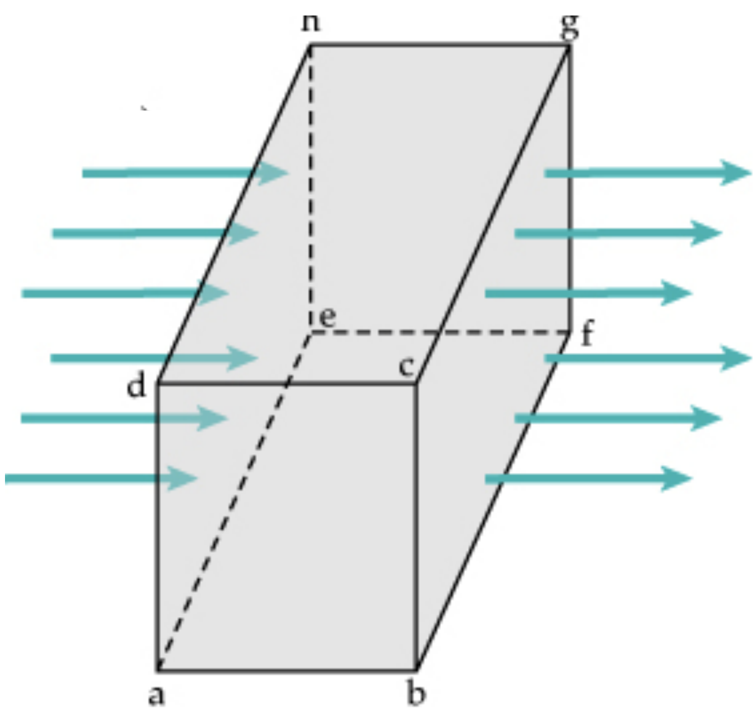
For a given electric field

Specified by field line diagram or by a function

$$\vec{E}(\vec{r}) = E_x(x,y,z)\hat{i} + E_y(x,y,z)\hat{j} + E_z(x,y,z)\hat{k}$$

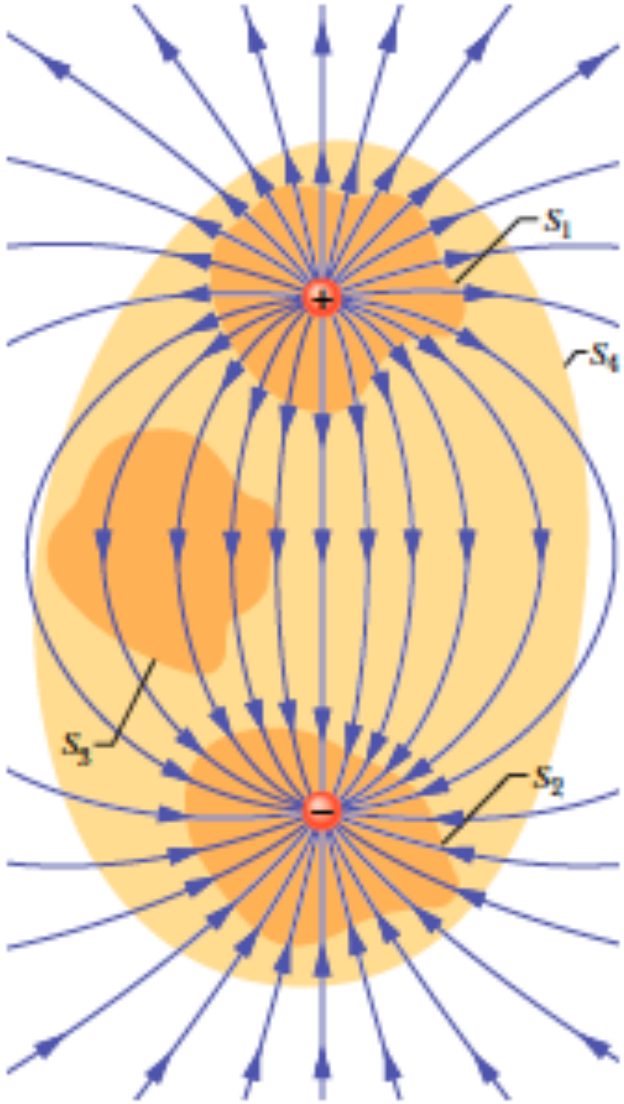
and a given closed surface S

“electric flux through the closed surface S”



Graphical approach to electric flux

Flux integral is proportional to
# of field lines out - # of field lines in



Graphical approach:

Flux integral is proportional to
# of field lines out - # of field lines in

Especially useful for cases in which
the flux is zero

or cases when we just want to know
if the flux is positive or negative

Flux of a vector field $\vec{F}(\vec{r})$ through surface
(see textbook Section 23-3 for more explanation)

simplest case:

- flat open surface of area A and normal vector $\hat{n}$
- vector field $\vec{F}(\vec{r}) = \vec{F}$ is the same at all points on the surface

Flux $\Phi = (\vec{F} \cdot \hat{n})A$

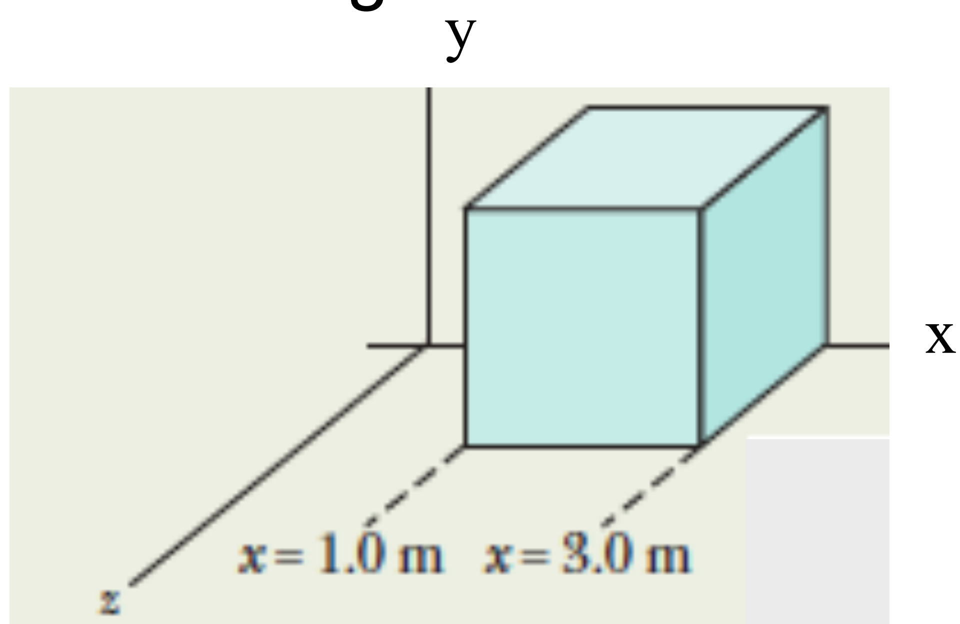
Recall fluid flow
If $\vec{F}(\vec{r}) = \vec{v}$, then Φ would be the flow rate through the flat surface (volume/time)

Flux of a vector field $\vec{F}(\vec{r})$ through closed surface

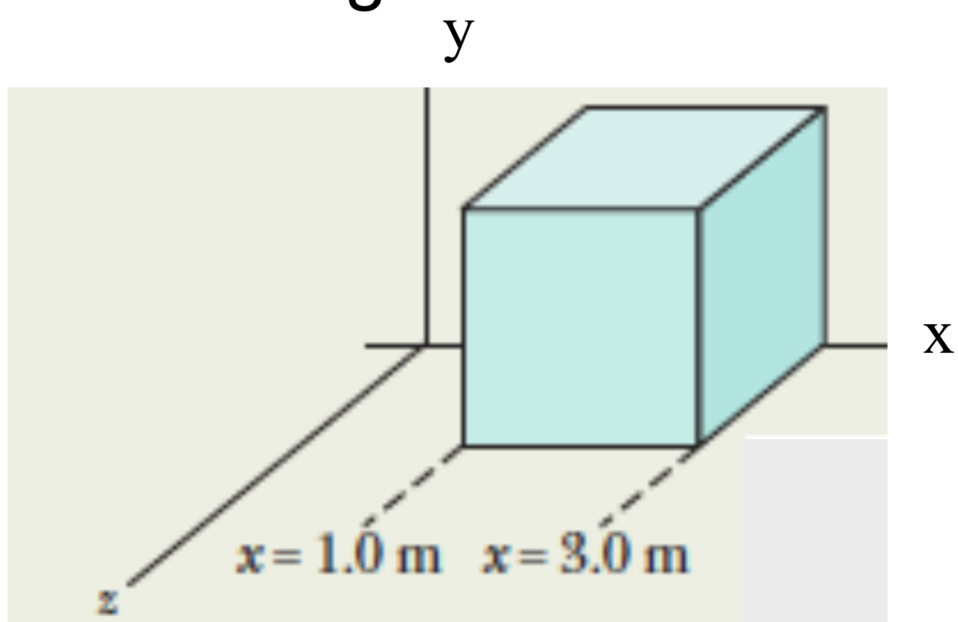
- Cube: 6 flat surface pieces each of area A
- $\hat{n}$ points from inside to outside
- vector field $\vec{F}(\vec{r})$ which **is uniform on each piece, that is $\vec{F}(\vec{r}) = \vec{F}_i$ at all points on piece i**
- Contribution of each piece is $(\vec{F}_i \cdot \hat{n}_i)A$

Flux through the cube $\Phi = \sum_{i=1}^{i=6} (\vec{F}_i \cdot \hat{n}_i)A$

What is the electric flux through the cube for $\vec{E}(\vec{r}) = 3.0x\hat{i}$?



What is the electric flux through the cube for $\vec{E}(\vec{r}) = 3.0x\hat{i}$?



24 N m^2/C

What if the surface is not made of flat pieces with uniform field on each piece?

Pieces are flat but field is not uniform on piece and/or

Surface is curved

What if the surface S is not made of flat pieces with uniform field on each piece?

Divide the surface S into tiny pieces

If tiny enough – flat with area dA and normal $\hat{n}$

-- compute $(\vec{E} \cdot \hat{n})dA = \vec{E} \cdot d\vec{A}$

Add up the contributions from the tiny pieces

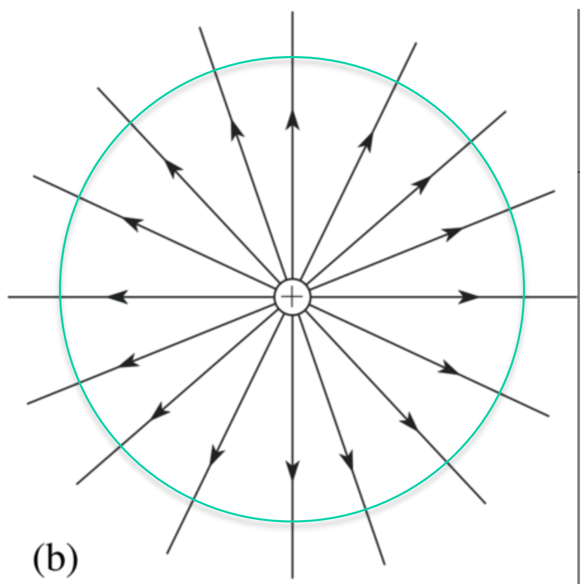
$$\oint_S \vec{E} \cdot d\vec{A}$$

A single point charge q: $\vec{E}(\vec{r})$ is given by Coulomb's law

Let's confirm that Gauss' law is true $\oint_S \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0$

S of radius R centered at point charge q

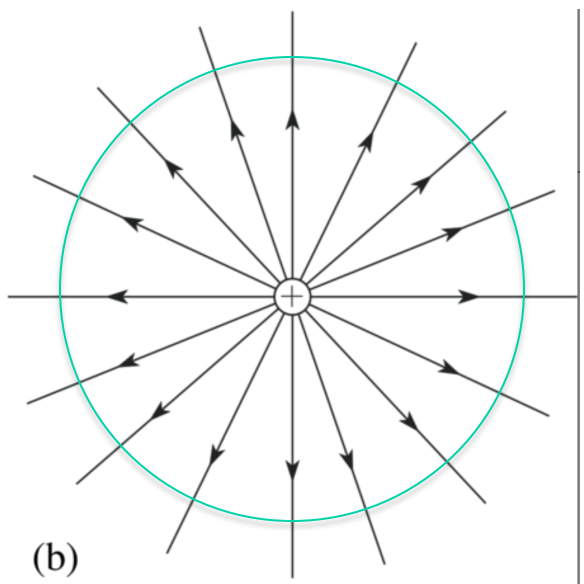
q_{enc} is just q; let's focus on computing the electric flux through S



(b)

A single point charge q: $\vec{E}(\vec{r})$ is given by Coulomb's law

Compute flux through spherical surface S of radius R centered at point charge q



(b)

Flux integral $\oint_S \vec{E} \cdot d\vec{A}$

$(\vec{E}(\vec{r}) \cdot \vec{n})dA$

$\hat{n} = \hat{r}$

$$(\frac{kq\hat{r}}{R^2} \cdot \hat{n})dA = \frac{kq}{R^2}dA$$

Surface integral is $kq/R^2 \, 4\pi R^2$

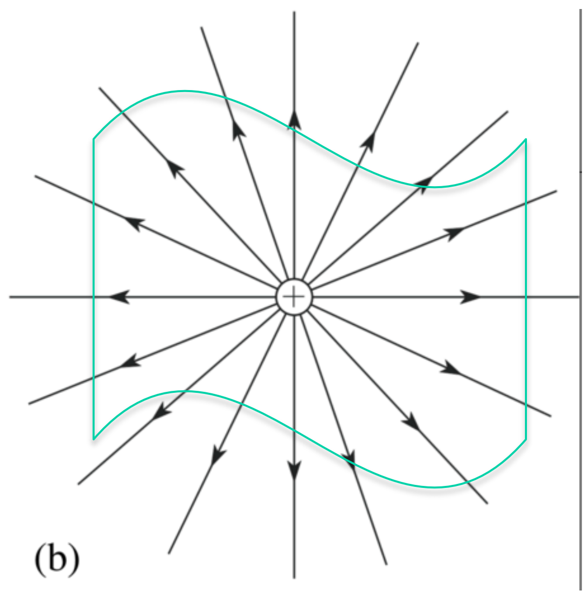
= flux integral = $4 \pi k q = q/\varepsilon_0$

THE SAME FOR ANY R!

(number of flux lines is the same!)

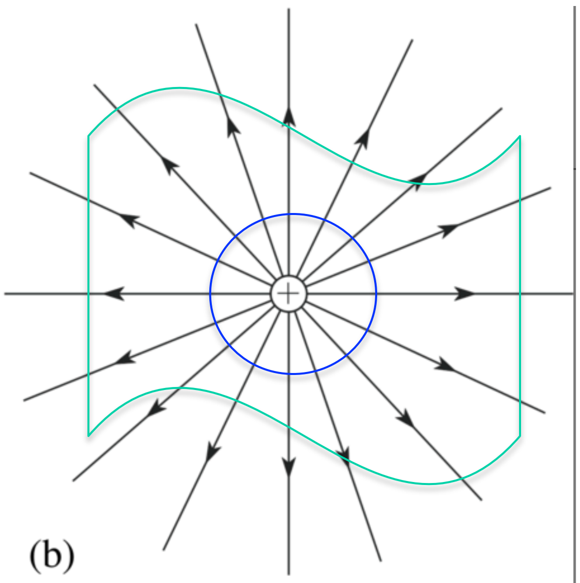
$$\oint_S \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0$$

What about other surfaces enclose q?
What if S is this weird-shaped surface?



(b)

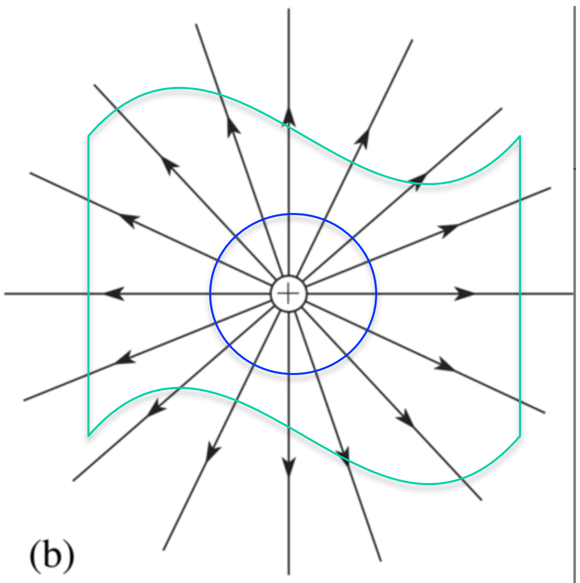
What about other surfaces that enclose q?
What if S is this weird-shaped surface?



(b)

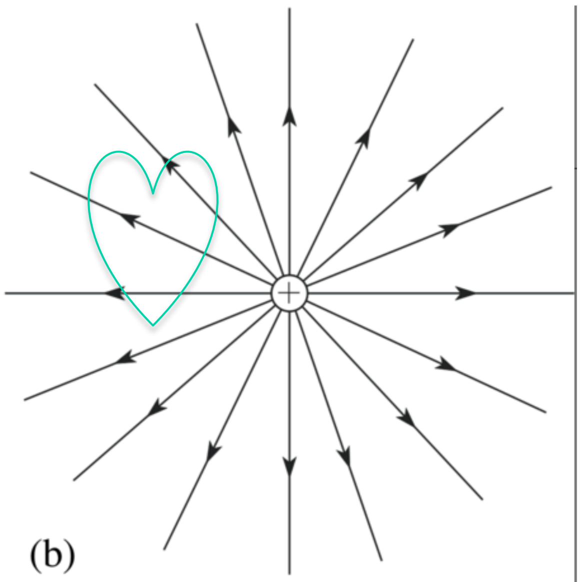
spherical surface S’ centered on q and completely inside S

What about other surfaces that enclose q?
What if S is this weird-shaped surface?



spherical surface S’ centered on q and completely inside S
We already showed that the electric flux through S’ is q/ε_0
Flux line counting: same flux lines go through S and S’
So electric flux through S’ is q/ε_0 – Gauss law works!

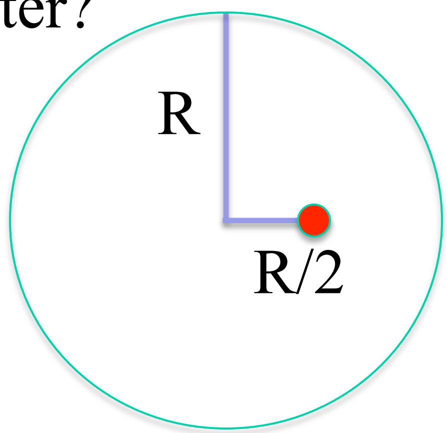
What about other surfaces that DON'T enclose q?
What if S is this weird-shaped surface?


(b)

Flux line counting: every flux line that goes in, also comes out
So electric flux through S is 0, and q_{enc}=0 – Gauss law works!

Poll question

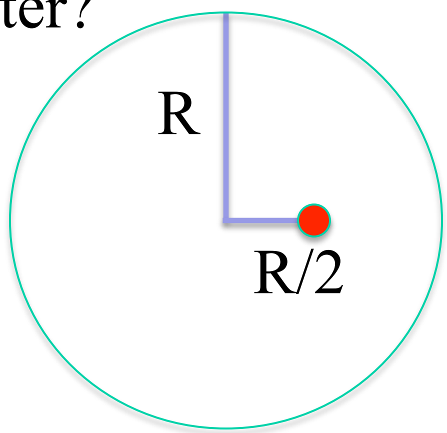
What is the electric flux through a spherical surface of radius R with point charge q a distance R/2 away from the center?



(A) 0
(B) $q/(2\varepsilon_0)$
(C) q/ε_0
(D) $2q/\varepsilon_0$
(E) Need to do a nasty integral to evaluate this

Poll question

What is the electric flux through a spherical surface of radius R with point charge q a distance R/2 away from the center?



(A) 0
(B) $q/(2\varepsilon_0)$
(C) q/ε_0
(D) $2q/\varepsilon_0$
(E) Need to do a nasty integral to evaluate this

Gauss' Law: another look

$$\oint_S \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0$$

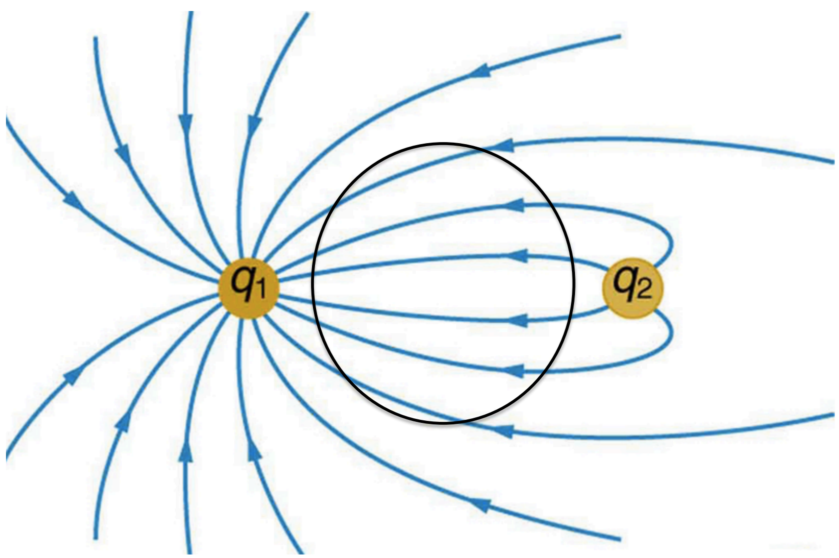
For a particular surface:
relates values of field on surface to the net charge inside

What can we say about the electric field on the surface if the right hand side is zero?

Gauss' Law: another look

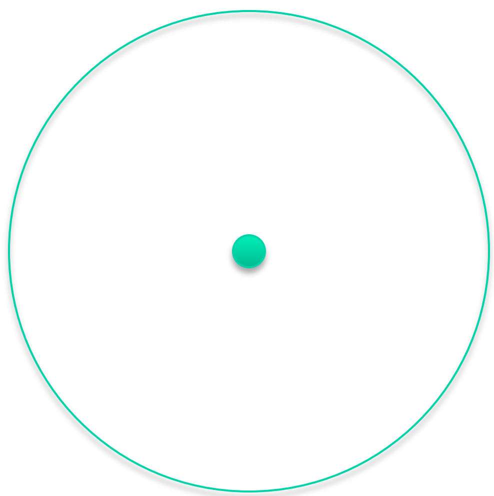
$$\oint_S \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0$$

If the right hand side is zero,
It DOESN'T mean that the field on the surface is zero!
In general, knowing the flux through a surface won't tell you the field at all points on the surface



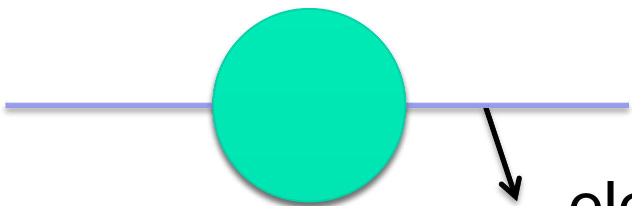
$$\oint_S \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0$$

BUT if we know the flux through a sphere of radius r centered on a point charge, we CAN find the field at all points on the sphere!

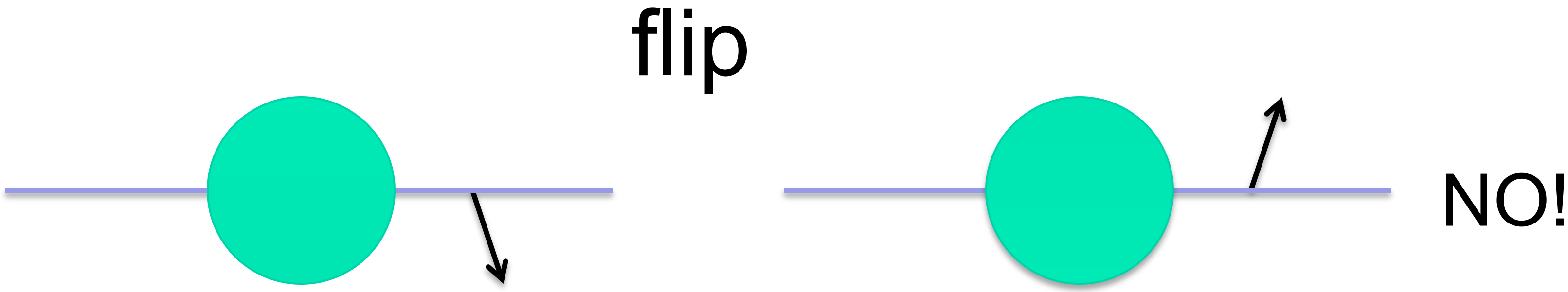


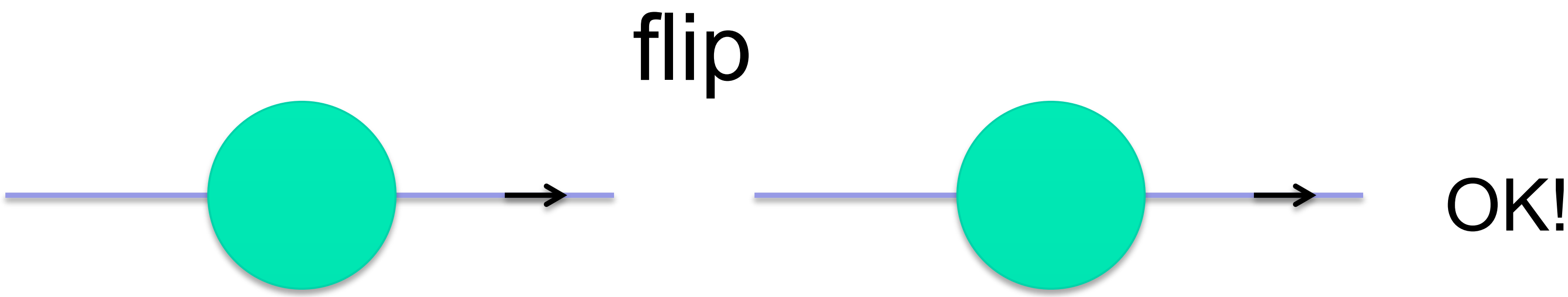
Get the necessary additional information about E(r) from SYMMETRY

Spherically symmetric charge distribution

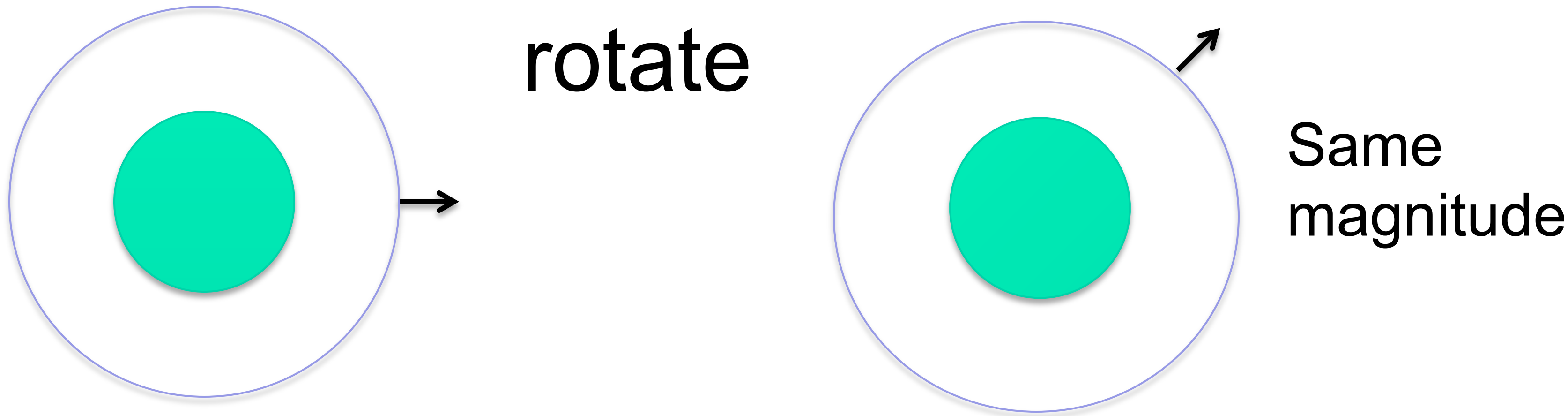


electric field?


flip
NO!



SO, direction of electric field must be RADIAL (along r)

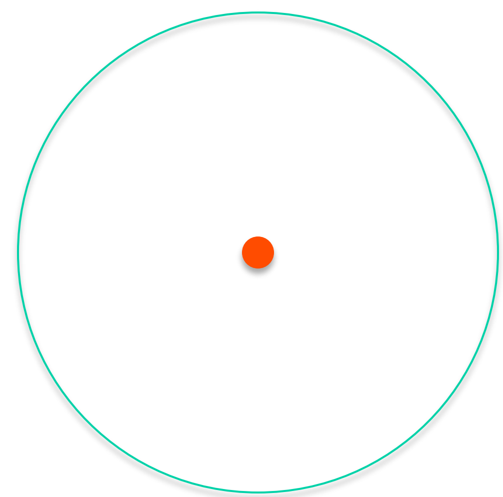


SO, magnitude of electric field only depends on the distance from the center!

$$\vec{E}(\vec{r}) = E(r)\hat{r}$$

Evaluate the flux integral
on sphere of radius r

$$\oint_S \vec{E} \cdot d\vec{A} = E(r)\pi r^2$$

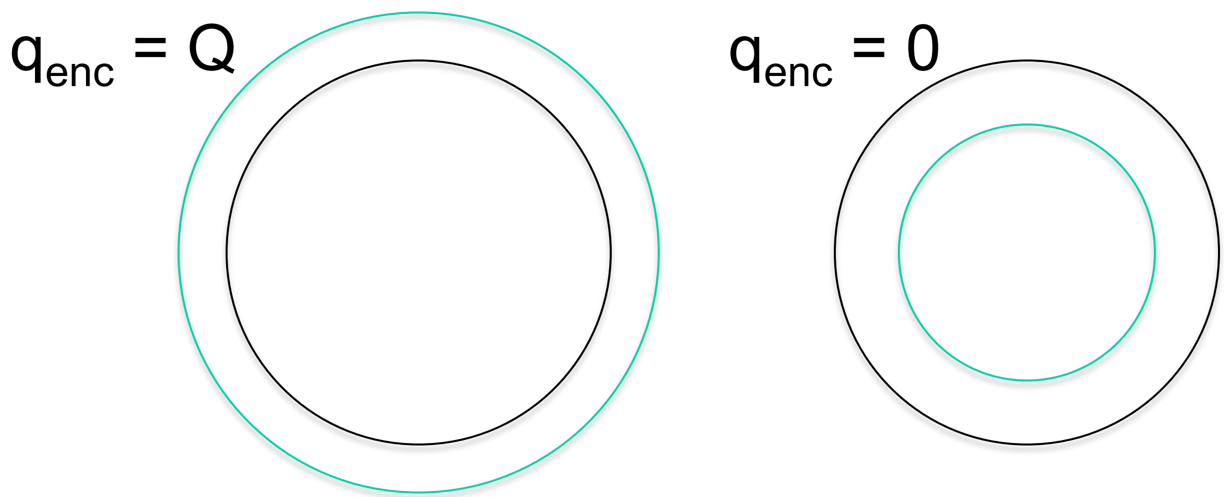


Gauss Law:
$E(r)\ 4\pi r^2 = q/\varepsilon_0$

$E(r) = q/(4\pi\varepsilon_0 r^2) = kq/r^2$
direction is radial
= Coulomb's law!

same argument works for any spherically symmetric charge distribution $\vec{E}(\vec{r}) = E(r)\hat{r}$

Use Gauss' law to prove the shell theorems
-field outside a uniformly charged spherical shell is the same as if the charge were concentrated at a point at the center
-field inside a uniformly charged spherical shell is zero



What have we gained by using Gauss' law instead of Coulomb's law?

Simple way of finding the field – works for spherical symmetry and other very symmetrical charge distributions

- Wires and symmetric infinite cylinders (HRW 10 23-4)

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \text{(line of charge).}$$

- Infinite flat uniform sheets of charge (HRW 10 23-5)

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(sheet of charge).}$$