Second homework is due next TUESDAY 11:59PM – Sept 15
Next spring Phys 228H is using HRW10 and Webassign
Yes to multiterm access

- Getting the most out of the homework

- A brief comment on grading

Course web site
http://www.physics.rutgers.edu/~karin/227H

Topics for today

Specifying charge distribution − point charges and continuous charge distributions
Electric field from a fixed charge distribution

Uniform electric field
force and torque on a dipole in a uniform field

Reformulation of Coulomb's law = Gauss' law
Electric flux

Specifying a fixed distribution of charges

Point charges Q_i, $\vec{r}_i$

Continuous charge distributions

Continuous charge distributions

Total charge Q uniformly spread over a wire of length L

Take a small piece of length d: what is the charge?
$Q\,(d/L) = (Q/L)\,d$

Define
$\lambda = Q/L$

"Line charge density" λ: charge/length (C/m)

Continuous charge distributions

Total charge Q uniformly spread over a surface of area A

Take a small piece of area A_1: what is the charge?
$Q\,(A_1/A) = (Q/A)\,A_1$

Define
$\sigma = Q/A$
"surface charge density" σ: charge/area (C/m^2)

Continuous charge distributions

Total charge Q uniformly distributed through a 3D region of volume V

Take a small chunk of volume V_1: what is the charge?
$Q\ (V_1/V) = (Q/V)\ V_1$

Define
$\rho = Q/V$
"charge density" ρ: charge/volume (C/m^3)

Electric field from a fixed distribution of charges

Point charges Q_i, $\vec{r}_i$

Sum the field from each point charge

$$\vec{E}(\vec{r}) = \sum_i \frac{kQ_i}{|\vec{r}-\vec{r}_i|^2} \frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|}$$

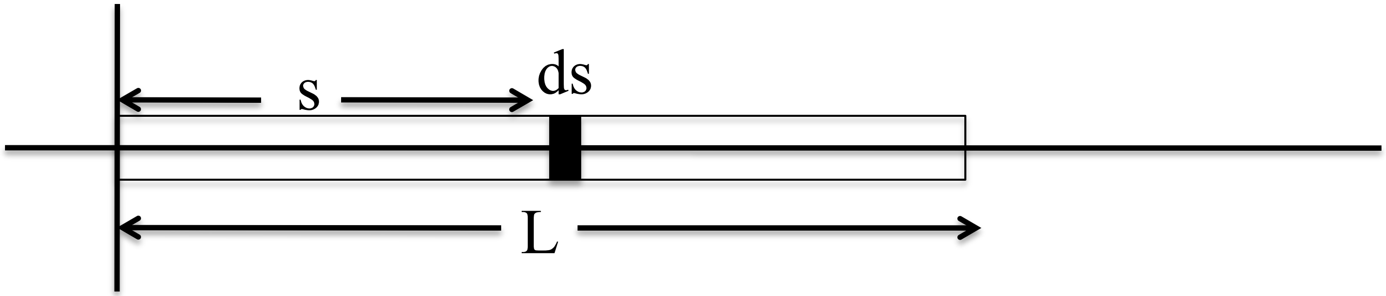
Continuous charge distributions

Divide charge distribution into teeny tiny pieces

Integrate the field from each piece in the charge distribution

The Electric Field due to a Continuous Charge : Example

Thin wire of length L has total charge Q distributed uniformly. Find E_x and E_y at any point on the x axis with x>L.



Linear charge density: $\lambda = Q/L$

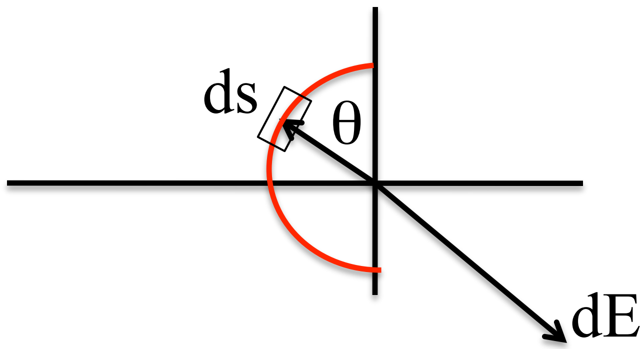
$dE_y = 0$ so $E_y = 0$

$dE_x = k\, dq / r^2 = k\, \lambda\, ds / (x-s)^2$ integrate from s=0 to s=L

$$E_x = \int dE_x = \int_{s=0}^{s=L} \frac{k\,dq}{r^2} = \int_{s=0}^{s=L} \frac{k\lambda}{(x-s)^2}\,ds = k\lambda (x-s)^{-1}\Big|_{s=0}^{s=L} = k\lambda\left(\frac{1}{x-L} - \frac{1}{x}\right) = k\lambda \frac{L}{x(x-L)}$$

LINE INTEGRAL over curve $\vec{c}(s) = (s, 0)$ with $0 \le s \le L$

The Electric Field due to a Continuous Charge : Example



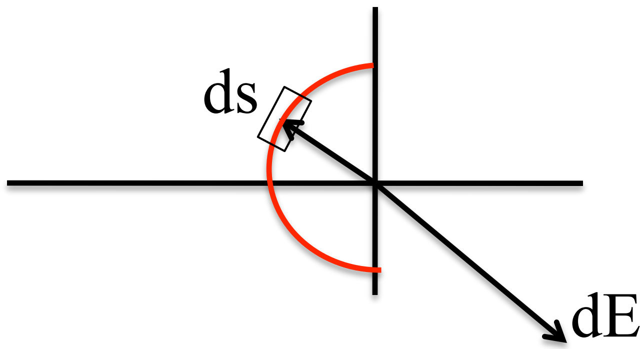
A uniformly charged wire of total charge Q is bent into semicircle of radius R.
Find E_x and E_y at center

Charge density $\lambda = Q/\pi R$ so that element ds of the wire has charge
$dq = \lambda ds$
Parametrize curve by θ
$ds = R d\theta$

$E_y = 0$ (cancellation, symmetry)

$dE_x = (k\, dq / R^2)(\sin\theta)$
θ runs from 0 to π

The Electric Field due to a Continuous Charge : Example



A uniformly charged wire of total charge Q is bent into semicircle of radius R.

Find E_x and E_y at center

Charge density $\lambda = Q/\pi R$ so that element ds of the wire has charge $dq = \lambda ds$

Parametrize curve by θ

$ds = R d\theta$

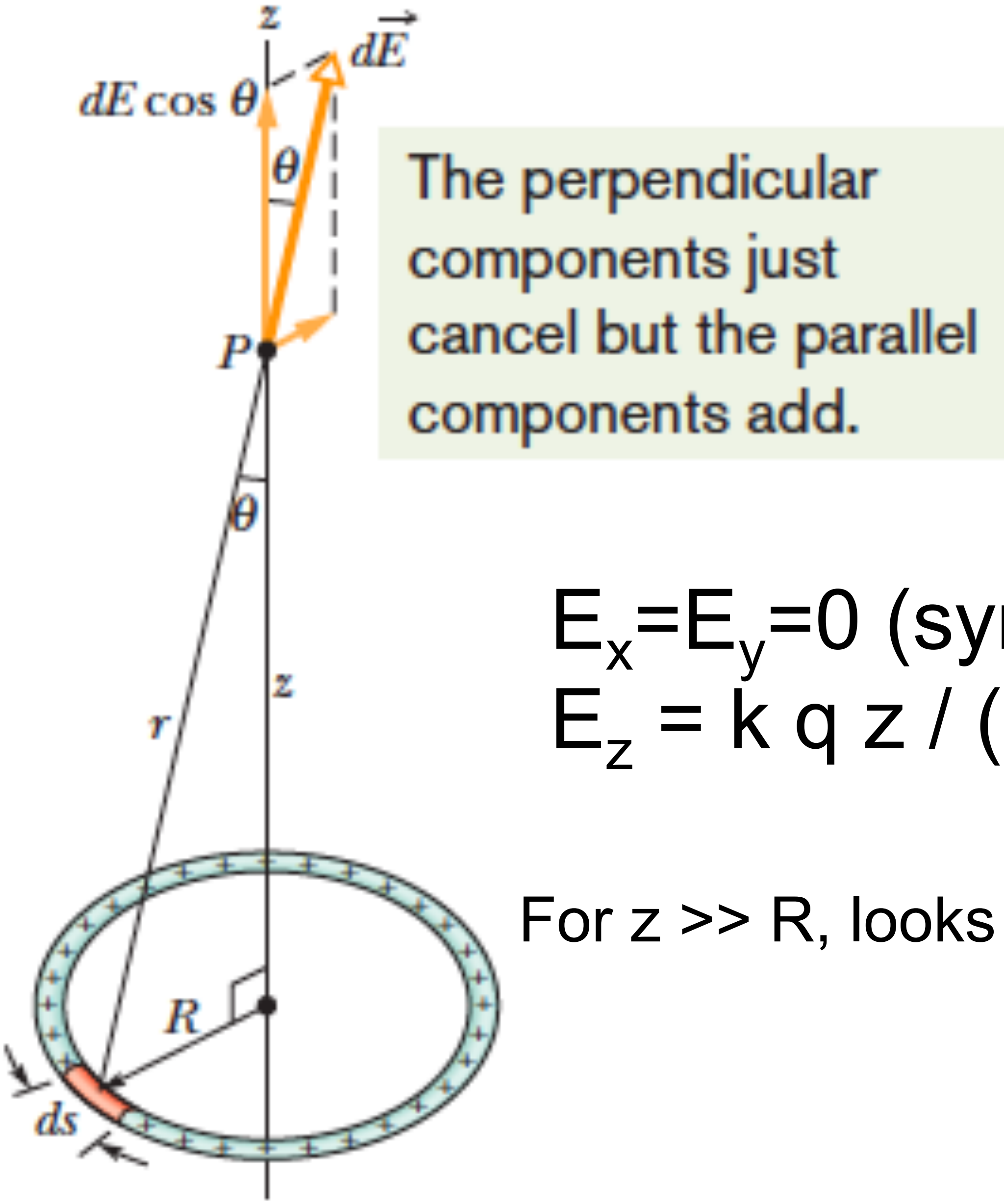
Integrate $dE_x = (k\lambda R d\theta / R^2)(\sin\theta)$ from $\theta = 0$ to $\theta = \pi$

$E_y = 0$

$E_x = 2\, k\, \lambda\, /\, R$ (to the right)

$$E_x = \int dE_x = \int_{\theta=0}^{\theta=180^\circ} \sin\theta \frac{k\lambda R d\theta}{R^2} = -\frac{k\lambda}{R}\cos\theta\Big|_{\theta=0}^{\theta=180^\circ} = -\frac{k\lambda}{R}(-1-(-1)) = \frac{2k\lambda}{R}$$

Electric field of uniformly charged ring on its central axis

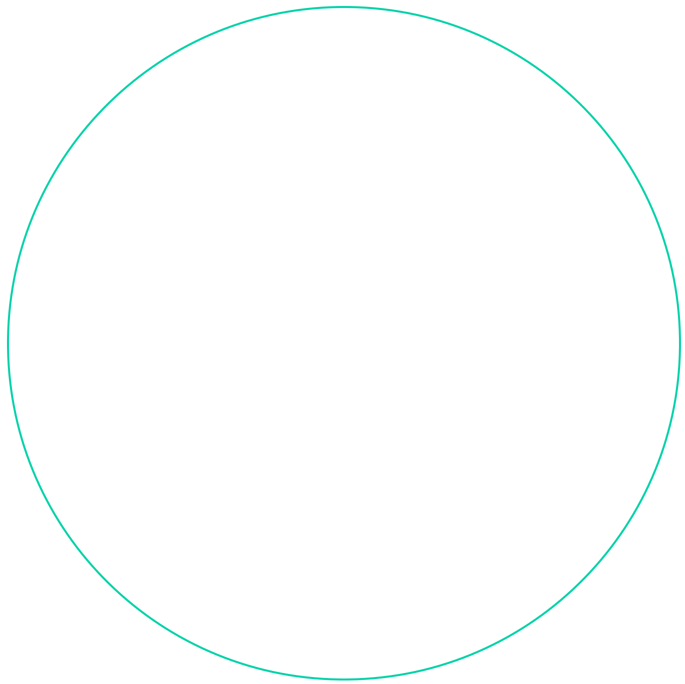


$E_x = E_y = 0$ (symmetry)

$E_z = k\, q\, z \,/\, (z^2 + R^2)^{3/2}$

For z >> R, looks like point charge q

Electric field of uniformly charged spherical shell Q, R



Gravitational force exerted by a uniform shell of mass

on a particle outside the shell

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.

on a particle inside the shell

A uniform shell of matter exerts no net gravitational force on a particle located inside it.

Force exerted by a uniform shell

on a particle outside the shell

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.

A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at its center.

on a particle inside the shell

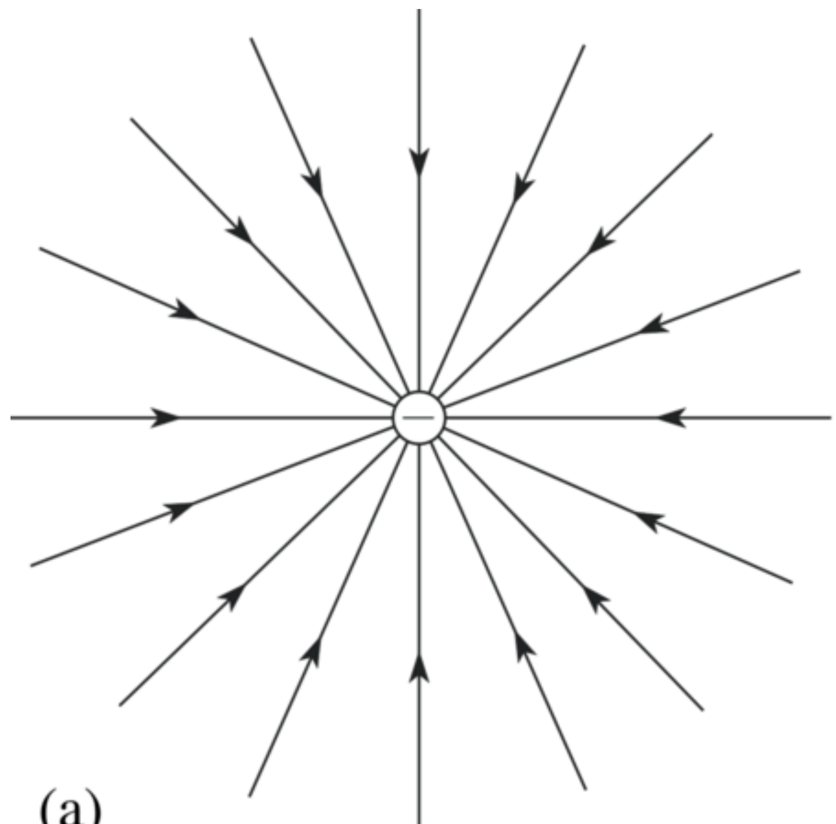
A uniform shell of matter exerts no net gravitational force on a particle located inside it.

If a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.

Electric field inside the shell: zero

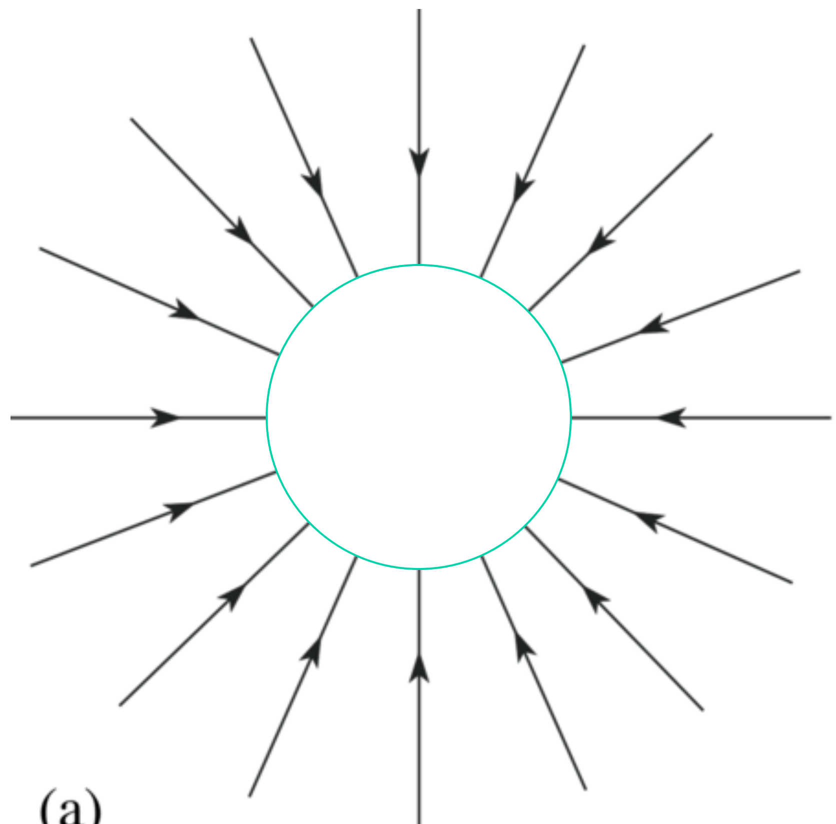
Outside: the same as if there were point charge Q at center

Electric field of a charge Q < 0



(a)

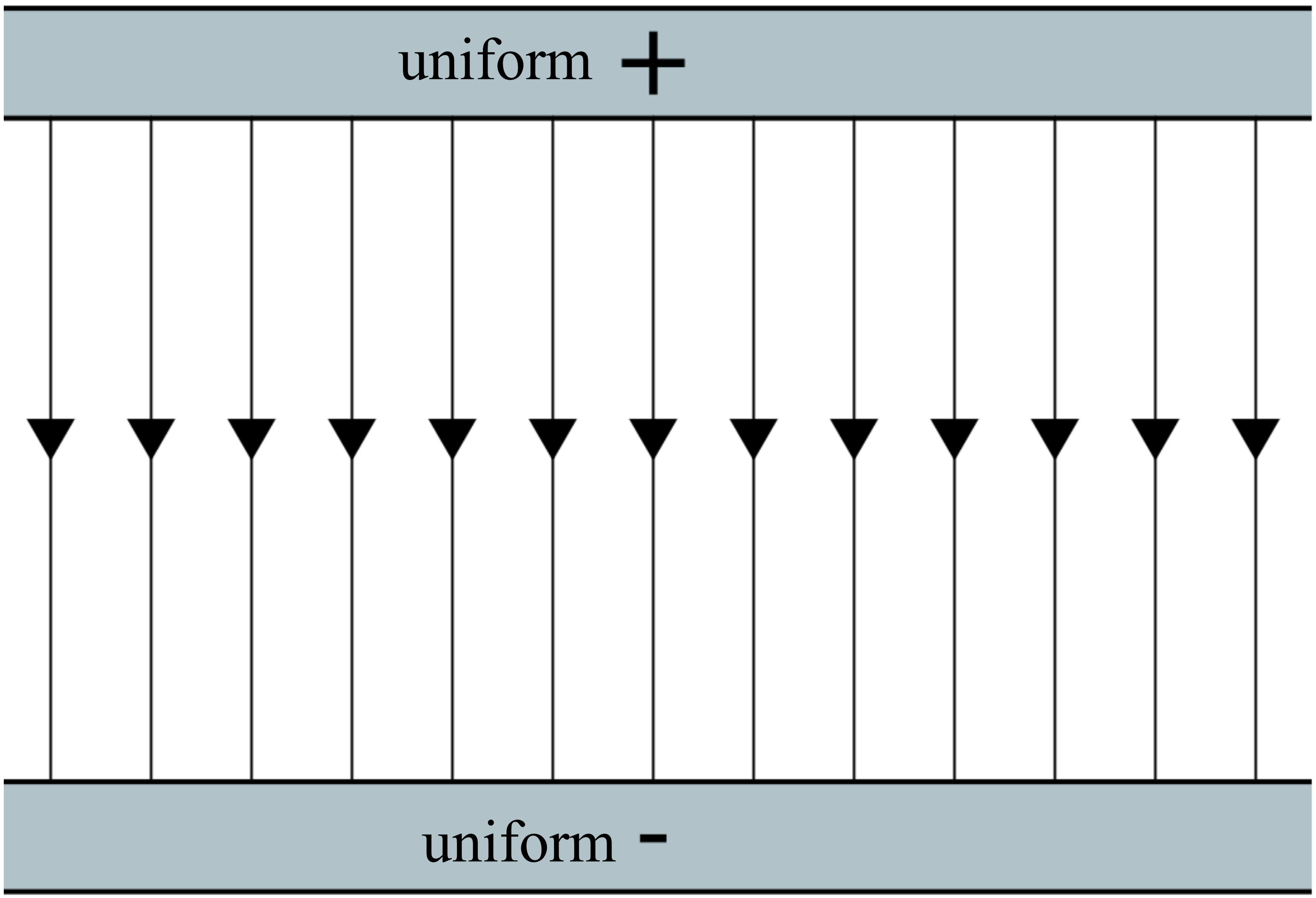
Electric field of a spherical shell with Q < 0



(a)

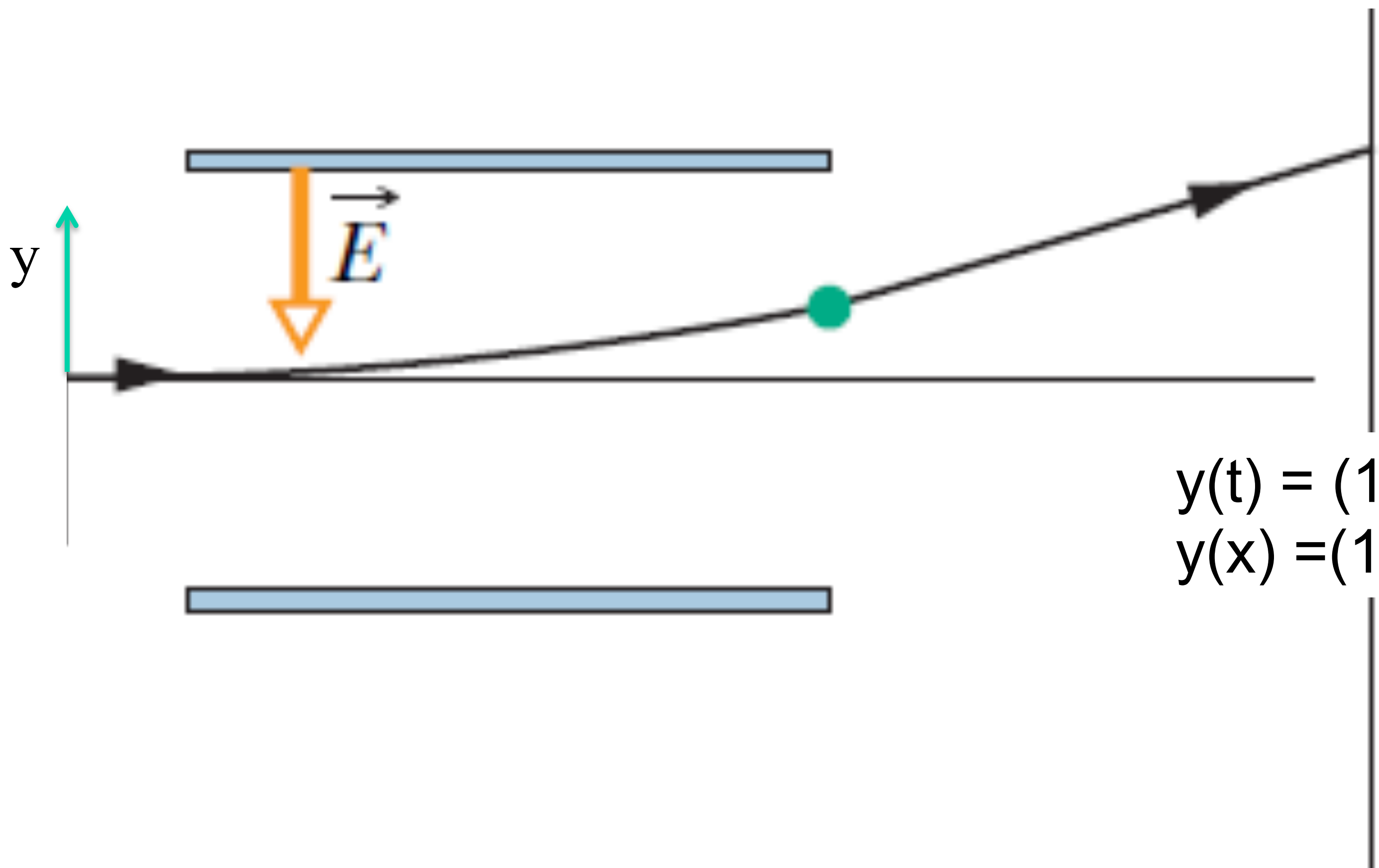
inside the shell: zero
Outside: the same as if there were point charge Q at center

The uniform electric field $\vec{E}(\vec{r}) = \vec{E}$



A Point Charge in an Electric Field : Motion in a uniform electric field

Given a uniform electric field **E**, how will charge q move?
Charge will feel force **F**=q**E** (same force for all positions)
F=m**a**, so **a** =**F**/m=q**E**/m (constant acceleration)

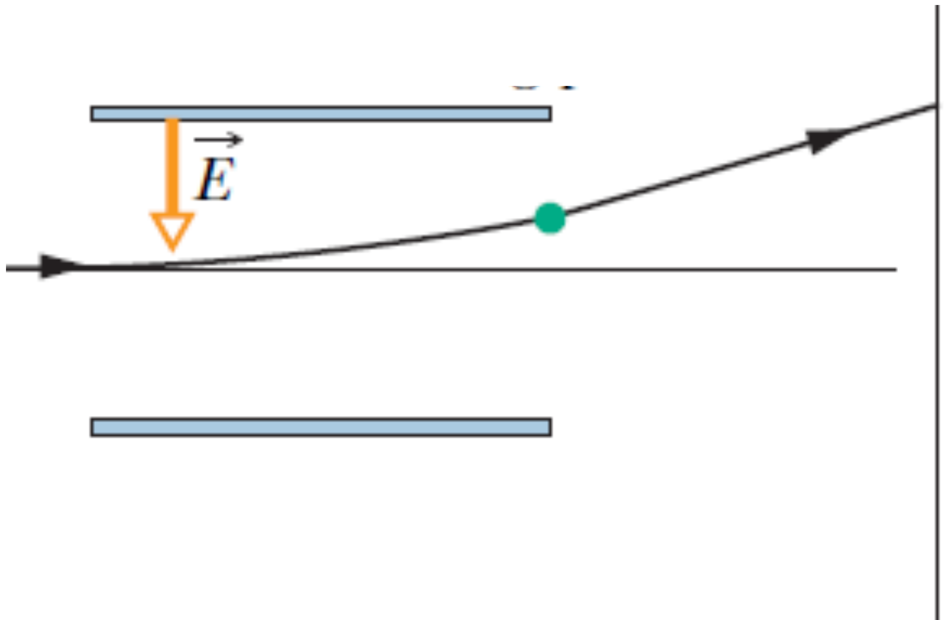


$y(t) = (1/2)at^2$

$y(x) = (1/2)a(x/v_{0x})^2$

A Point Charge in an Electric Field : Motion in a uniform electric field

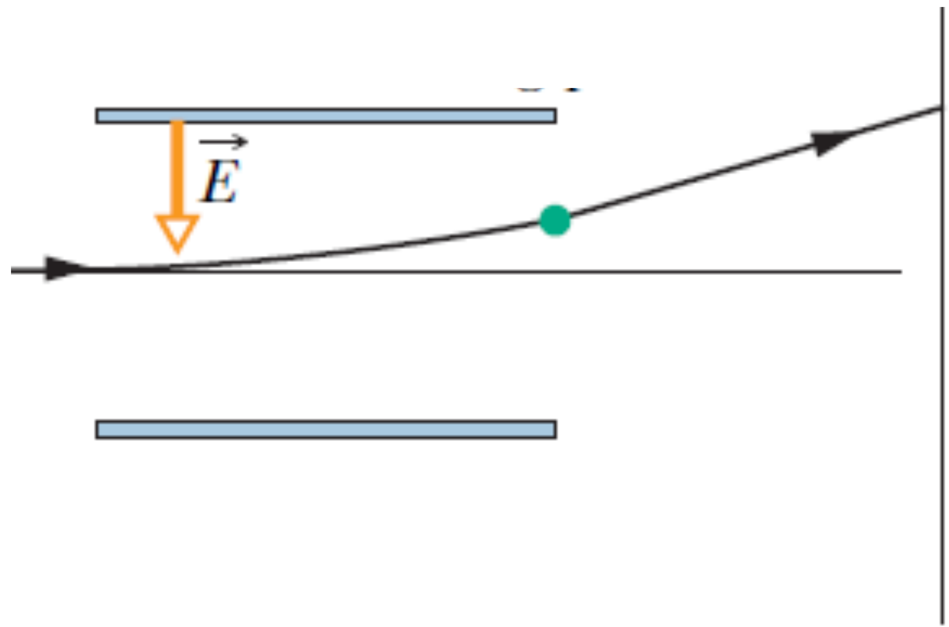
Poll
Question 1



The charge on the green particle in the figure is
(a) Positive
(b) Negative
(c) Zero
(d) There is not enough information to determine its charge
(e) If I put any of the other answers I would be just guessing

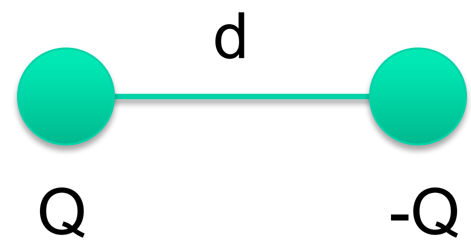
A Point Charge in an Electric Field : Motion in a uniform electric field

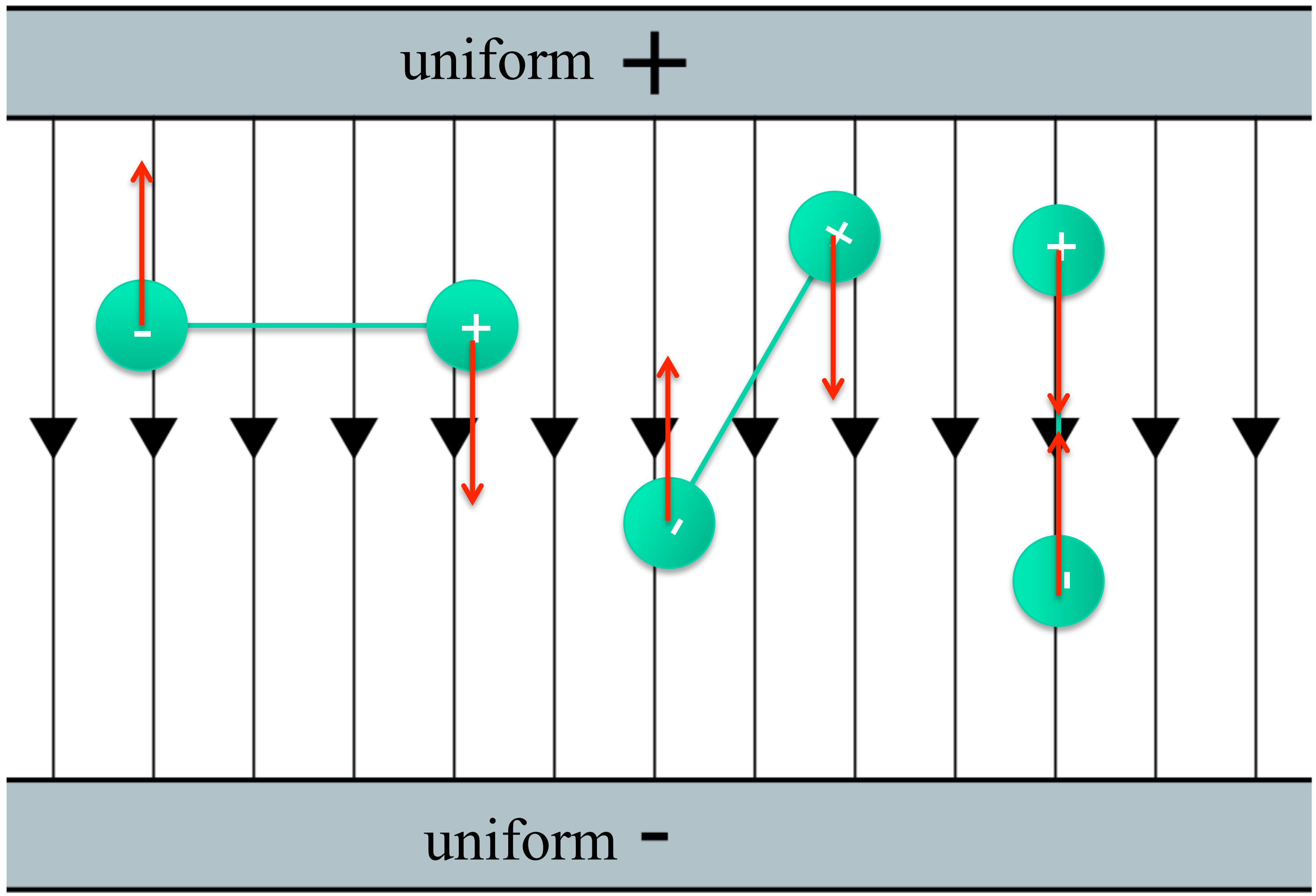
Poll
Question 1



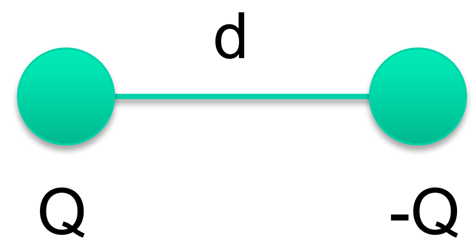
The charge on the green particle in the figure is
(a) Positive
(b) Negative – force is up, opposite to electric field
(c) Zero
(d) There is not enough information to determine its charge
(e) If I put any of the other answers I would be just guessing

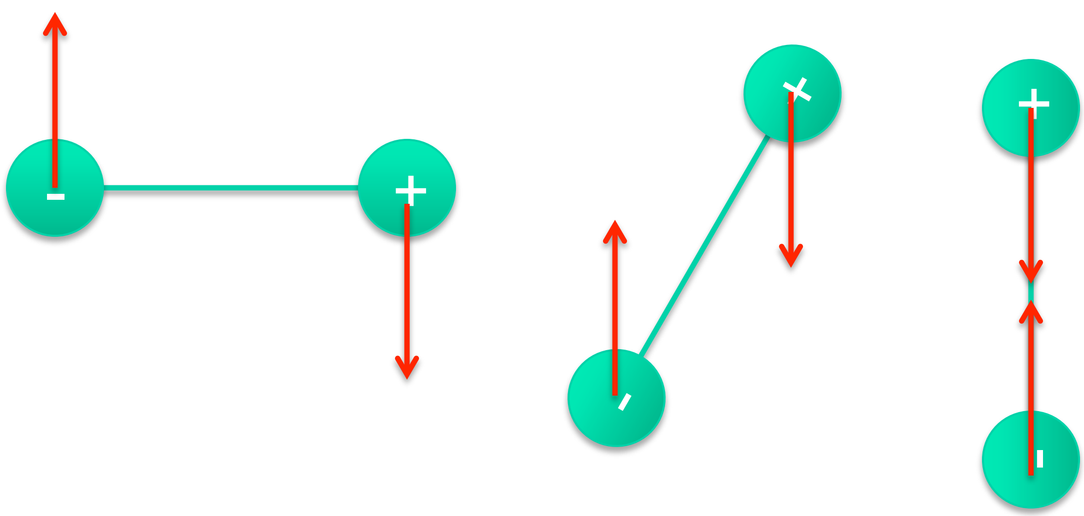
Force on dipole in uniform field



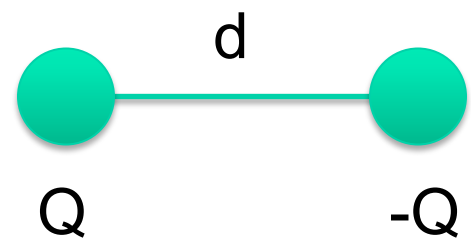


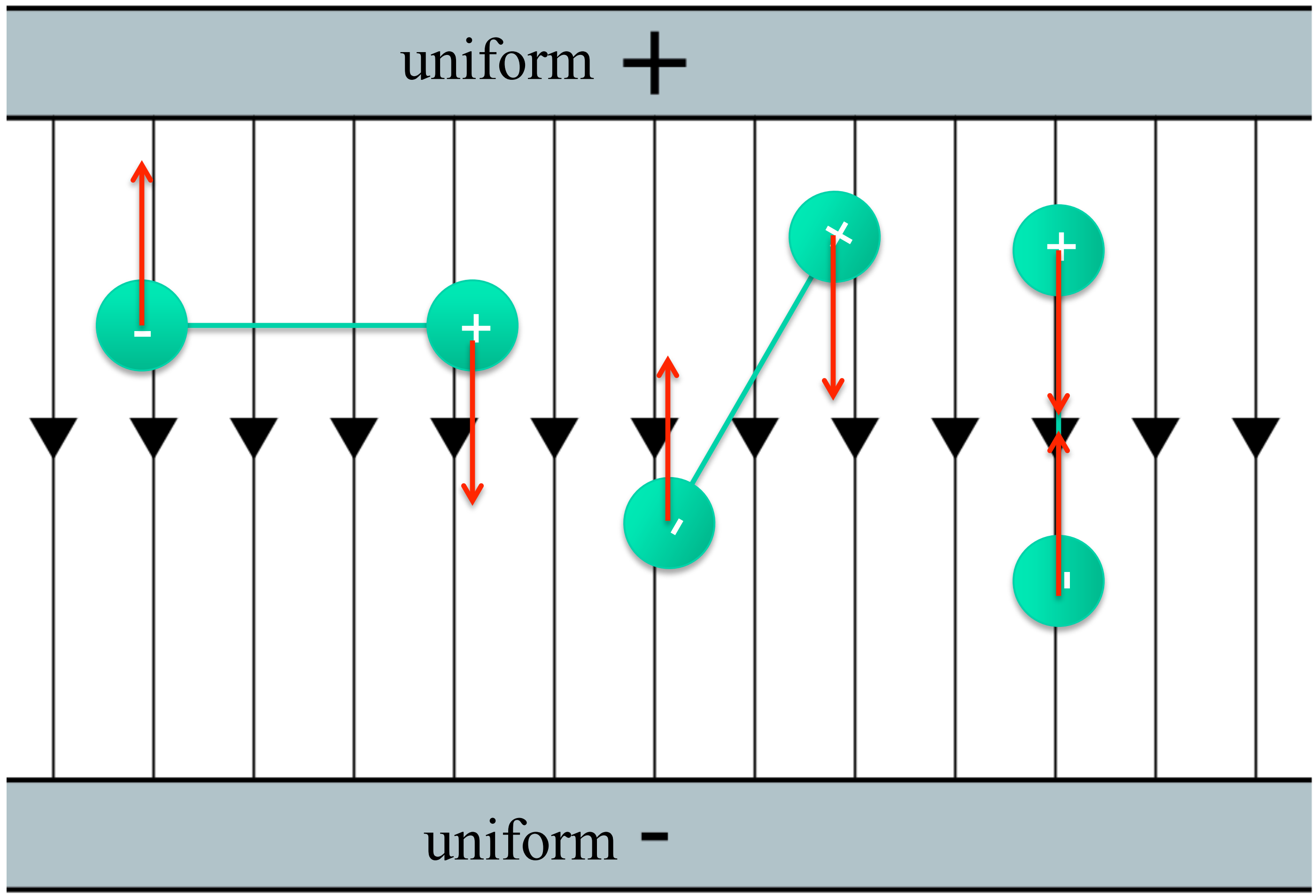
Force on dipole in uniform field



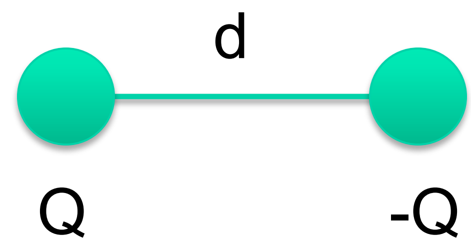


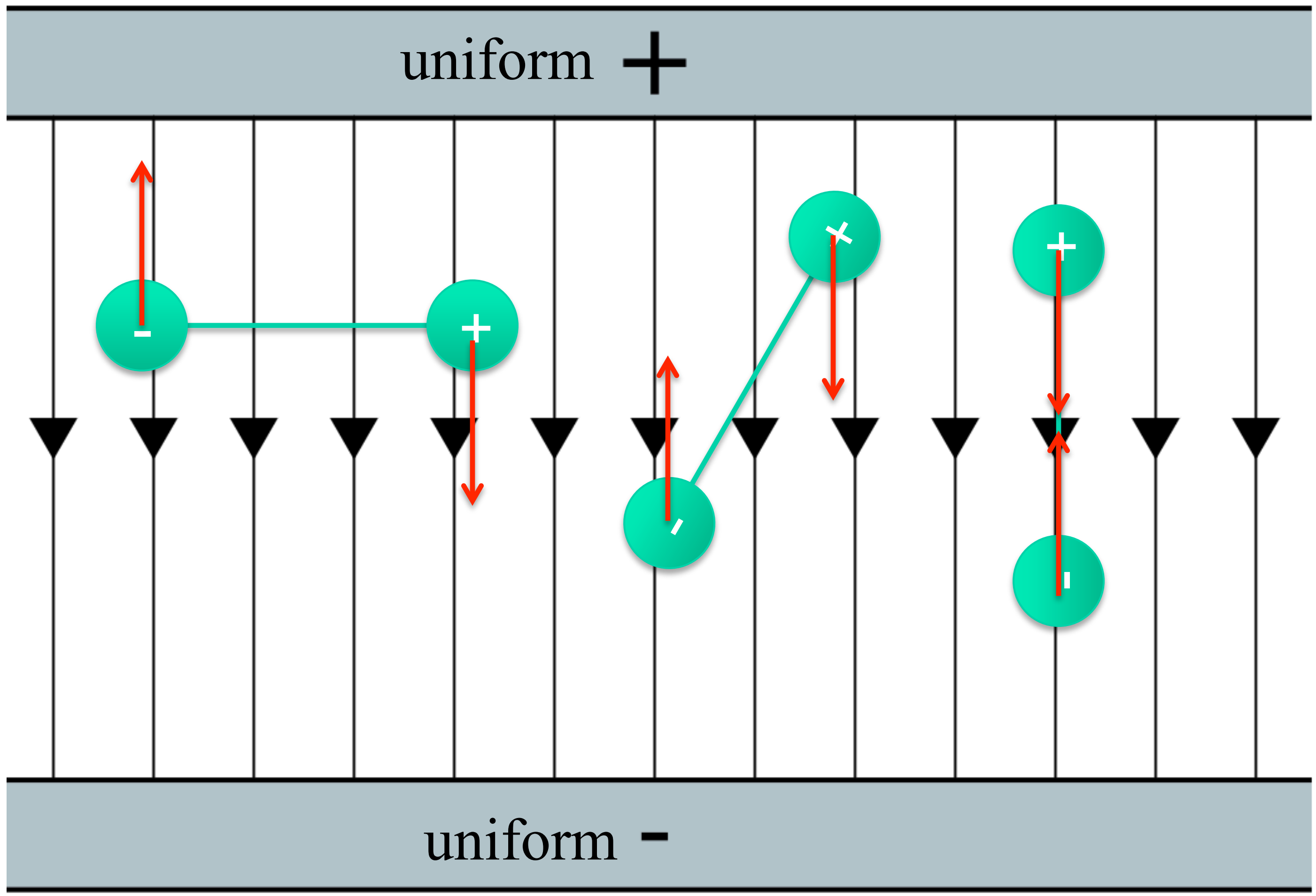
Force on dipole in uniform field
NET FORCE IS ALWAYS ZERO



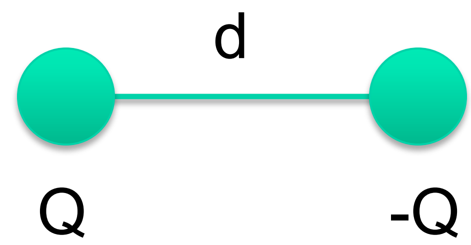


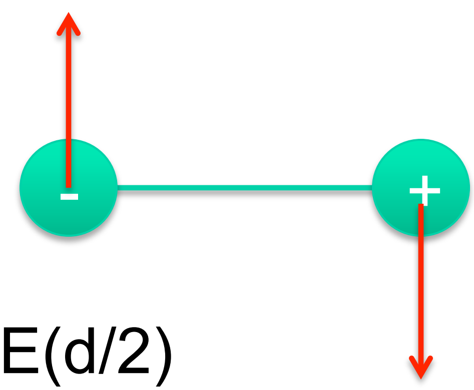
TORQUE on dipole in uniform field
torque on dipole in uniform field



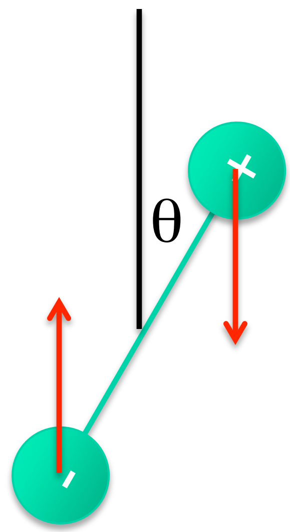


TORQUE on dipole in uniform field

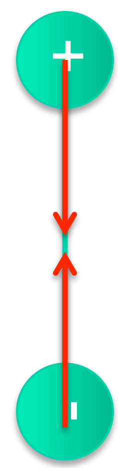




2 QE(d/2)
into the screen

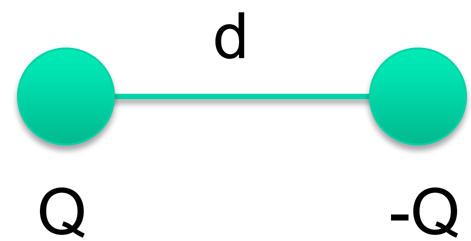


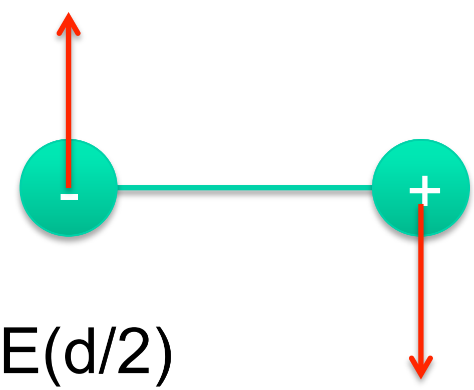
2 QE(d sinθ /2)
into the screen



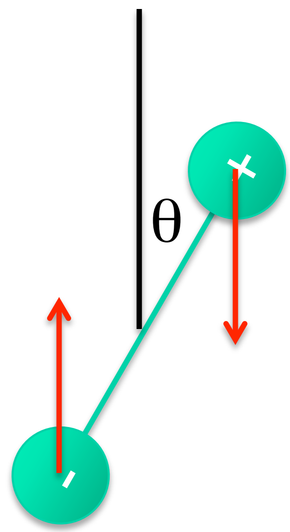
zero

TORQUE on dipole in uniform field
Q d sinθ
turns dipole to align with the field

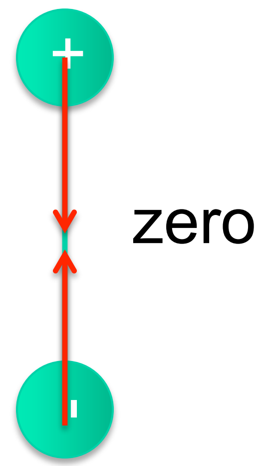




2 QE(d/2)
into the screen



2 QE(d sinθ /2)
into the screen



As the dipole turns in the electric field, the electric torque does work

We can keep track of the work done using a potential energy function U

$$U = -W = -\int_{90^\circ}^{\theta} \tau \, d\theta = \int_{90^\circ}^{\theta} pE \sin\theta \, d\theta. \tag{22-36}$$

Evaluating the integral leads to

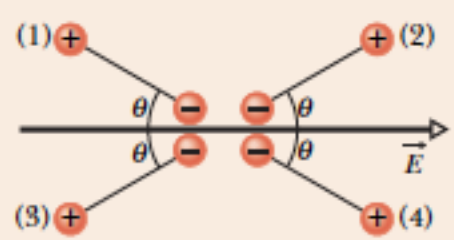
$$U = -pE\cos\theta. \tag{22-37}$$

We can generalize this equation to vector form as

$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).} \tag{22-38}$$

CHECKPOINT 4

The figure shows four orientations of an electric dipole in an external electric field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first.

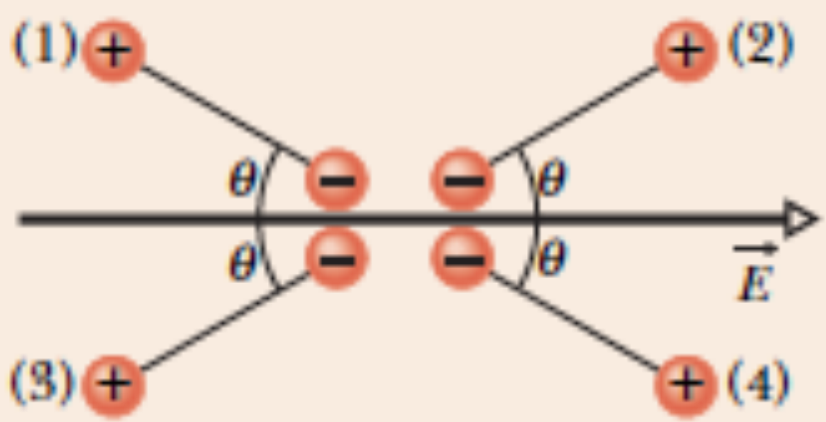


As the dipole turns in the electric field, the electric torque does work

We can keep track of the work done using a potential energy function U

CHECKPOINT 4

The figure shows four orientations of an electric dipole in an external electric field. Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first.



Charge distribution determines electric field
sum, line integral, surface integral, volume integral

--shouldn’t there be an easier way to prove the shell theorem?
--does the electric field determine the charge distribution?
--how do you find the charge distribution from the electric field?

Charge distribution determines electric field
sum, line integral, surface integral, volume integral

--shouldn't there be an easier way to prove the shell theorem?
--does the electric field determine the charge distribution?
--how do you find the charge distribution from the electric field?

REFORMULATION OF COULOMB'S LAW

$$\oint_S \vec{E} \cdot d\vec{A} = q_{enc} / \varepsilon_0$$ GAUSS' LAW