Physics 618: Applied Group Theory

Fall, 2009

September 1, 2009

1. What the course is about

"A man who is tired of group theory is a man who is tired of life." – Sidney Coleman

This is a course about groups and their representations. This is a vast topic, with an unbelievably wide spectrum of applications to physics. We will cover basic definitions and examples, and also illustrate these with more advanced applications.

2. Boundary Conditions

This course is primarily intended for advanced undergraduate students and graduate students intending to specialize in theory. There will be some bias towards particle theory, although there is much here that is useful to the nuclear and condensed matter theorist.

We try to keep prerequisites to a minimum. Occasionally I will introduce examples based on more advanced material, but the main development will be elementary. Occasionally a knowledge of basic differential geometry and topology is useful, particularly for the material on Lie groups.

3. Tentative Plan

Approximate plan for topics to cover. (This year I might introduce new topics and skip over some of the topics listed below. On a previous occasion I covered about 11 of the subsections.)

3.1. Abstract Group Theory: basic structure theory of groups. Extensions and central extensions. Classification of finitely generated abelian groups.

3.2. Brief review of linear algebra and some functional analysis. Spectral theorem, Jordan decomposition, and other canonical forms. Basic definitions of representation theory

3.3. Groups and Symmetries: Orbits and orbifolds. Discrete subgroups of $SU(2)$ *and* $SO(3)$ *and symmetries of solids. Crystallographic groups. Rubik's cube.*

3.4. Group actions on manifolds

3.5. Representations of finite groups. Symmetric functions and bosonization in 1+1 dimensions.

3.6. Survey of matrix groups: GL, SL, SO, SU, Sp etc. Some geometry and topology of *these groups.*

3.7. Lorentz groups and rotation groups in low dimensions: quaternions and octonions

3.8. Lie algebras from Lie groups: Lie's theorem, Baker-Campbell-Hausdorff formula,

3.9. Harmonic oscillators: Symplectic and metaplectic groups.

3.10. Survey of some Lie algebras and Lie superalgebras.

3.11. Conformal and superconformal algebras. Their relation to deSitter and anti-deSitter spaces (as used in the AdS/CFT conjecture). Wigner-Inonu contraction.

3.12. Clifford algebras and spinors: Real and Complex. All signatures, all dimensions. Mod 8 periodicity. Relation of products of spinors to antisymmetric tensors

3.13. Some remarks on Poincare supersymmetry and superconformal algebras. Supersymmetric quantum mechanics

3.14. Structure of semisimple Lie algebras: Root systems. Cartan classification. Kac-Moody and affine Lie algebras.

3.15. Highest weight representations of semisimple Lie algebras

3.16. Induced representations

3.17. Unitary Representations of the Lorentz and Poincaré groups

3.18. Representations of supersymmetry algebras

3.19. Nonlinear sigma models: Quantum field theories defined by group manifolds and homogeneous spaces.

3.20. Geometry and topology of Lie groups

3.21. Affine Lie algebras and Kac-Moody algebras

4. Administrative

1. Notes for all the lectures will be handed out. There is a list of useful references and textbooks at the end of this handout.

2. The grade for those taking the course for credit will be based on a short paper and possibly a presentation given at the end of the semester. I will hand out topics towards the middle of the course.

3. I will probably not hand out problem sets to be graded. However, the lecture notes contain plenty of exercises. You are encouraged to do them.

4. As a courtesy to others, PLEASE DO NOT EAT OR DRINK DURING CLASS.

5. Some sources

There is no formal textbook. The following is a list of sources I have used. There are many other texts available. Go to the library and find your favorite.

Basic math texts:

1. Herstein, Topics in Algebra, ch.2 : basic group theory, for the mathematically inclined.

2. N. Jacobsen, Basic Algebra I, also for the mathematically inclined.

3. D. Joyner, *Adventures in Group Theory*, a gentler introduction with lots of fun applications.

4. J.-E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Springer GTM

5. R. Carter, G. Segal, and I. MacDonald, *Lectures on Lie Groups and Lie Algebras*, London Mathematical Society Students Texts, 32. Beautifully concise.

Books written for physicists:

6. Miller, Symmetry groups and applications: Much of the lecture material was drawn from this book.

7. I.V. Schensted, A Course on the Application of Group Theory to Quantum Mechanics: very readable.

8. M. Hamermesh, Group Theory: a good reference

9. Wu-Ki Tung, Group Theory in Physics

10. H. Georgi, Lie Algebras in Particle Physics: Group representation theory for particle physicists.

11. R.N. Cahn, *Semi-Simple Lie Algebras and Their Representations*, Frontiers in Physics

12. J. Fuchs and C. Schweigert, *Symmetries, Lie Algebras and Representations: A graduate course for physicists*, Cambridge

13. J. Fuchs, *Affine Lie Algebras and Quantum Groups*, Cambridge

14. L. O' Raifeartaigh, *Group Structure of Gauge Theories*, Cambridge. Gives applications to unified field theory model building.

15. J.D. Talman and E.P. Wigner, Special Functions: This book explains the group theoretic approach to the theory of special functions.

16. C.J. Isham, Modern Differential Geometry for Physicists: a gentle introduction to the differential geometry of Lie groups and their cosets.

another good, and unusual summary of group theory is in

17. D. Mermin, Rev. Mod. Phys. **51** (1979)591. which has much interesting information.

For the theory of matrices two references are:

18. F.R. Gantmacher, *Applications of the Theory of Matrices*, Interscience, 1959

19. P. Lancaster, *Theory of matrices*, Academic Press, 1969.

For supersymmetry:

1. J. Bagger and J. Wess, *Supersymmetry and Supergravity.* Princeton

2. P. Freund, *Supersymmetry.* Cambridge; Nice summary of super-Lie algebras

3. J. Lykken - TASI lectures hepth/9612114

4. P. West, *Introduction to supersymmetry and supergravity.*

5. M. Sohnius, "Introducing supersymmetry," Phys. Rep. **128**(1985) 39.

6. J. Strathdee, "Extended superpoincare supersymmetry" Int. J. Mod. Phys. **A2**(1987) 273

7. A. Van Proeyen, "Tools for supersymmetry," hep-th/9910030

8. A. Bilal, "Introduction to supersymmetry," hep-th/0101055

9 S. Weinberg, *Quantum Theory of Fields, vol. 3*

For some history and culture associated with group theory see

1. H. Weyl, *Symmetry*

2. E.T. Bell, *Men of Mathematics*