Driven-dissipative dynamics of atomic clocks

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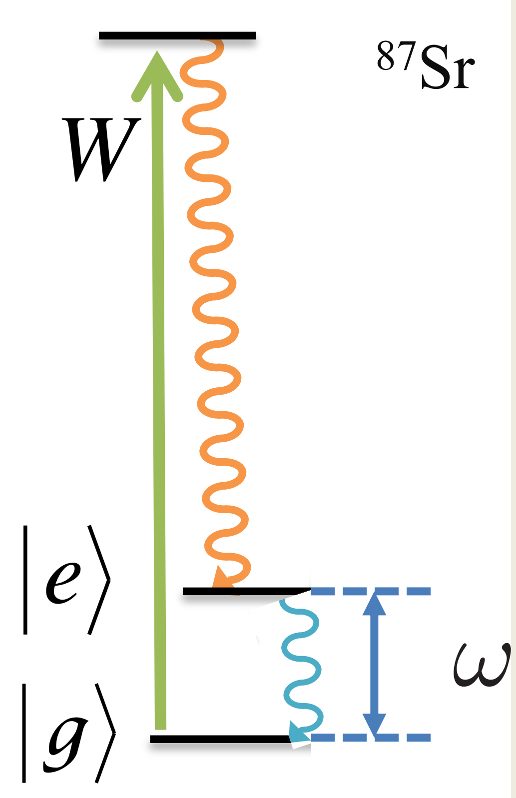
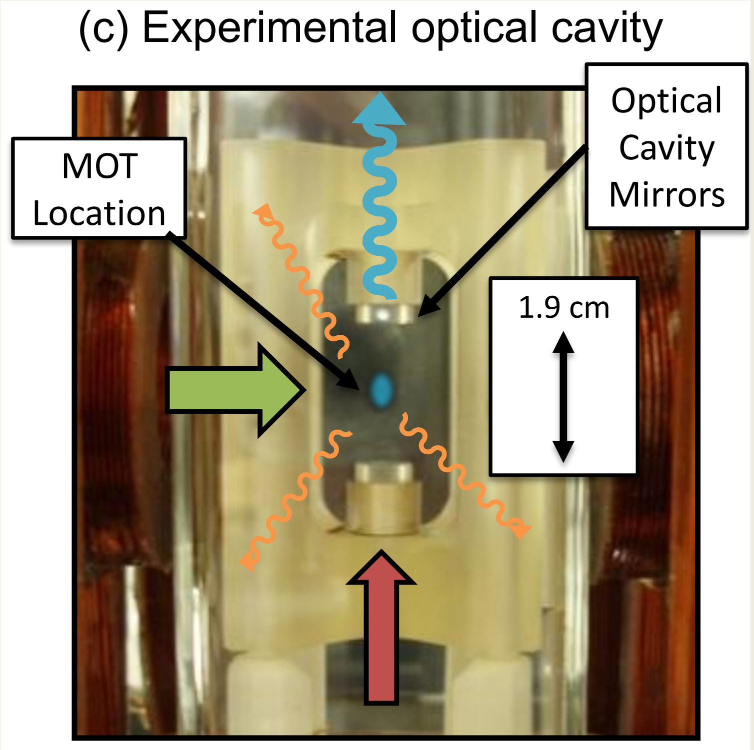
Ultra-Precise Superradiant Atomic Clock from JILA





J.K. Thompson et. al., JILA (2013)

- Proof-of-principle: ultracold ^{87}Rb atoms ($N \sim 10^6$) in a bad cavity
- Actual clock: ^{87}Sr or ^{171}YB instead

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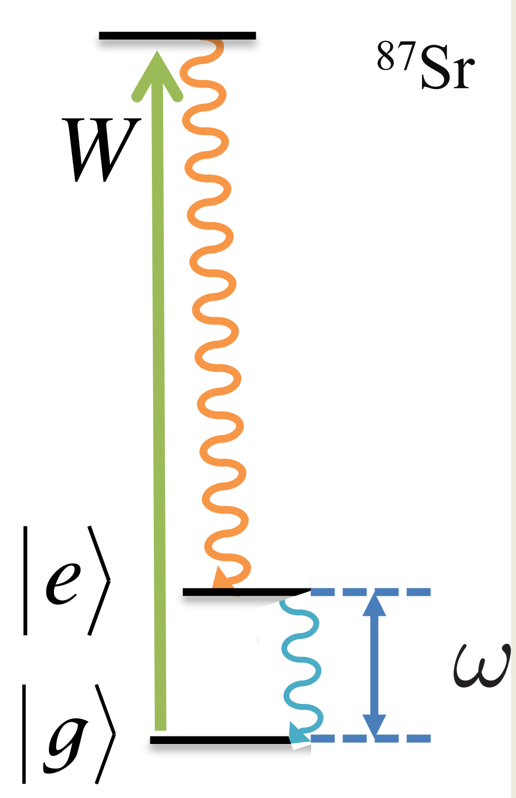

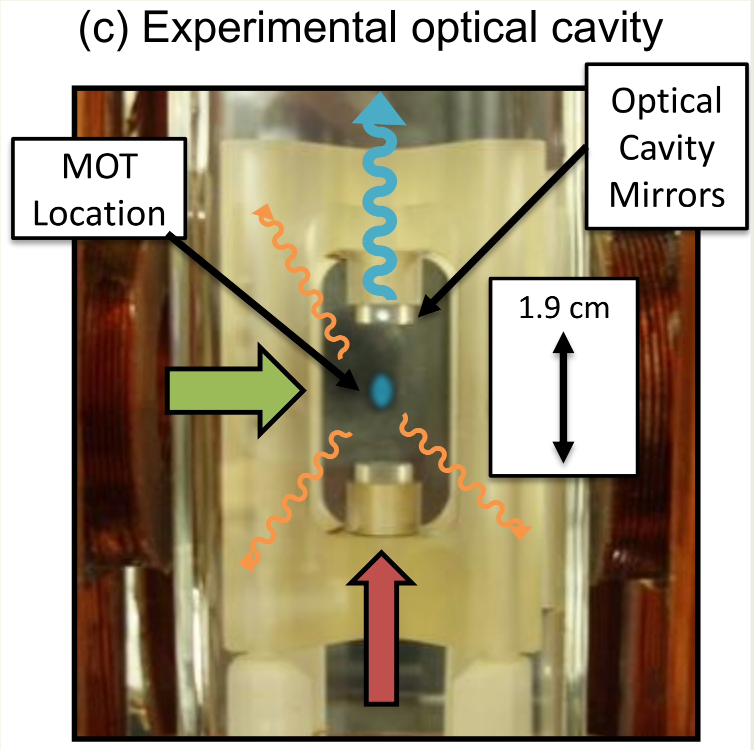

- Incoherent pumping with rate W

Atom + Cavity Hamiltonian in the rotating frame of the cavity field:

$$\hat{H} = \omega \hat{S}_z + \frac{\Omega}{2}\left(\hat{a}^\dagger \hat{S}_- + \hat{a}\hat{S}_+\right)$$

Cavity mode: $\hat{a}, \hat{a}^\dagger$

Dicke Hamiltonian

Collective spin: $\hat{S}_z = \frac{1}{2}\sum_{j=1}^{N} \hat{\sigma}_j^z, \quad \hat{S}_\pm = \frac{1}{2}\sum_{j=1}^{N} \hat{\sigma}_j^\pm$

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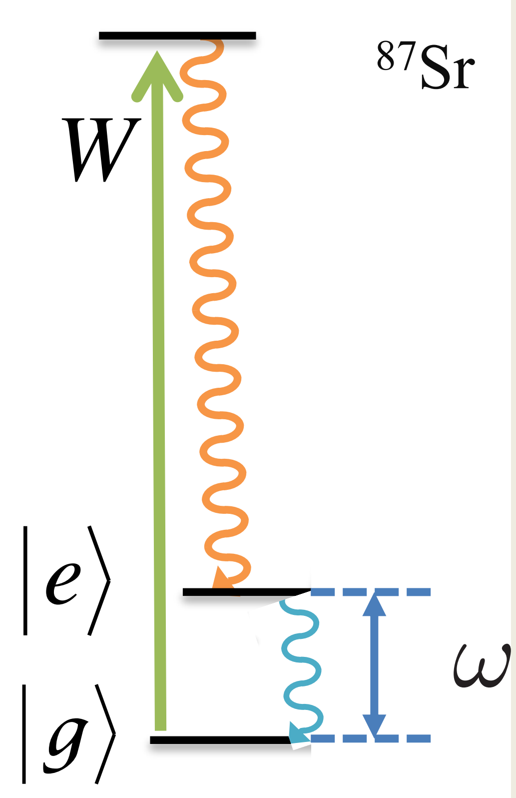

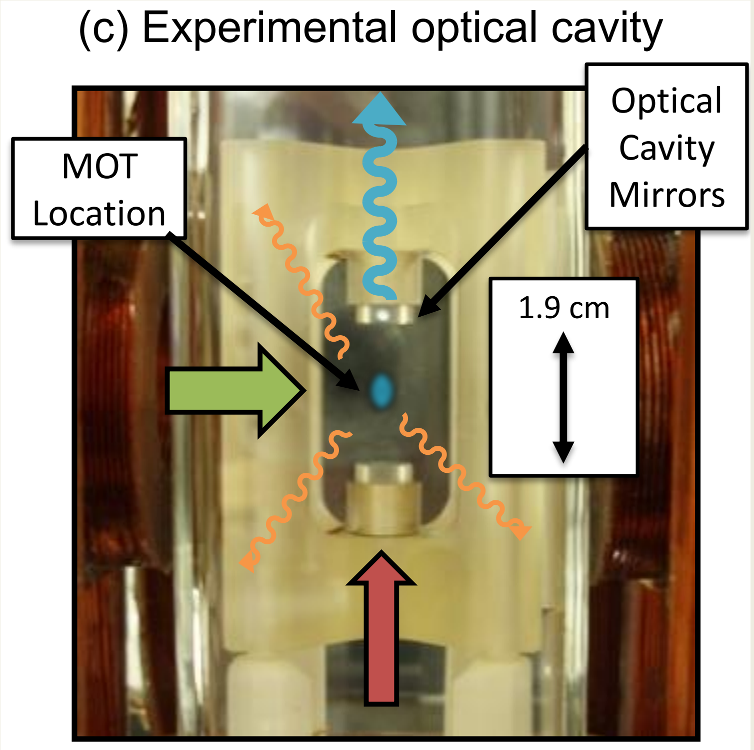

- Incoherent pumping with rate W
- Extremely bad cavity (cavity photon decay rate κ – largest rate in the problem)

Master Equation: $$\dot{\rho} = -\imath[\hat{H}, \rho] + W \sum_{j=1}^{N} \mathcal{L}[\hat{\sigma}_{j+}]\rho + \kappa\mathcal{L}[\hat{a}]\rho$$

Lindblad super-operators: $$\mathcal{L}[\hat{O}]\rho = \frac{1}{2}\left(2\hat{O}\rho\hat{O}^\dagger - \hat{O}^\dagger\hat{O}\rho - \rho\hat{O}^\dagger\hat{O}\right)$$

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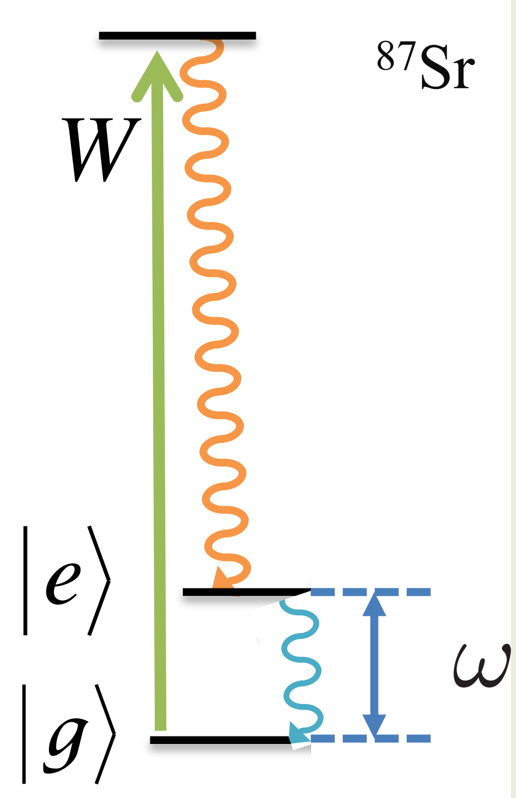

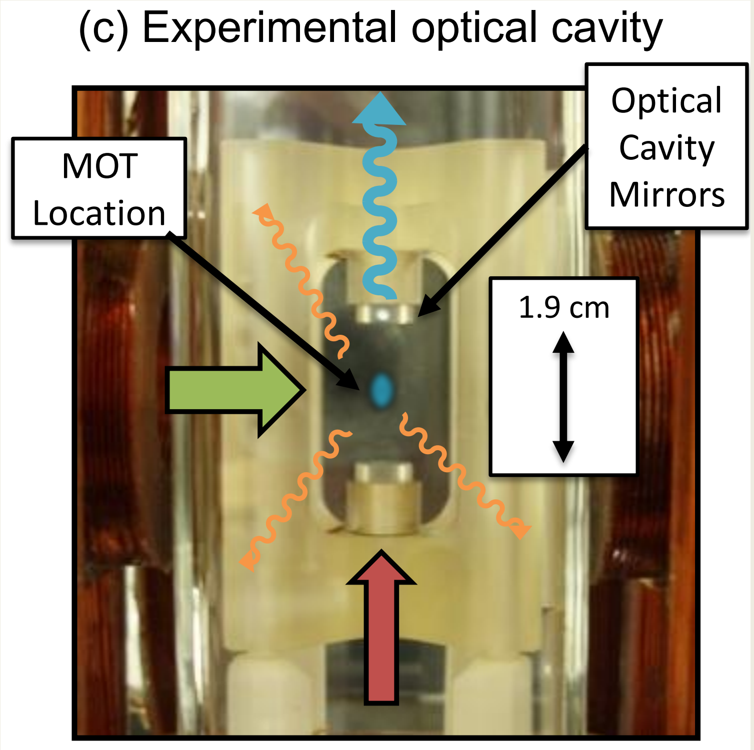

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Adiabatic elimination: (bad cavity limit) $$\hat{a} = -i\frac{\Omega}{\kappa}\hat{S}_-, \quad t \gg \kappa^{-1}$$

Bonifacio et. al., Phys. Rev. A (1971)

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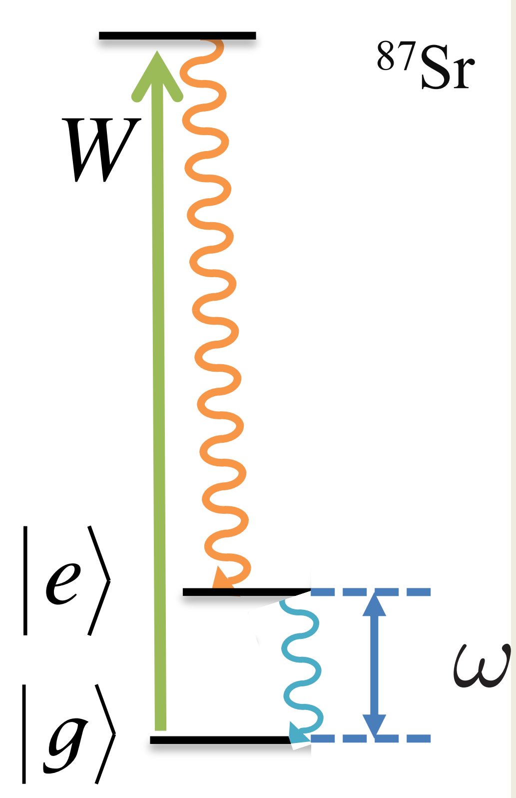

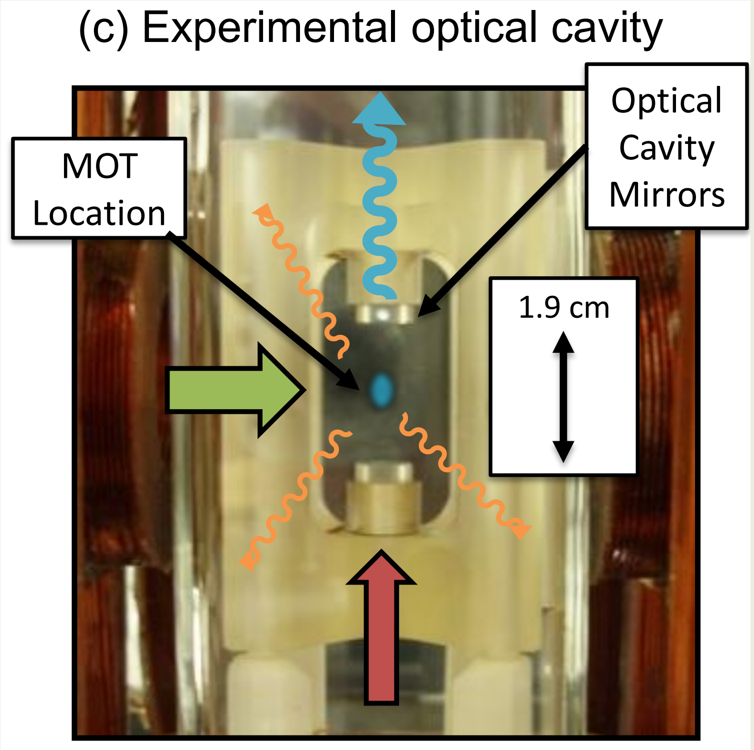

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Collective decay rate $\Gamma_c = \frac{\Omega^2}{\kappa}$

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System Size Expansion in $1/\sqrt{N}$ ⟹ Mean-Field Equations, $\boldsymbol{s} = \frac{2}{N}\langle \hat{\boldsymbol{S}} \rangle$

$$\dot{s}_+ = (i\omega - \frac{W}{2})s_+ + \frac{1}{2}s_z s_+$$

$$s_\pm = s_x \pm i s_y$$

$$\dot{s}_z = W(1 - s_z) - \frac{1}{2}s_+ s_-$$

Without pumping, $W = 0$: $\frac{d\mathbf{s}}{dt} = \mathbf{H} \times \mathbf{s} + \lambda(\mathbf{H} \times \mathbf{s}) \times \mathbf{s}$ $\mathbf{H} = \omega \hat{z}$

Landau-Lifshitz equation

Master Equation: $\dot{\rho} = -\imath[\omega \hat{S}_z, \rho] + W \sum_{j=1}^{N} \mathcal{L}[\hat{\sigma}_{j+}]\rho + \Gamma_c \mathcal{L}[\hat{S}_-]\rho$

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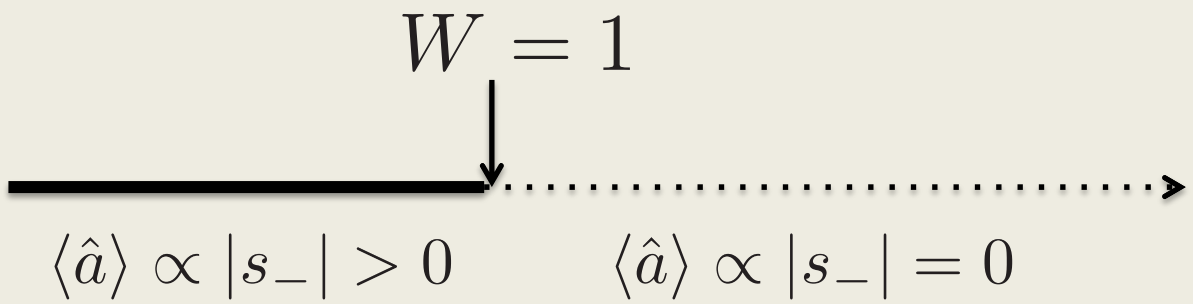
- ❖ Rotating frame: $\omega \to 0$
- ❖ One parameter (W) – phase diagram is 1D
- ❖ Single attractor – fixed point

Phase Diagram (1D): Two phases



Phase Diagram (1D): Two phases

$$W = 1$$

$\langle \hat{a} \rangle \propto |s_-| > 0$ $\qquad$ $\langle \hat{a} \rangle \propto |s_-| = 0$

$$\langle \hat{a} \rangle \propto |s_-| > 0$$

Macroscopic population of cavity mode $\Rightarrow$ Superradiance Intensity $\propto N^2$ $\Rightarrow$ Ultra-stable, submilihertz linewidth optical laser

Accurate frequency source $\Rightarrow$ Accurate time measurement $\Rightarrow$ Atomic Clock

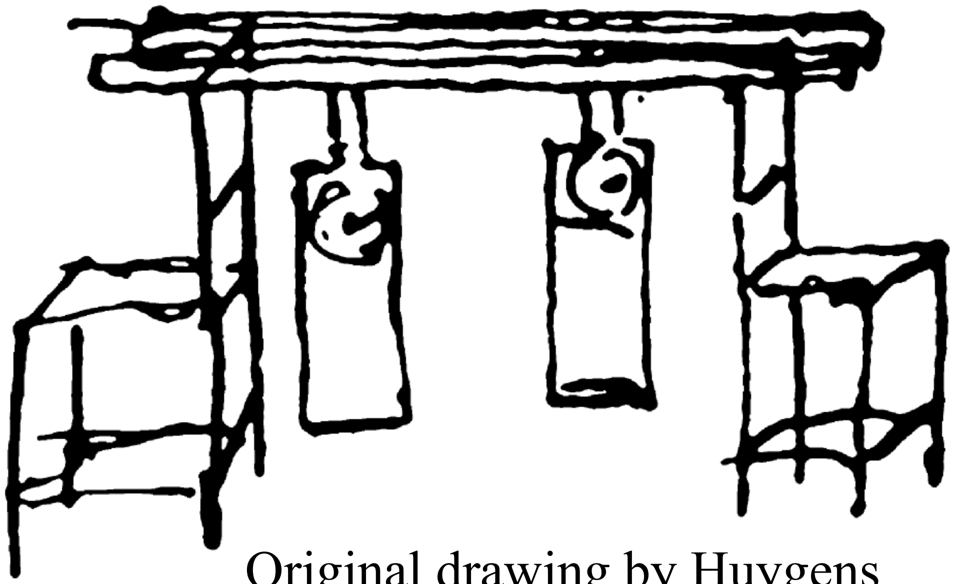
Q: Do atomic clocks synchronize?

Synchronization:

- ❖ First observed by Huygens in maritime clocks 1665 – "an odd kind of sympathy"
- ❖ The opposite swings of the pendulums coincide, when two pendulum clocks are hung from same support.
- ❖ Anti-phase synchronization
- ❖ Reason: "…imperceptible motion of the beam…"

Christiaan Huygens

Letters to de Sluse, 1665

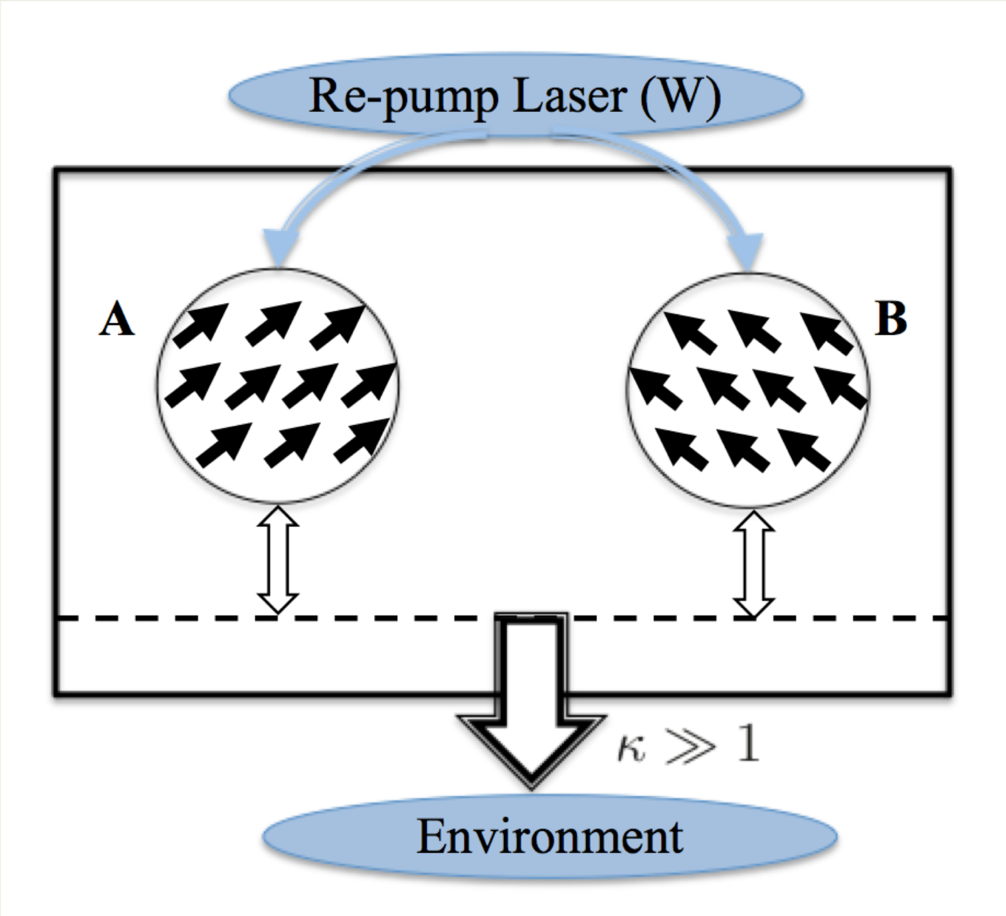


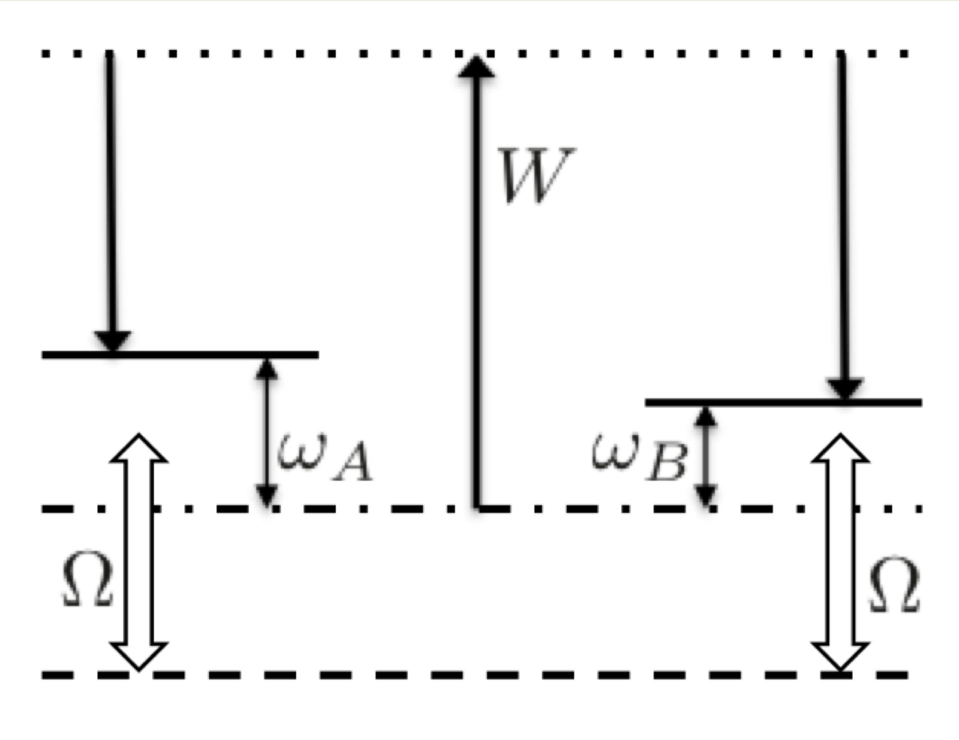
Original drawing by Huygens

✓ Also observed by J. W. Strutt (3rd Baron Rayleigh): "When two organ-pipes of same pitch stand side by side…cause the pipes to speak in absolute unison, in spite of inevitable small differences."

✓ Frequency Locking

Two atomic clocks in a bad cavity



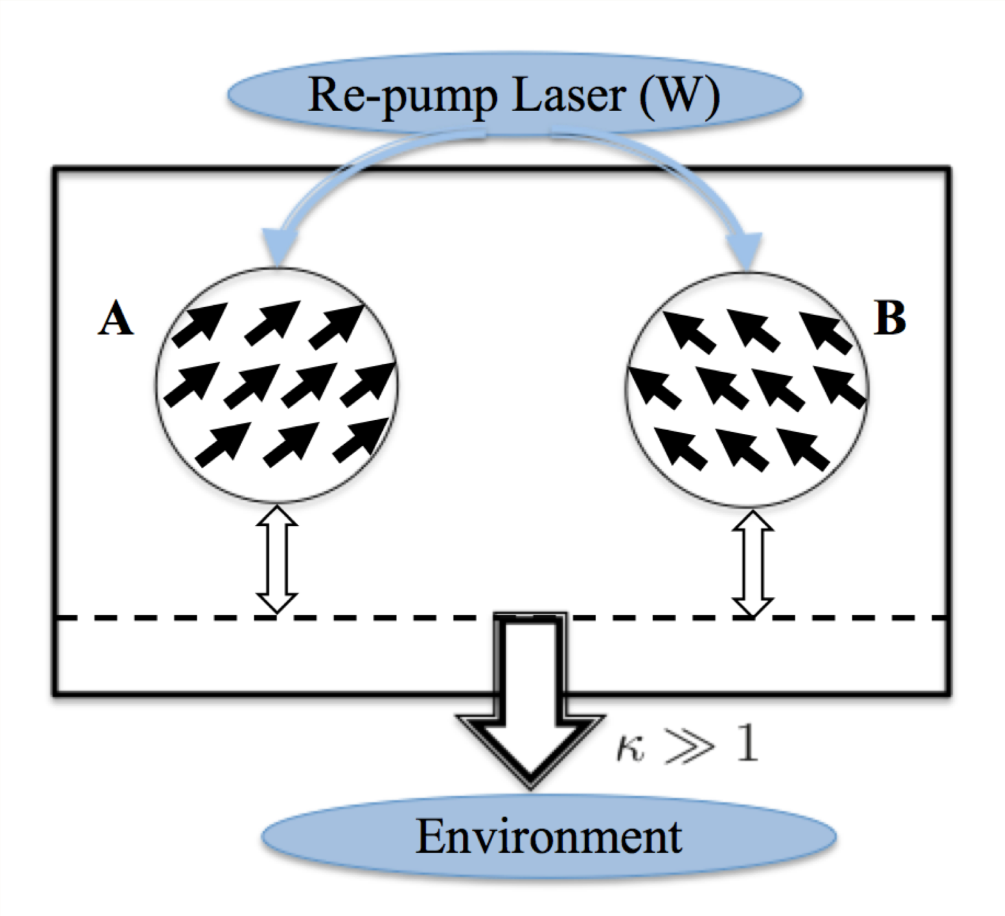


Two dimensionless parameters: W, $\delta = \omega_A - \omega_B$ ⇨ 2D phase diagram

Re-pump rate

Detuning

Two atomic clocks in a bad cavity

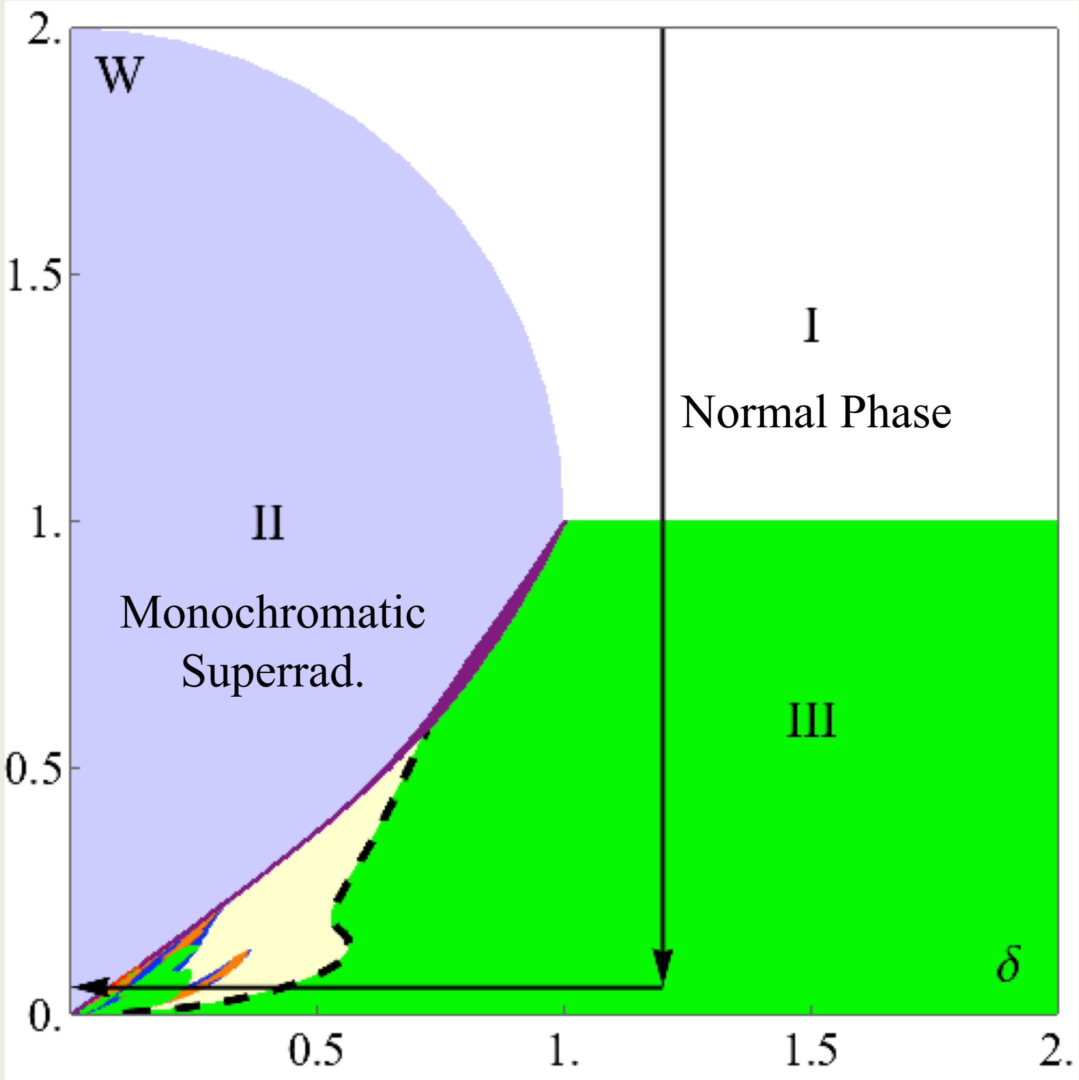


Experiment: Weiner et. al., PRA (2017)

Theory: Xu et al., PRL (2014)
Roth & Hammerer, PRA (2016)
Li et. al., Comm. Non. Sci (2017)
Shankar et. al., PRA (2017)
⋮

The phase diagram consists of same two phases as for one clock???

Our result: complete, exact nonequilibrium phase diagram



W – re-pump rate, δ - detuning

- Incredibly rich phase diagram. 5 new phases in addition to the 2 one-clock phases! Dynamics extremely rare in other driven-dissipative systems.
- Normal phase (no radiation): I
- Monochromatic superradiance: II

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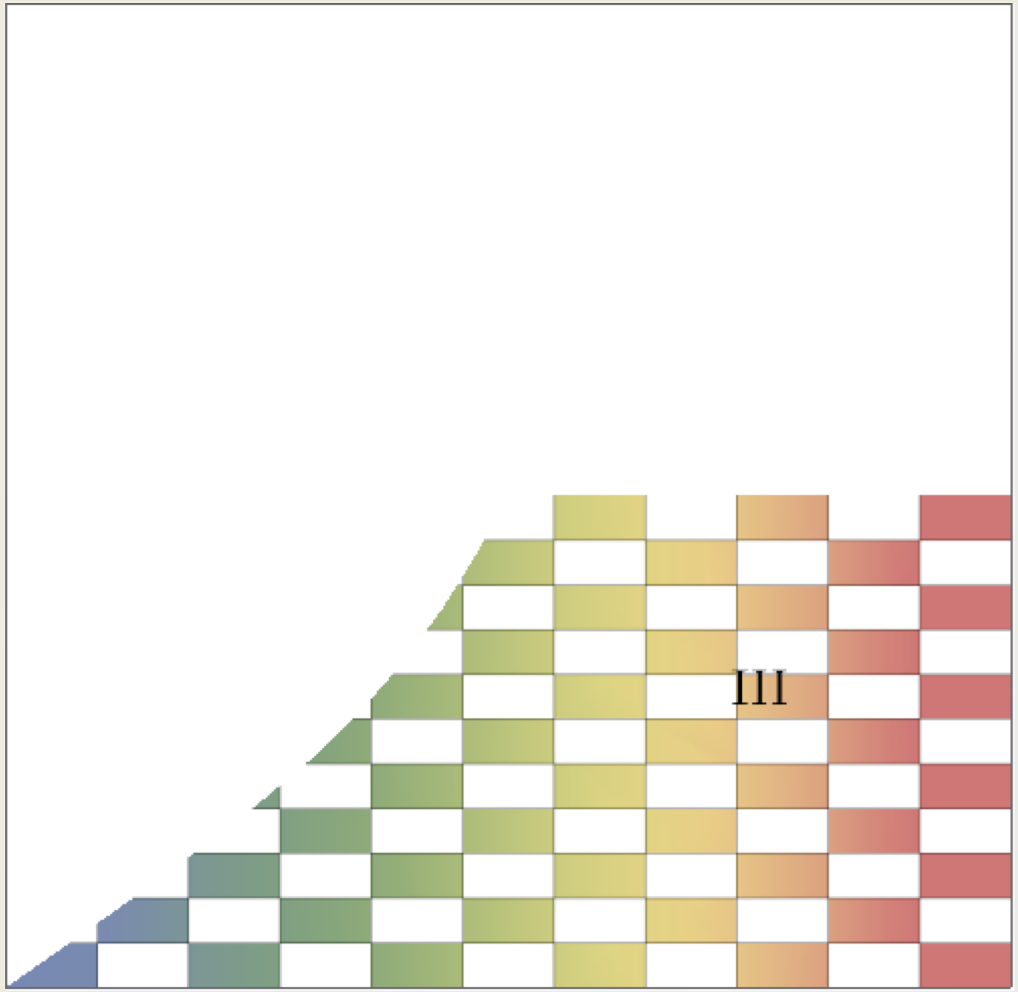

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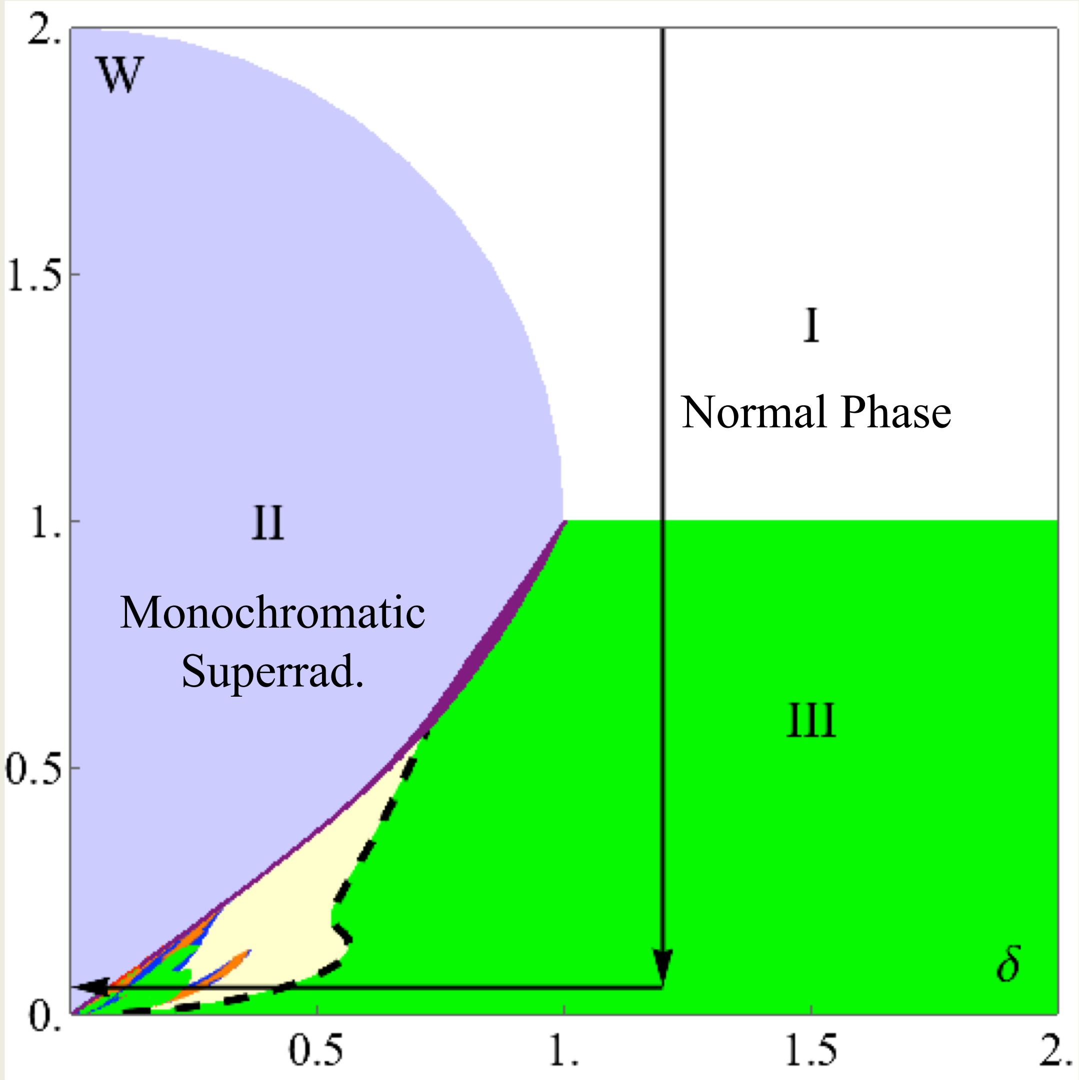

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- Amplitude-modulated superradiance: III
  - ✓ Symmetric Limit Cycle (green inside III)
  - ✓ Symmetry-broken Limit Cycle (yellow to the left of dashed line inside III)

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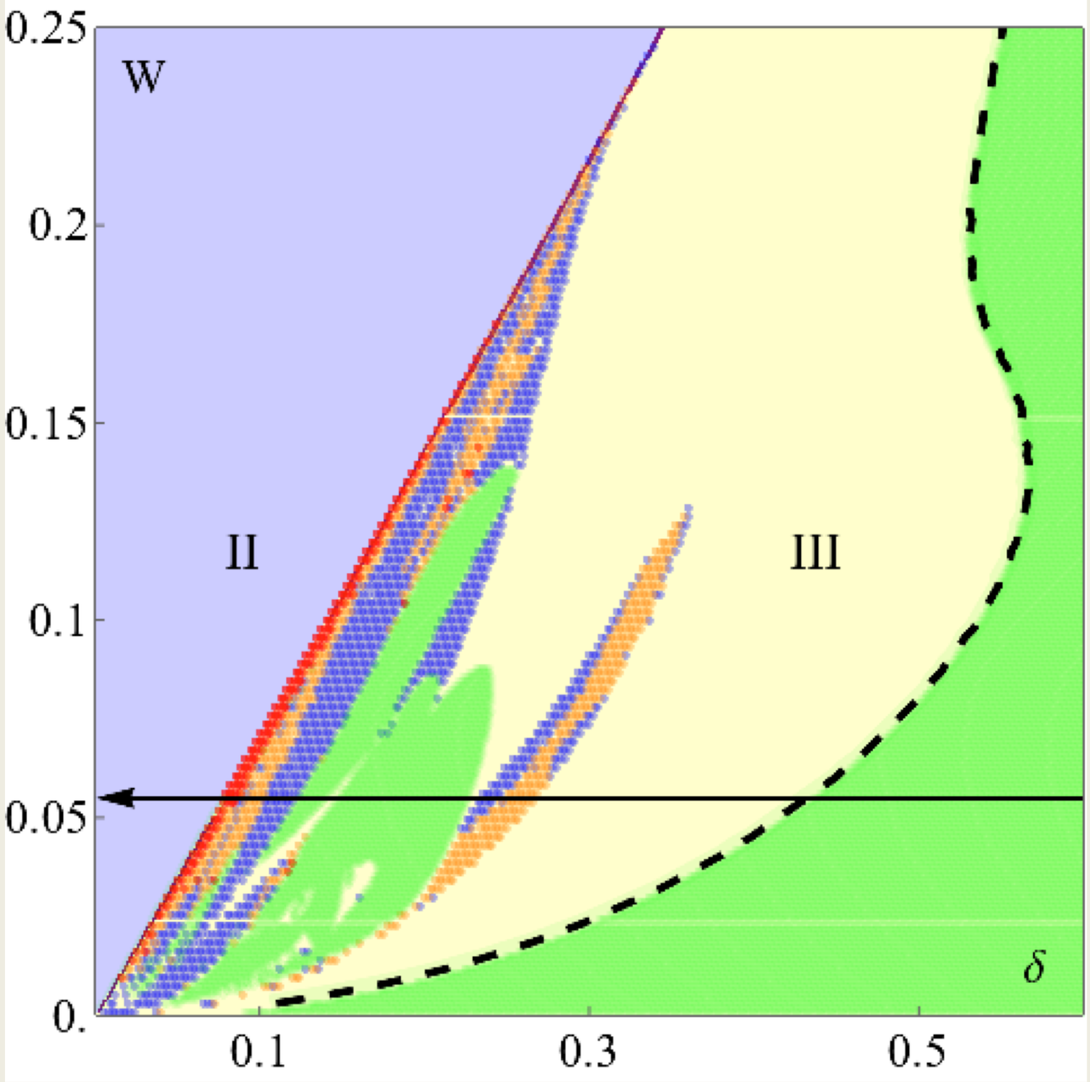

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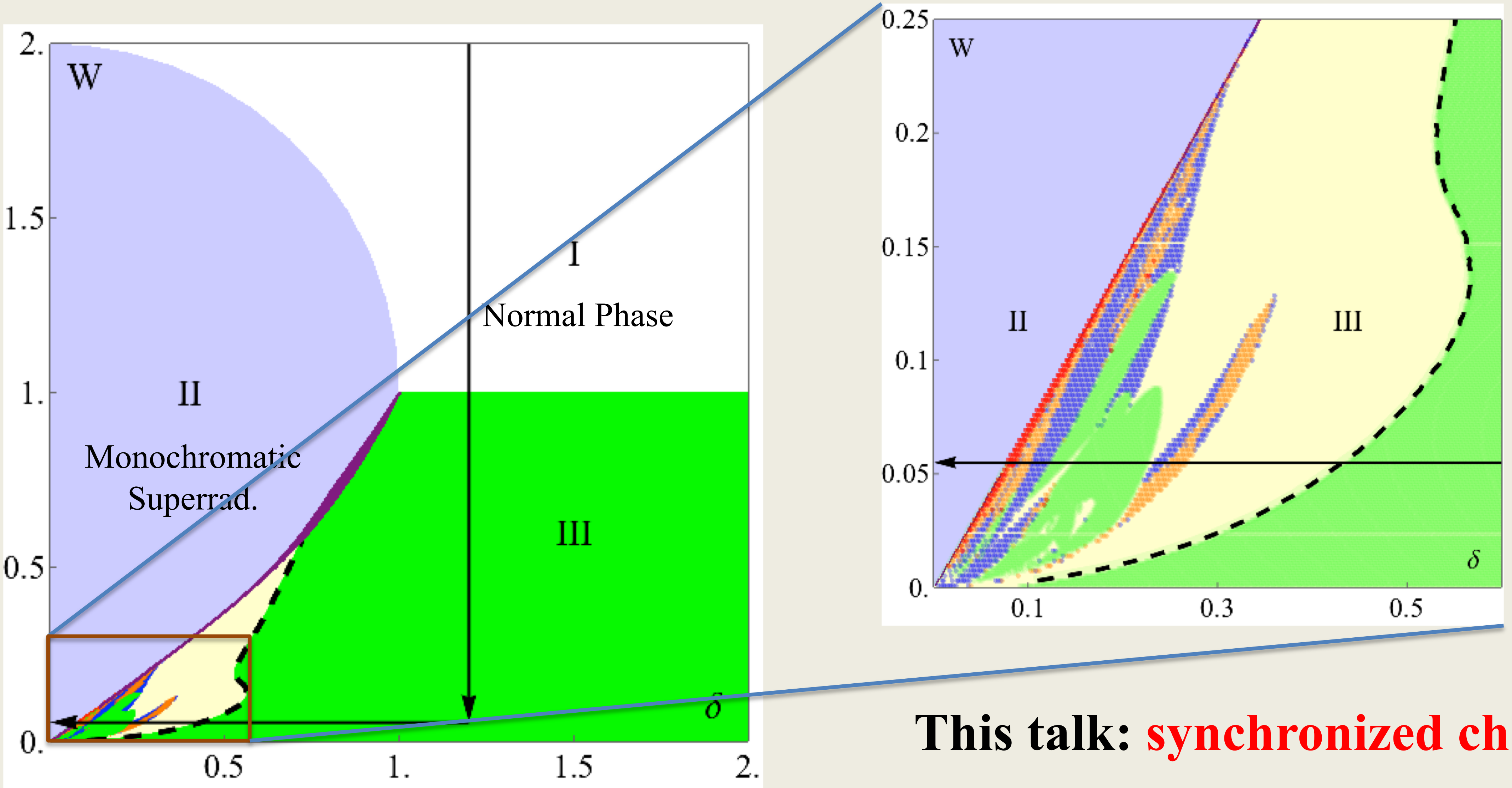
- Amplitude-modulated superradiance: III
  - ✓ Symmetric Limit Cycle (green inside III)
  - ✓ Symmetry-broken Limit Cycle (yellow to the left of dashed line inside III)

  - ✓ Quasiperiodic attractor (dark blue points)
  - ✓ Chaos (orange points)
  - ✓ **Red points: synchronized chaos!** Extremely rare with bidirectional coupling. Never seen or predicted in cavity QED.

Appearance, disappearance (via quasiperiodic route to chaos) and restoration of synchronization.

Three different kinds of synchronized dynamics.

Our result: complete, exact nonequilibrium phase diagram



This talk: synchronized chaos

W – re-pump rate, δ - detuning

Two atomic clocks in bad cavity: equations of motion

$$\dot{s}^j_+ = (i\omega_j - \frac{W}{2})s^j_+ + \frac{1}{2}s^j_z l_+$$

$$\dot{s}^j_z = W(1 - s^j_z) - \frac{1}{4}s^j_+ l_- - \frac{1}{4}s^j_- l_+$$

$$j = A, B \quad \boldsymbol{l} = \boldsymbol{s}^A + \boldsymbol{s}^B$$

Two coupled Landau-Lifshitz equations + pumping

cf. one clock

$$\dot{s}_+ = (i\omega - \frac{W}{2})s_+ + \frac{1}{2}s_z s_+$$

$$\dot{s}_z = W(1 - s_z) - \frac{1}{2}s_+ s_-$$

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$$\dot{s}_+^j = (i\omega_j - \frac{W}{2})s_+^j + \frac{1}{2}s_z^j l_+$$

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$$j = A, B \quad \boldsymbol{l} = \boldsymbol{s}^A + \boldsymbol{s}^B$$

Two coupled Landau-Lifshitz equations + pumping

Symmetries:

✓ Axial symmetry about z-axis

$$s_+^j \to s_+^j e^{i\varphi}, \quad s_-^j \to s_-^j e^{-i\varphi}$$

✓ $\mathbb{Z}_2$ symmetry (similar to particle hole)
In a frame rotating with mean frequency

$$\omega_A = -\omega_B = \delta/2$$

$$s_x^A \leftrightarrow s_x^B, \quad s_z^A \leftrightarrow s_z^B, \quad s_y^A \leftrightarrow -s_y^B$$

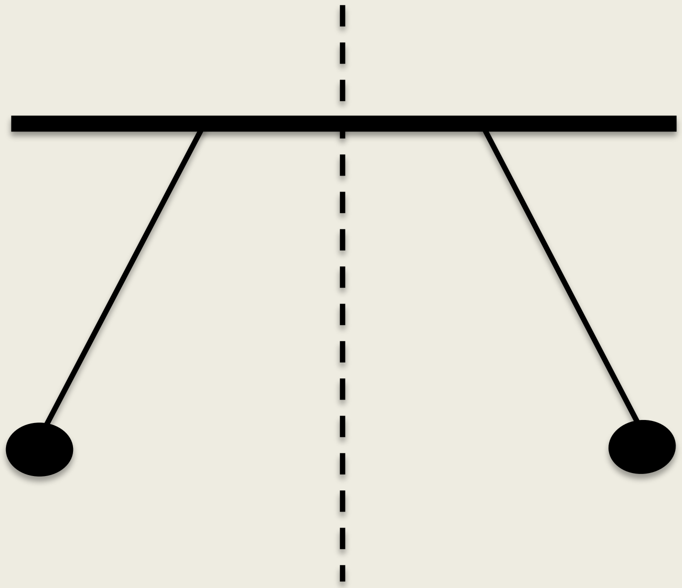
✓ Steady states (attractors, phases) can brake one symmetry or both

Synchronization of atomic clocks

- Clocks A & B are synchronized when the steady state is $\mathbb{Z}_2$ symmetric. Then, spins corresponding to the two atomic ensembles follow one another

$$s_x^A = s_x^B, \quad s_y^A = -s_y^B, \quad s_z^A = s_z^B$$

- Compare with anti-phase synchronization of classical clocks. The pendula are at opposite apexes at the same time.



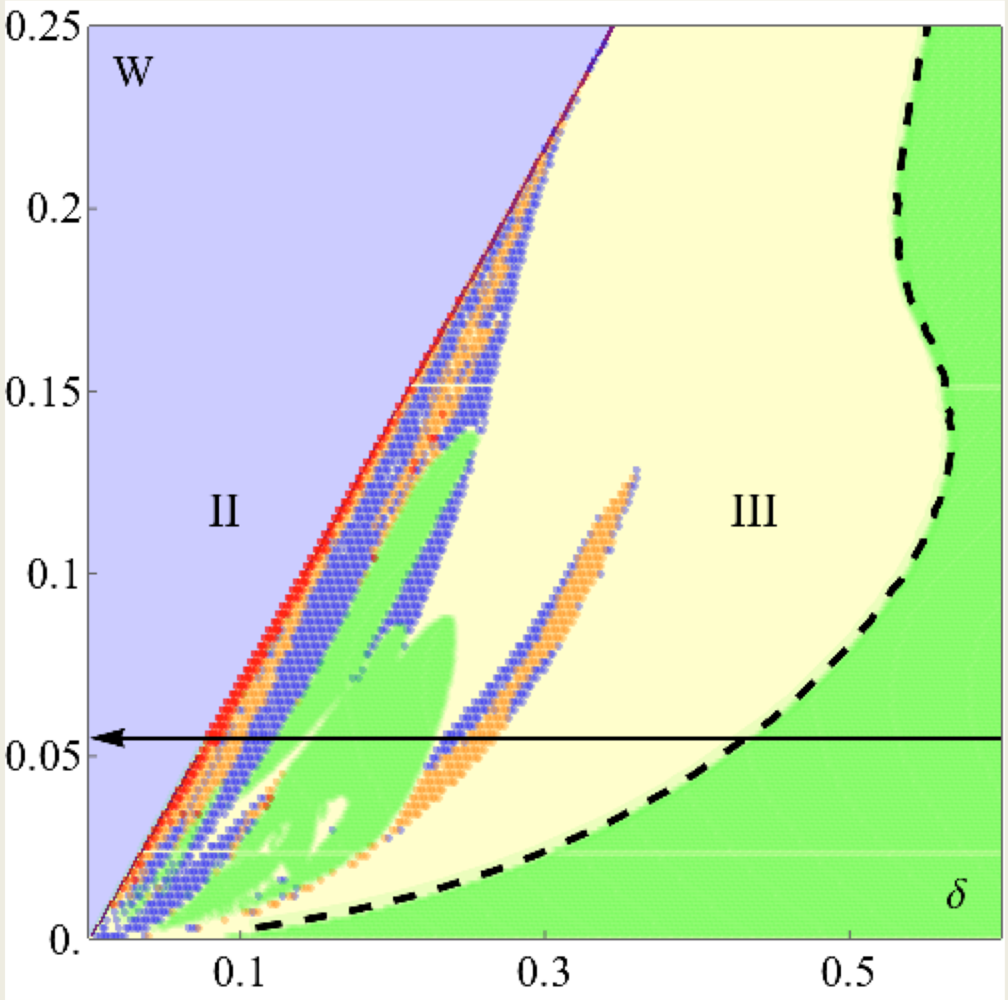
For atomic clocks we replace pendula with spins and the median (dashed line) with the *xz*-plane.

This talk: synchronized chaos

- ✓ Origin
- ✓ Experimental signature
- ✓ Applications

Chaotic synchronization: Origin

- Chaotic phases (orange and red). Maximum Lyapunov exponent is positive.
- Ordinary chaotic trajectories do not posses $\mathbb{Z}_2$ symmetry. They occupy 6D regions of the 6D phase space.



- $\mathbb{Z}_2$ symmetry is restored for the chaotic trajectories in the red part of the phase diagram. They occupy 3D regions.

Chaotic synchronization: Origin

To determine the origin we study the equations of motion restricted to the $\mathbb{Z}_2$-symmetric synchronization submanifold, 6D ⟶ 3D

$$s_x^A = s_x^B \equiv s_x$$
$$s_y^A = -s_y^B \equiv s_y$$
$$s_z^A = s_z^B \equiv s_z$$

$$\dot{s}_x = -\frac{\delta}{2}s_y - \frac{W}{2}s_x + s_z s_x$$
$$\dot{s}_y = \frac{\delta}{2}s_x - \frac{W}{2}s_y$$
$$\dot{s}_z = W(1 - s_z) - s_x^2$$

- Distinct from one clock equations: 2 parameters instead of 1
- The phase diagram is 2D

Chaotic synchronization: Origin

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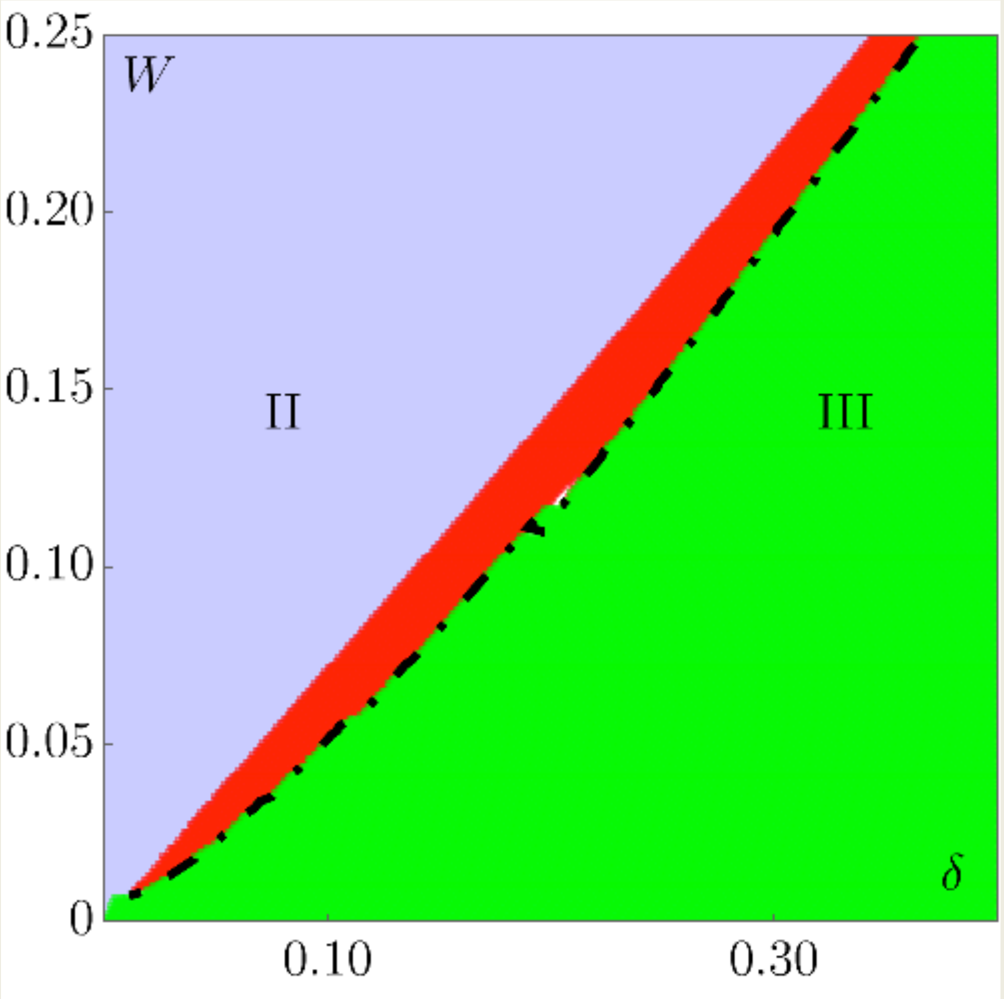
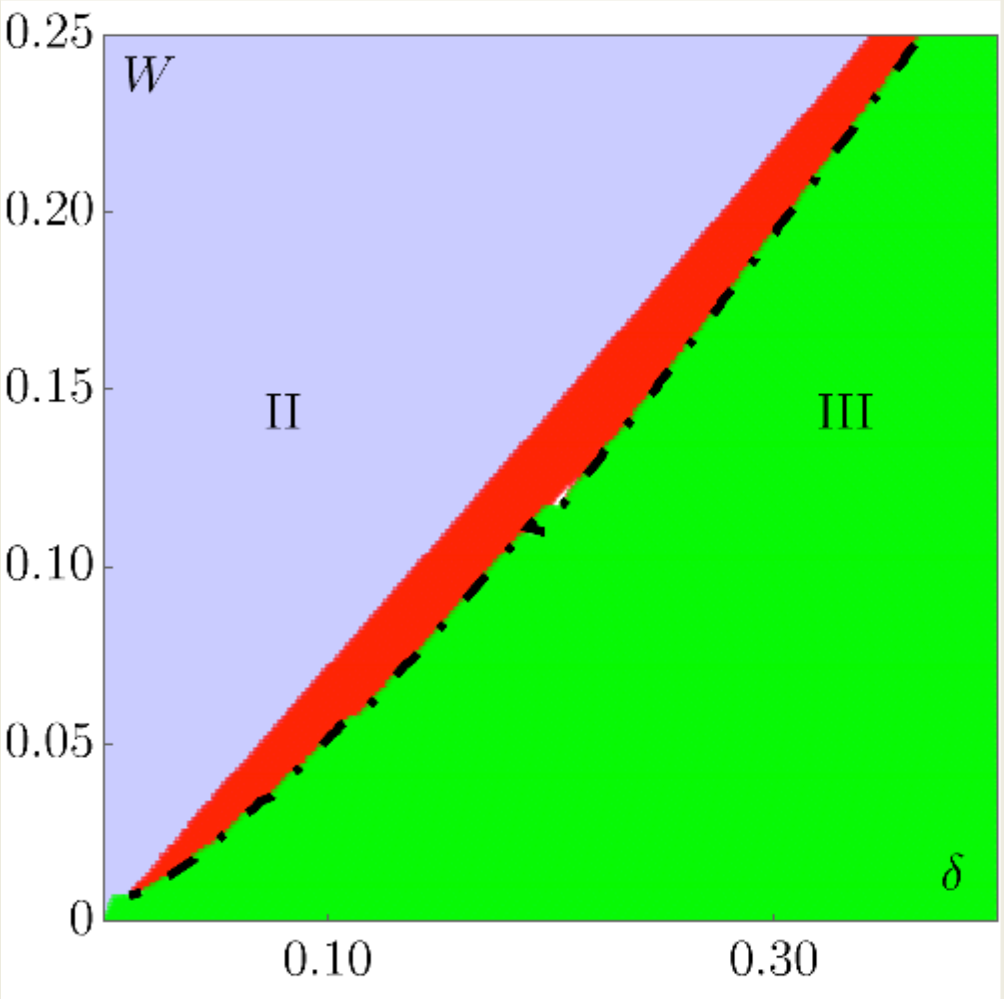


- Distinct from one clock equations: 2 parameters instead of 1
- The phase diagram is 2D, but with fewer phases than without $\mathbb{Z}_2$: fixed point, limit cycle & chaos
- Synchronized chaos originates directly from the $\mathbb{Z}_2$ symmetric limit cycle

We determine the nature of the transition from the $\mathbb{Z}_2$ symmetric limit cycle to synchronized chaos via Floquet stability analysis



$$\frac{d\Delta s_x}{dt} = -\frac{\delta}{2}\Delta s_y - \frac{W}{2}\Delta s_x + \Delta s_z s_x + s_z \Delta s_x$$

$$\frac{d\Delta s_y}{dt} = \frac{\delta}{2}\Delta s_x - \frac{W}{2}\Delta s_y$$

$$\frac{d\Delta s_z}{dt} = -W\Delta s_z - 2 s_x \Delta s_x$$

$$\Delta \boldsymbol{s}_{n+1} = \mathbb{M} \cdot \Delta \boldsymbol{s}_n$$

3 Floquet multipliers:

$$(r_1, r_2, r_3)$$

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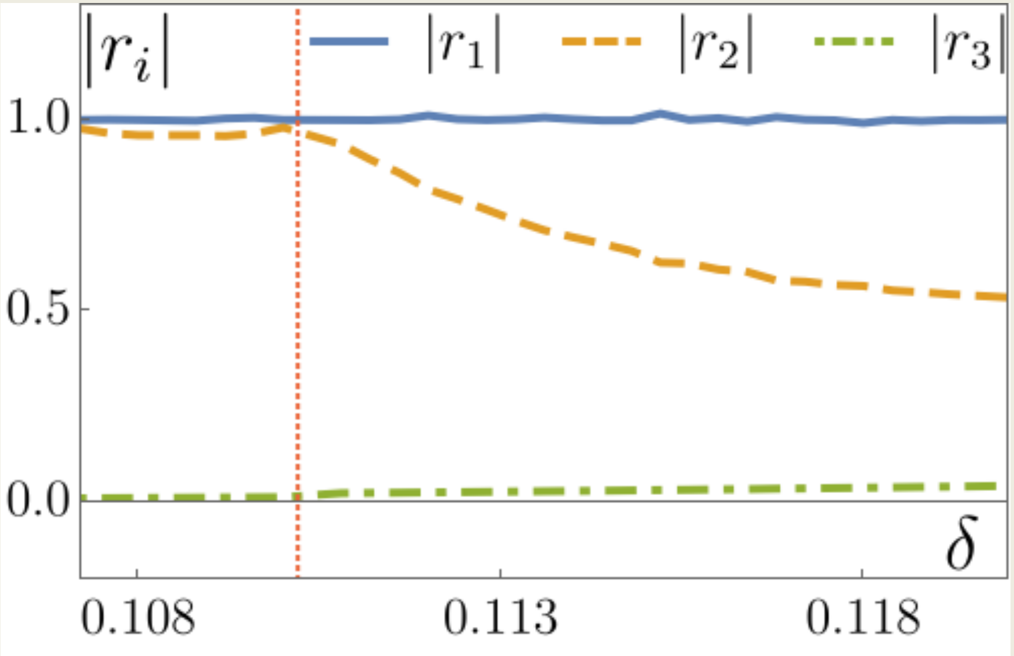

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3 Floquet multipliers: (r_1, r_2, r_3)

$r_1 \equiv 1$, because $\Delta \boldsymbol{s} = \dot{\boldsymbol{s}}$ is a solution

$r_3 \ll 1$ (numerical observation)

$r_2 - 1$ changes sign across the transtion

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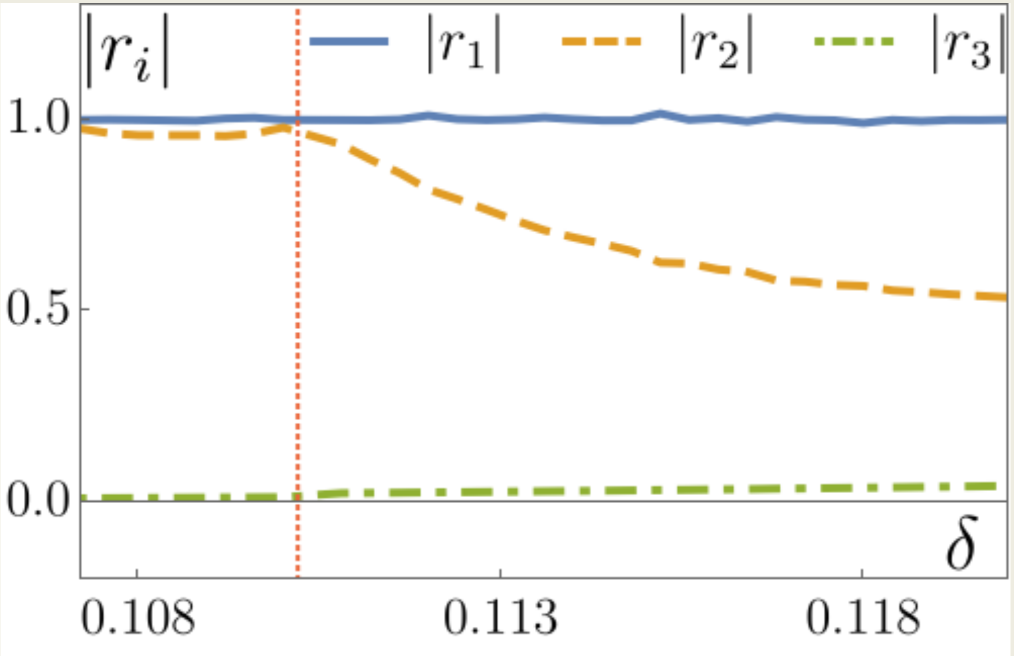

$\Longrightarrow$ Tangent Bifurcation

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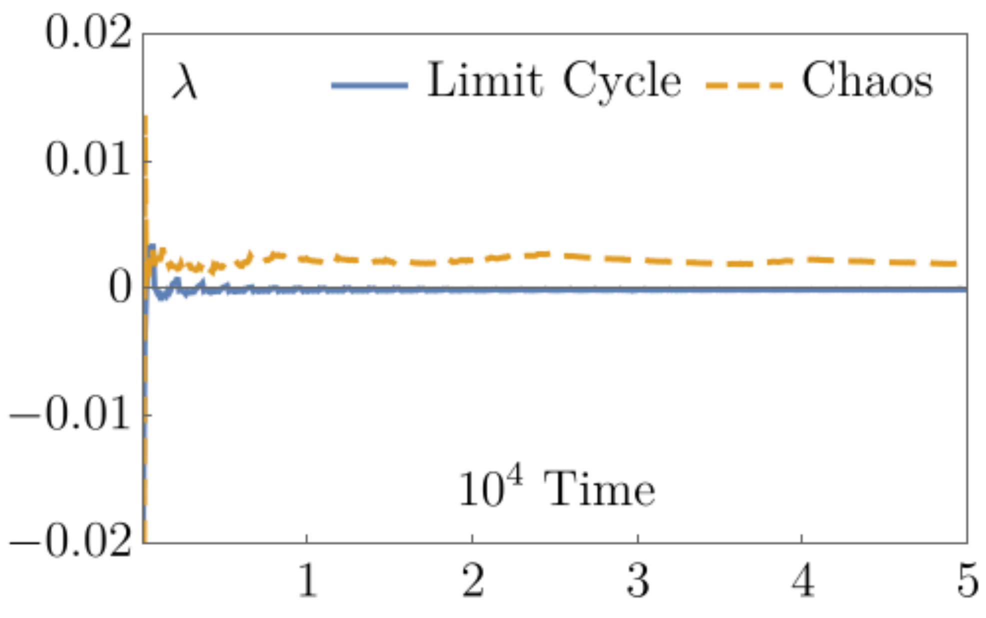
Chaotic synchronization: Origin - Tangent Bifurcation Intermittency

❖ Synchronized chaos originates directly from the $\mathbb{Z}_2$-symmetric limit cycle

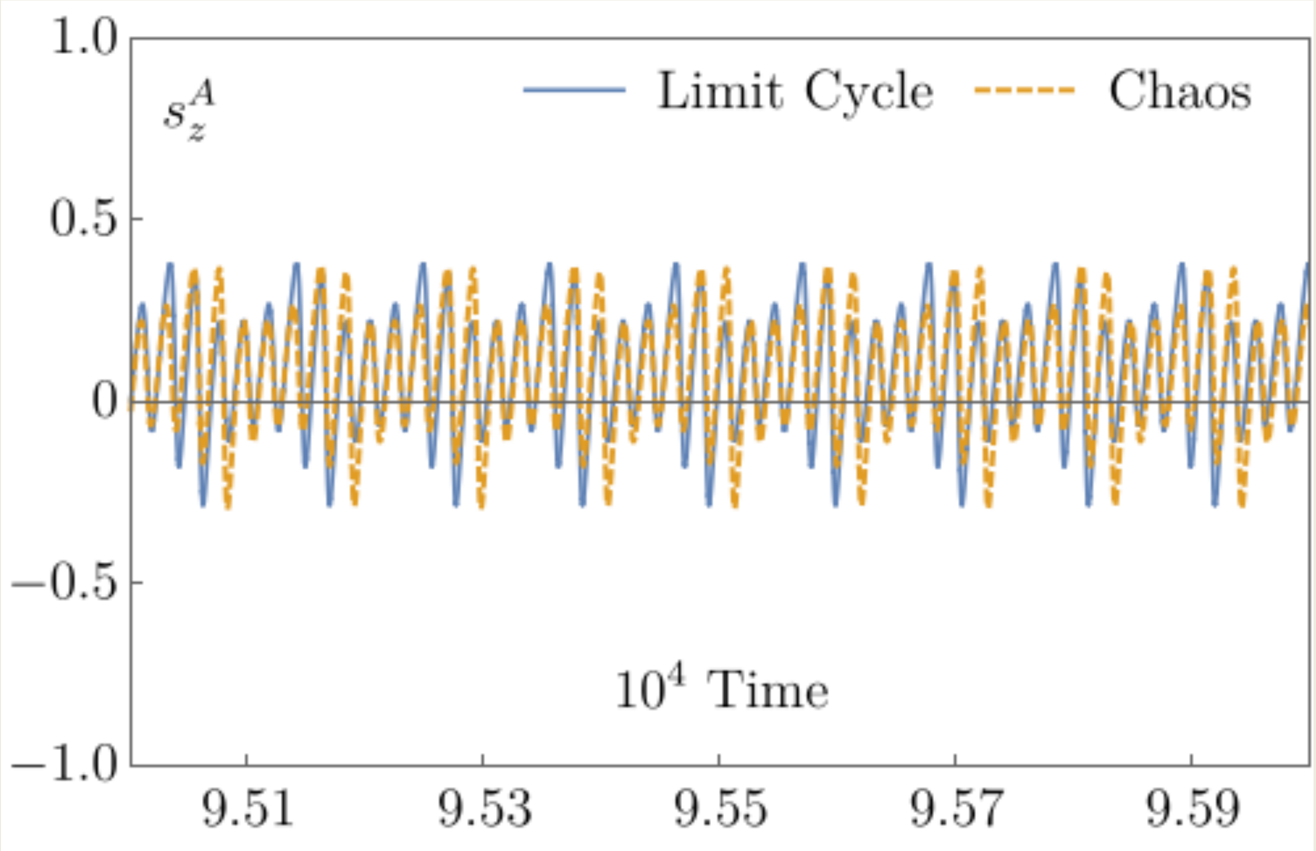
❖ Near transition, synchronized chaotic dynamics stay close to the $\mathbb{Z}_2$ symmetric limit cycles.

Lyapunov exponents:



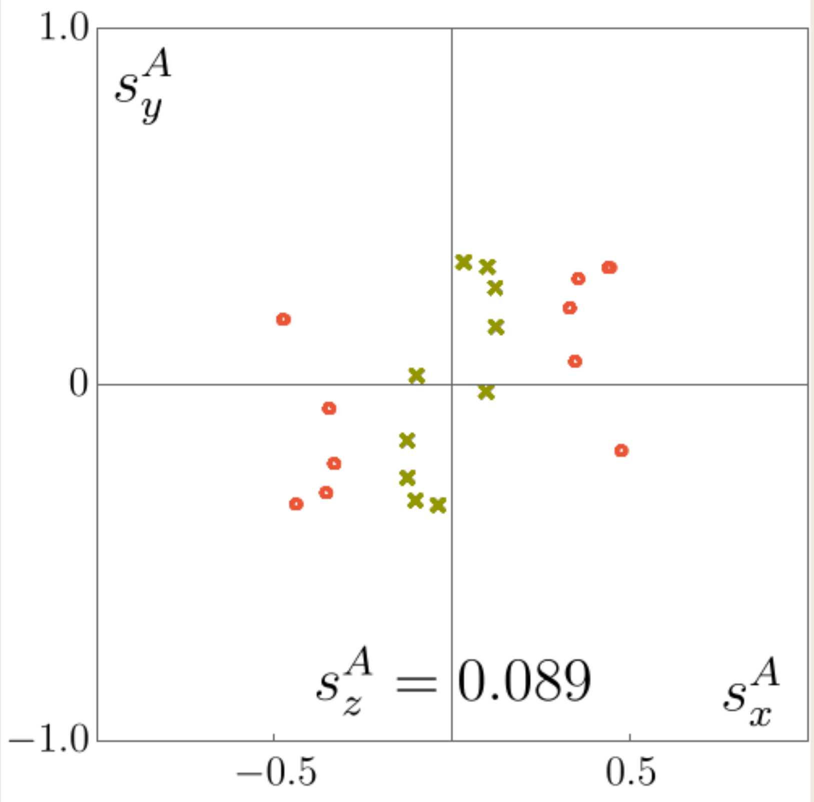
LC: $\delta = 0.107, W = 0.055$

SC: $\delta = 0.106, W = 0.055$

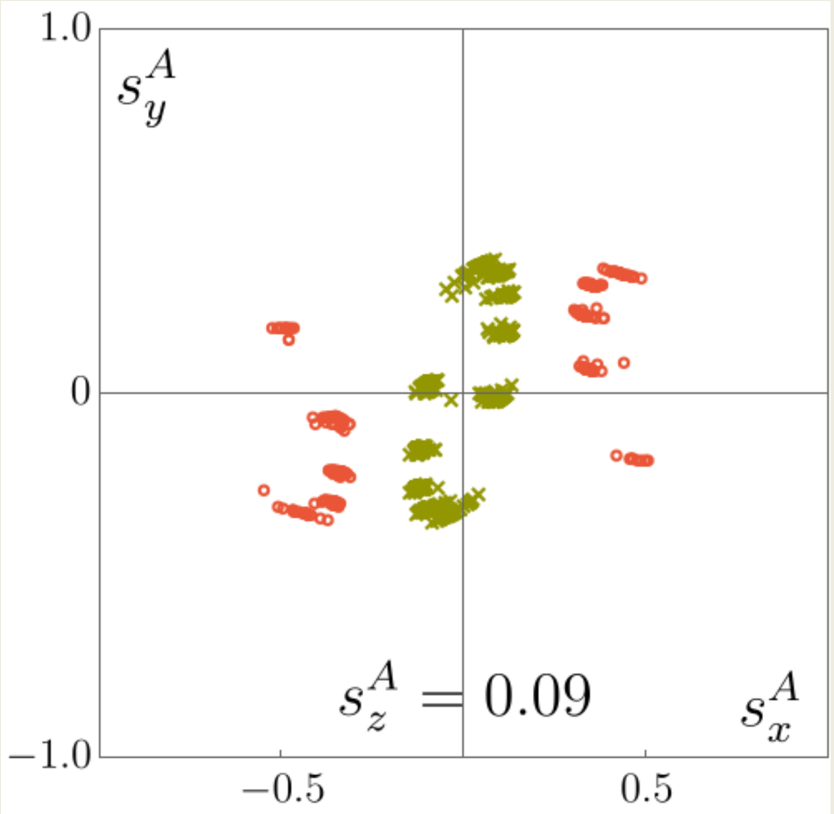


Chaotic synchronization: Origin - Tangent Bifurcation Intermittency

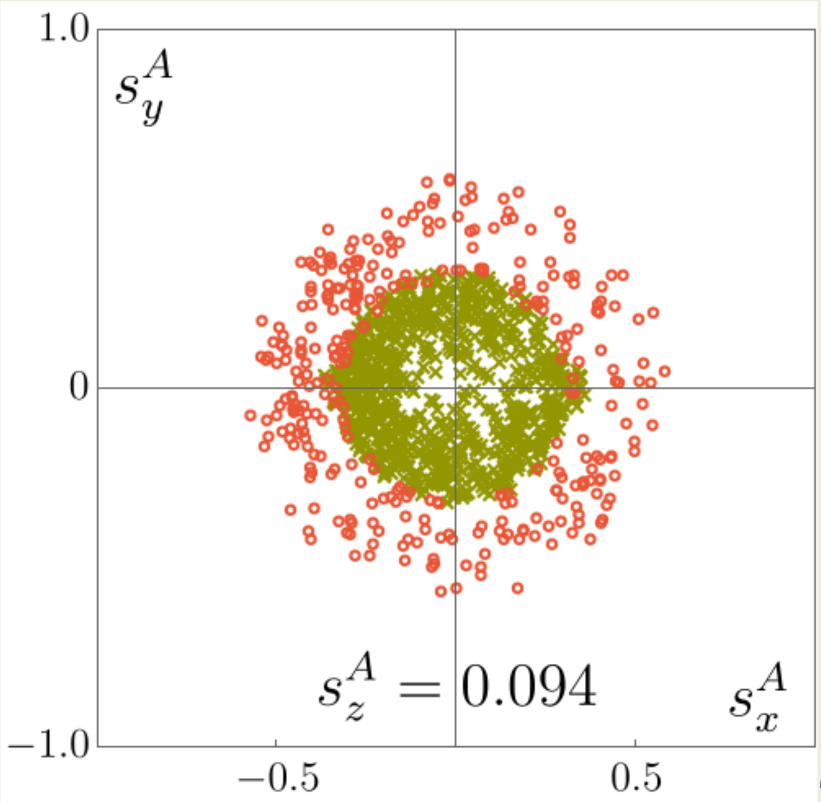
Poincare sections:



$\mathbb{Z}_2$-symmetric limit cycle



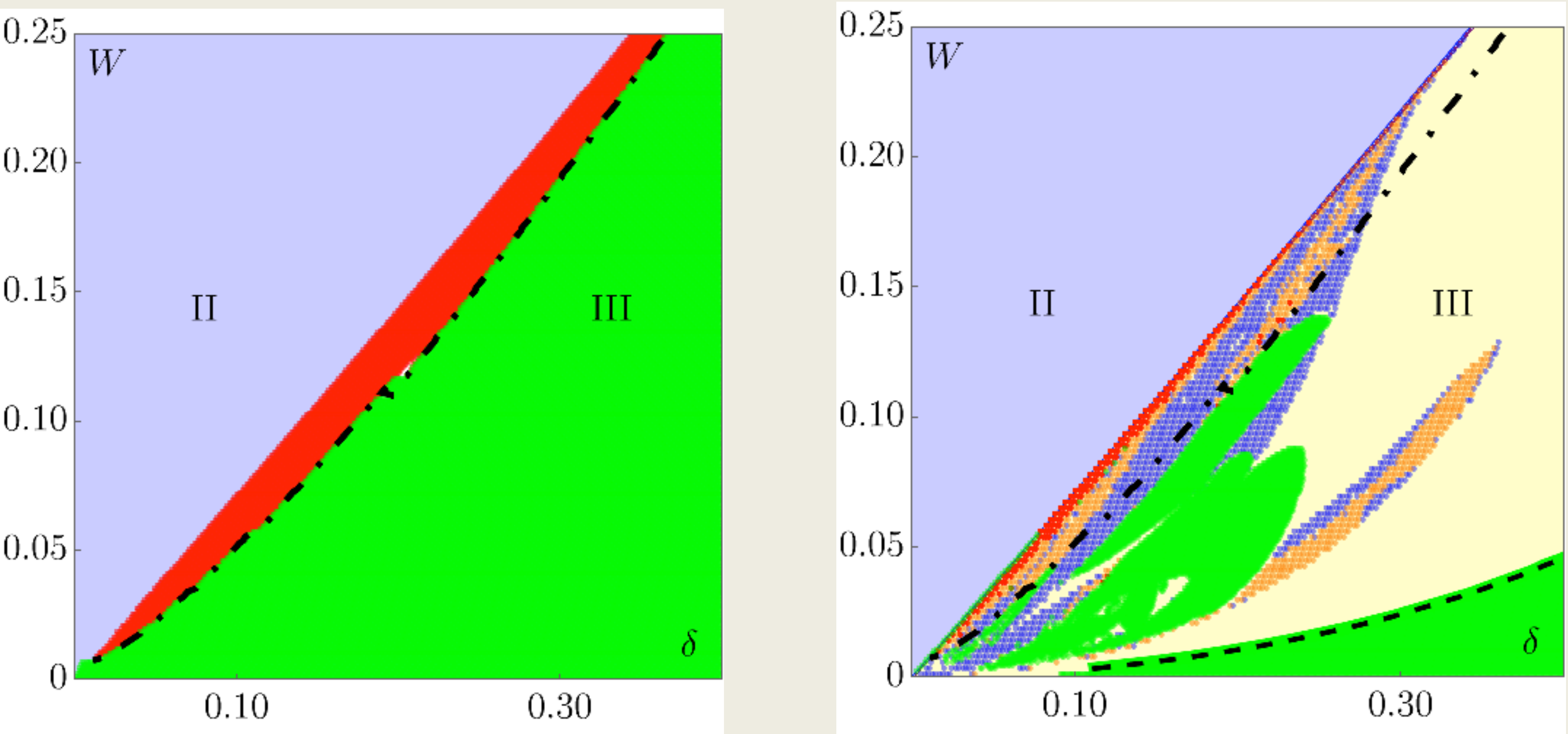
Synchronized chaos



Ordinary (6D) chaos

Back to 6D: stability of synchronized chaos

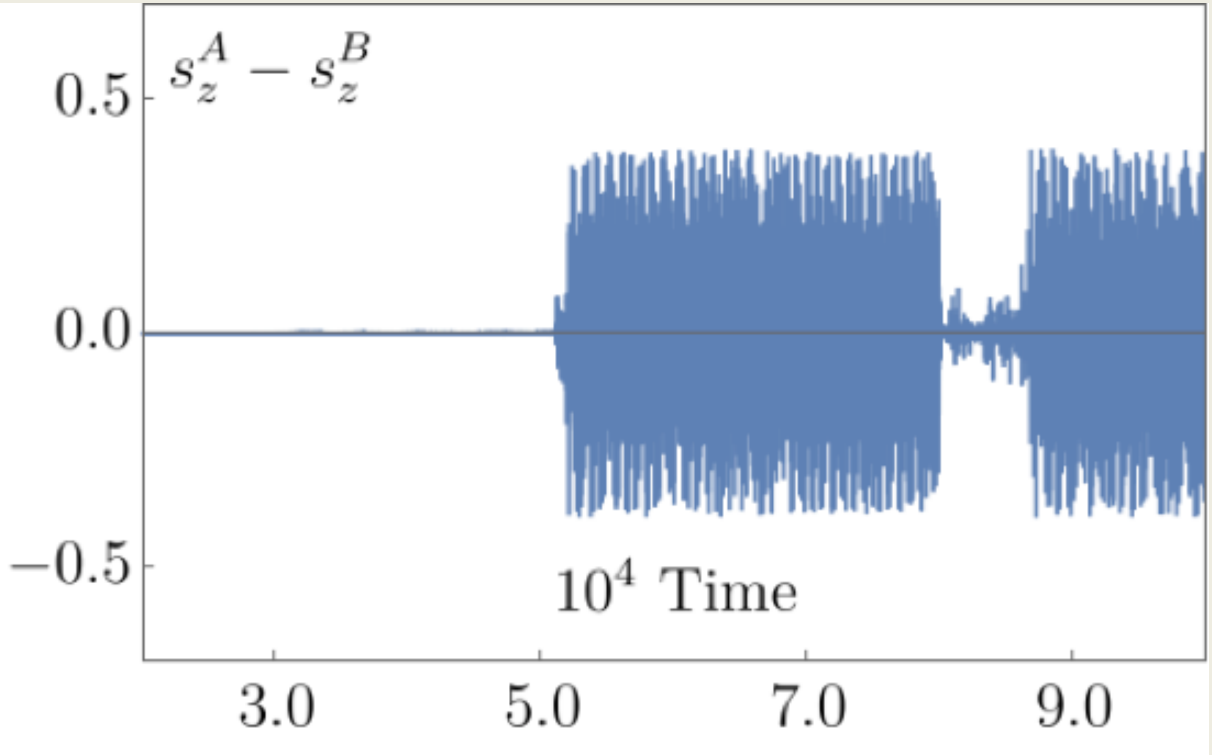
At inception, synchronized chaos is unstable in the full 6D phase space.
Compare the full 6D and $\mathbb{Z}_2$-restricted dynamics.

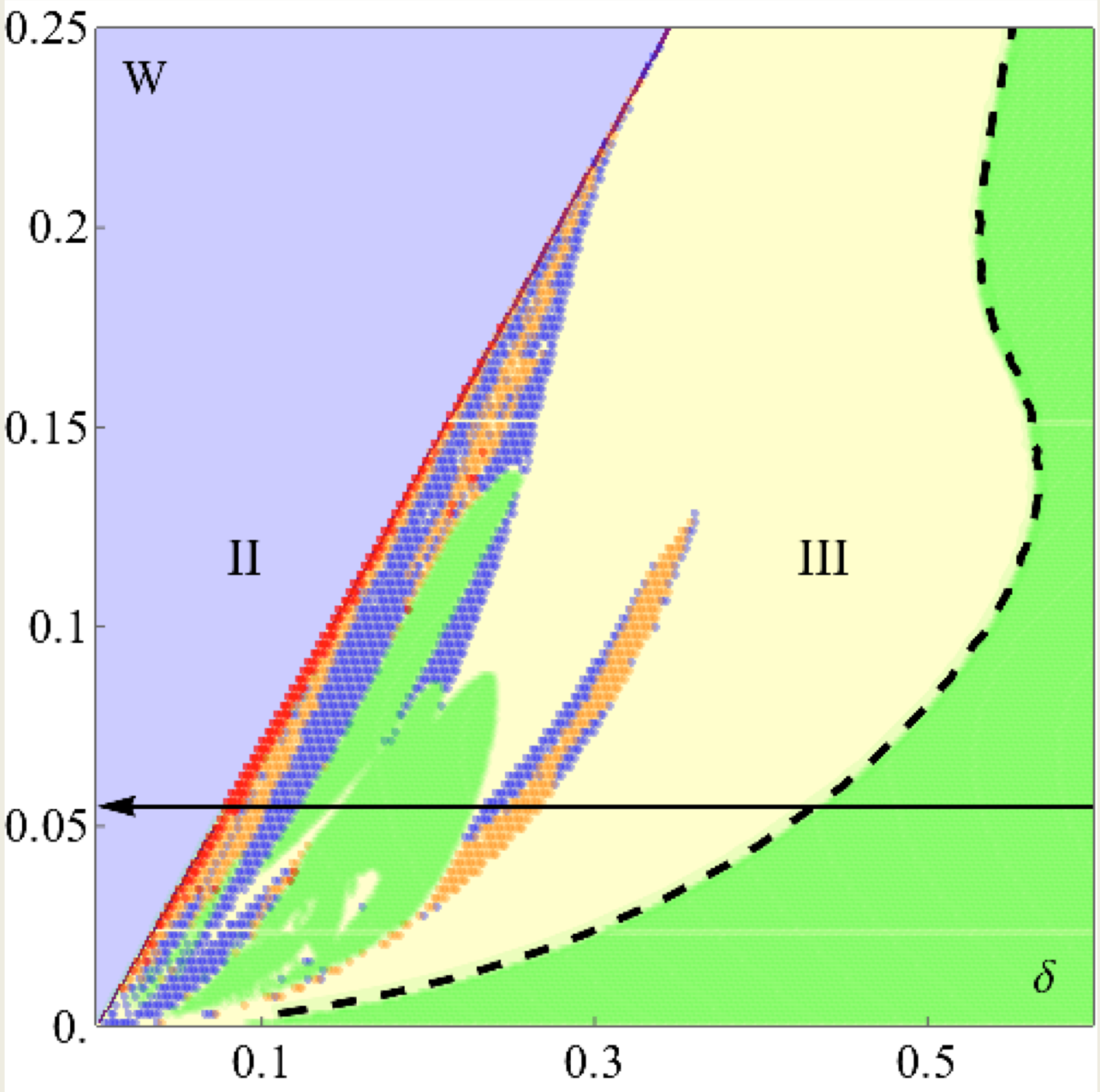


On-off intermittency

As we decrease δ keeping W fixed, chaos synchronizes via on-off intermittency

$$W = 0.055, \delta = 0.08021$$





$s_z^A - s_z^B = 0$ for synchronized chaos

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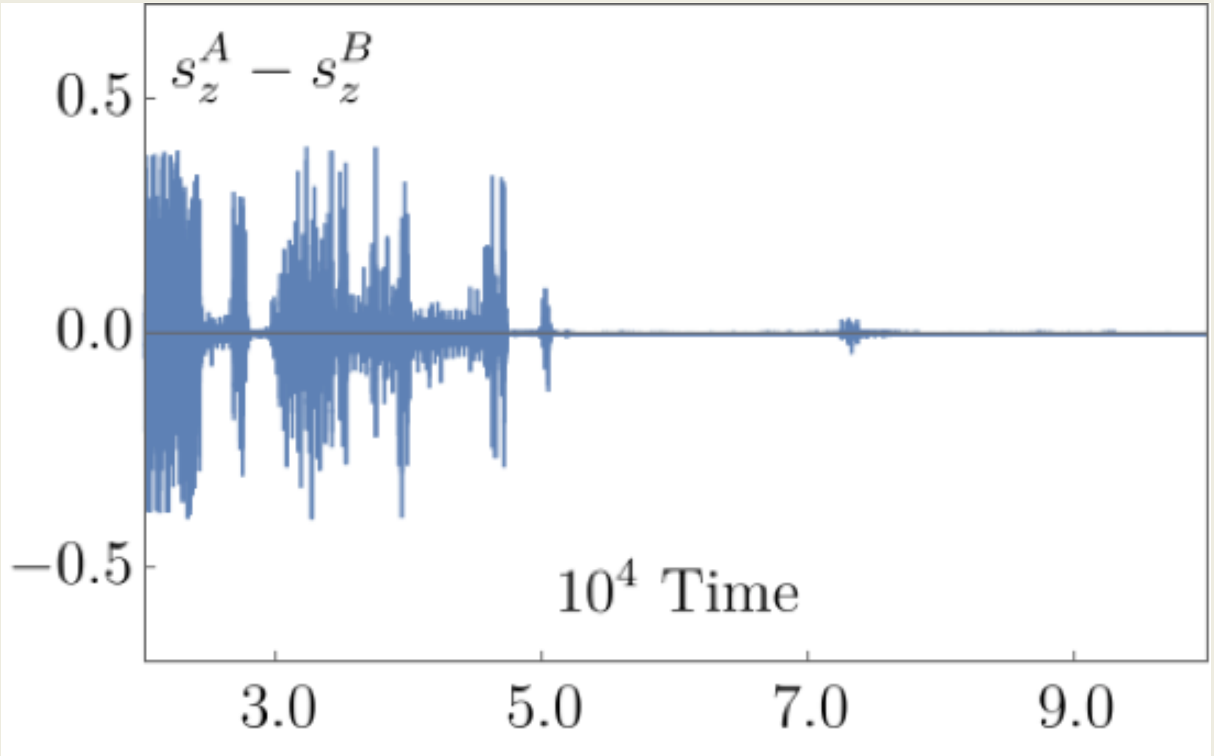
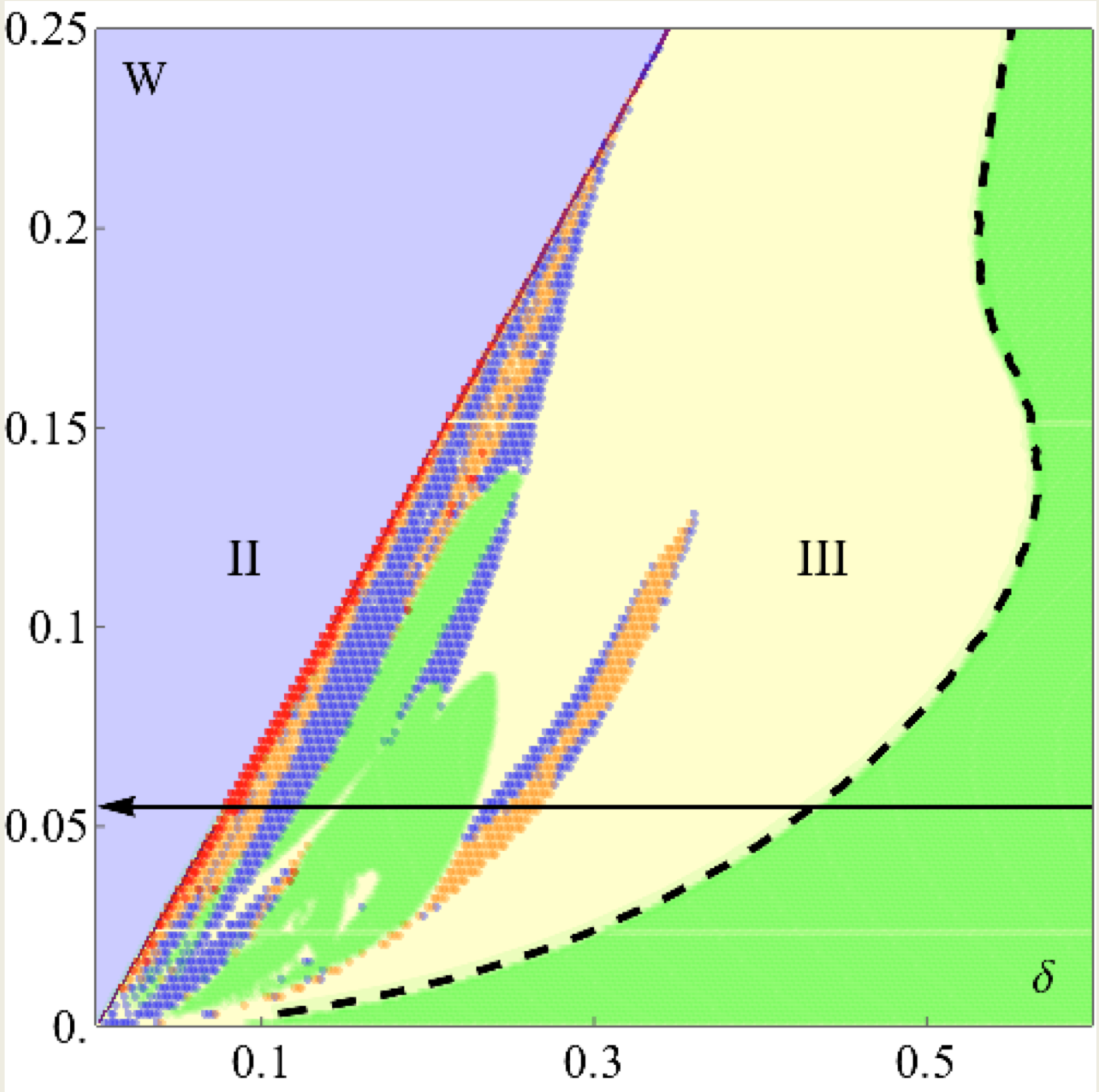
$$W = 0.055, \delta = 0.08010$$



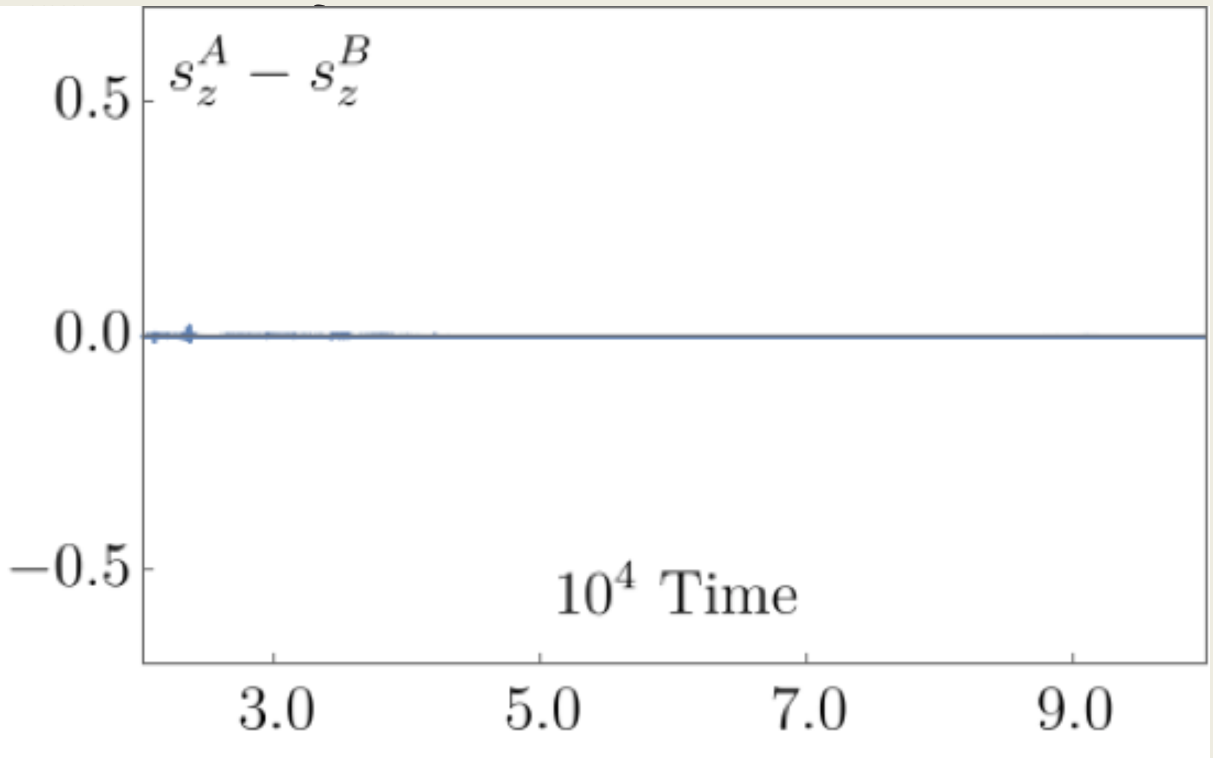


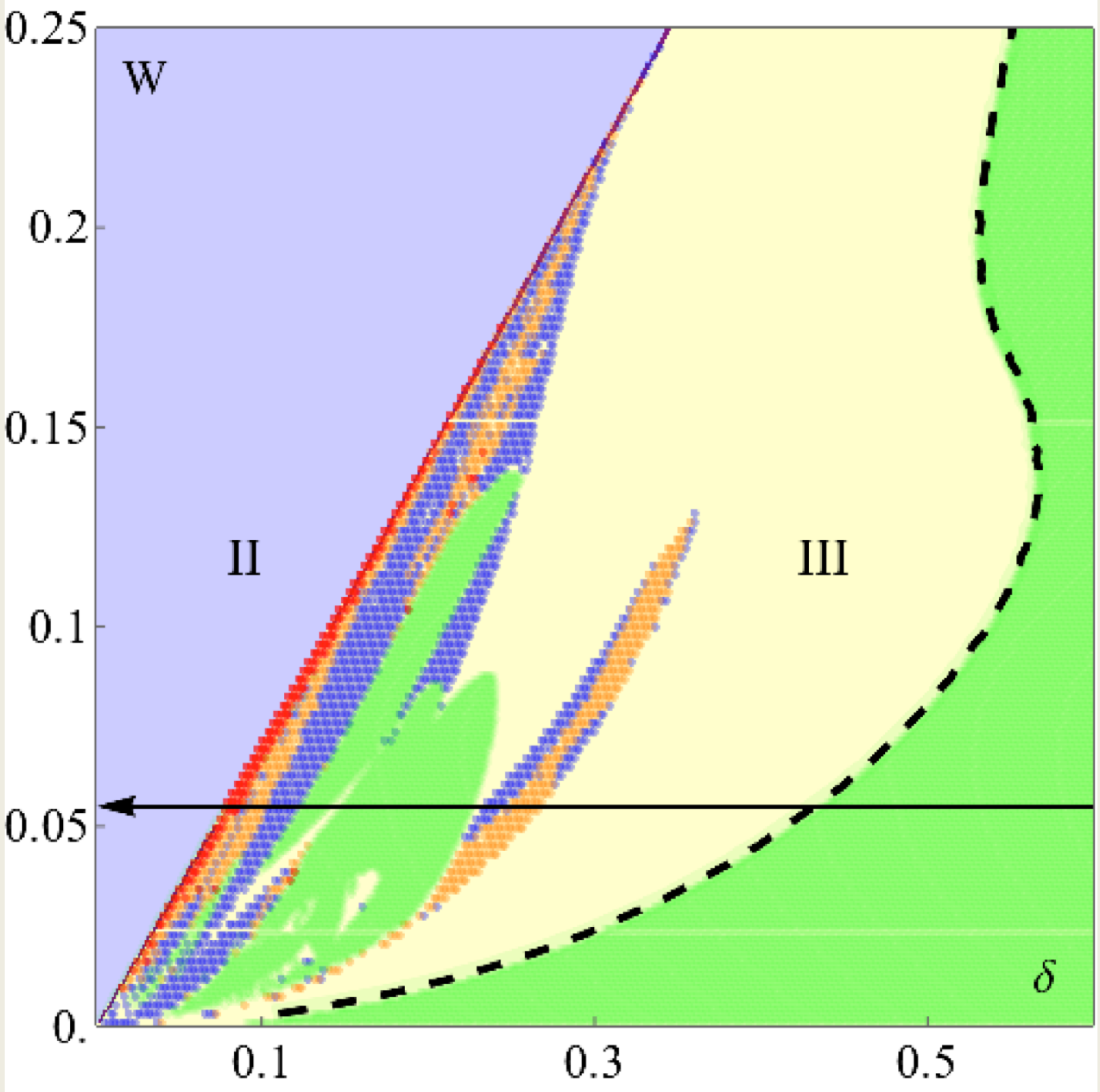
The frequency and magnitude of chaotic outbursts progressively decrease with δ

On-off intermittency

As we decrease δ keeping W fixed, chaos synchronizes via on-off intermittency

$$W = 0.055, \delta = 0.08000$$



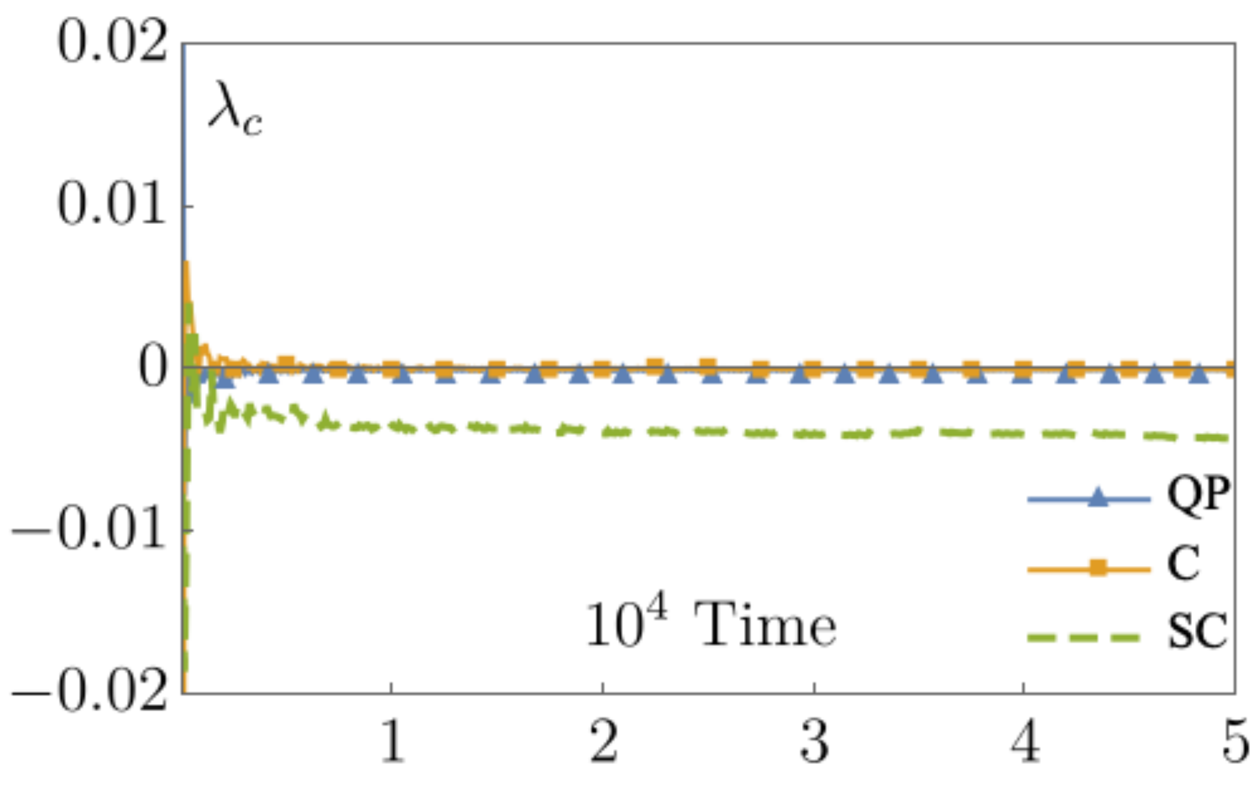


The frequency and magnitude of chaotic outbursts progressively decrease with δ

Stable synchronized chaos

As we decrease δ keeping W fixed, chaos synchronizes via on-off intermittency

$$W = 0.055, \delta = 0.08000$$



The conditional Lyapunov exponent (maximum Lyapunov exponent for directions transverse to the synchronization manifold) is negative

Chaotic synchronization: Experimental Signature

Observable: Power spectrum of radiated electric field. Measured with Michelson interferometry.

Proportional to $|l_{-}(f)|^2$

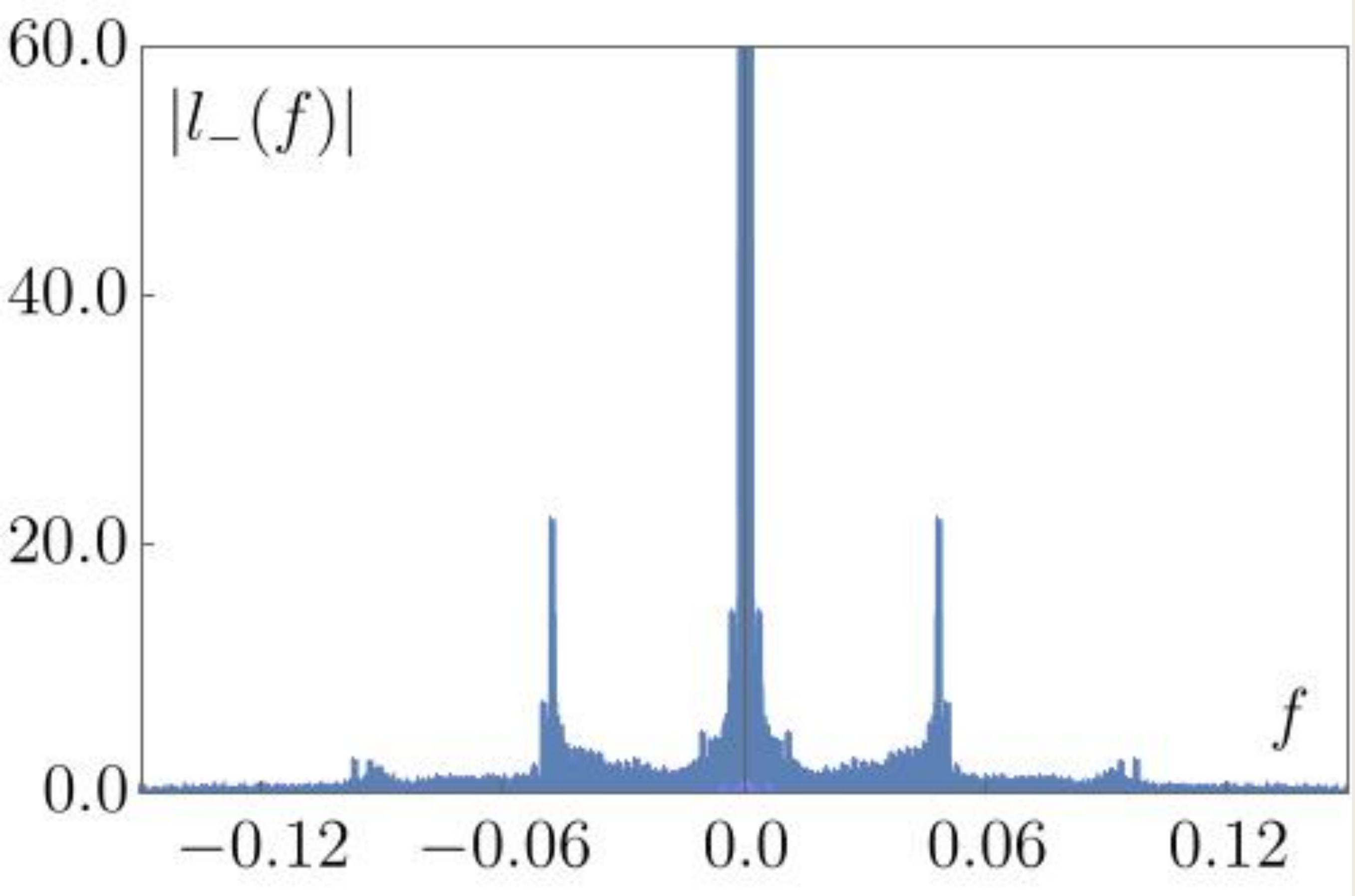
- Each phase leaves a unique signature in the radiated power spectrum

Chaotic synchronization: Experimental Signature

Observable: Power spectrum of radiated electric field. Measured with Michelson interferometry.

Proportional to $|l_-(f)|^2$

Synchronized chaos: $\delta = 0.080, W = 0.055$



- ✓ Chaos: continuous spectrum
- ✓ Synchronization ($\mathbb{Z}_2$ symmetry): reflection symmetry about the origin

$$\mathbb{Z}_2 \text{ symmetry} \Rightarrow l_y(t) = 0$$

$$\Rightarrow l_-(t) \text{ - real}$$

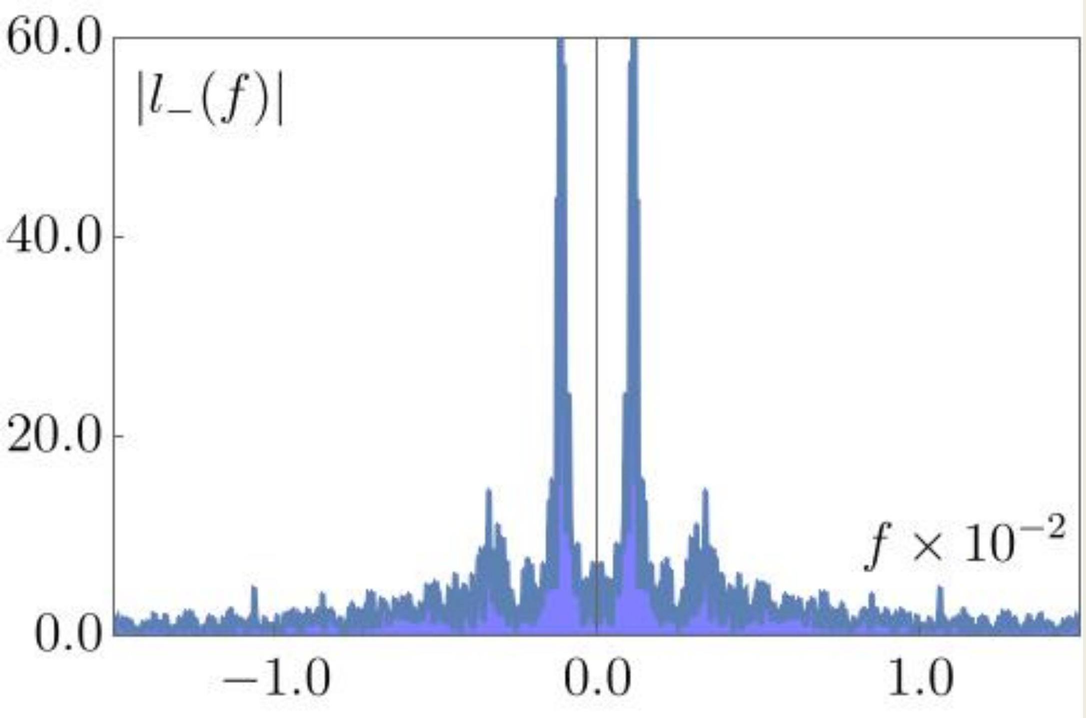
$$s_y^A = -s_y^B, \quad l_y = s_y^A + s_y^B = 0$$

Chaotic synchronization: Experimental Signature

Observable: Power spectrum of radiated electric field. Measured with Michelson interferometry.

Proportional to $|l_-(f)|^2$

Synchronized chaos:



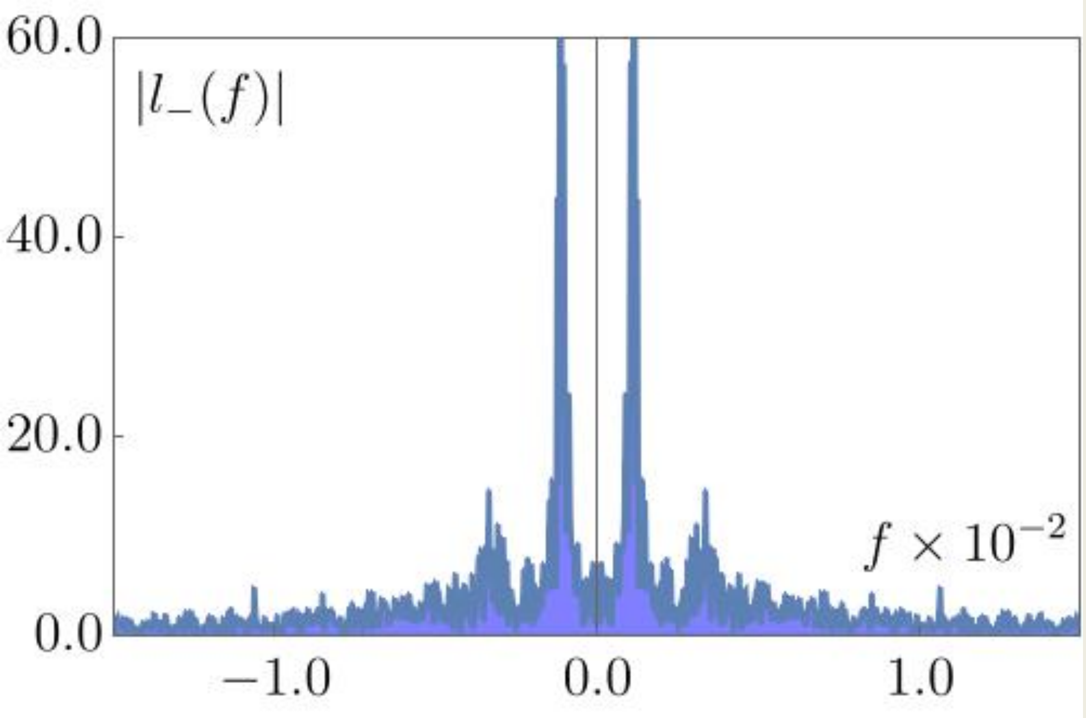
No peak at the origin – leftover from the $\mathbf{Z}_2$ symmetric limit cycle, which only has odd harmonics

Compare synchronized (left) and unsynchronized (right) chaos

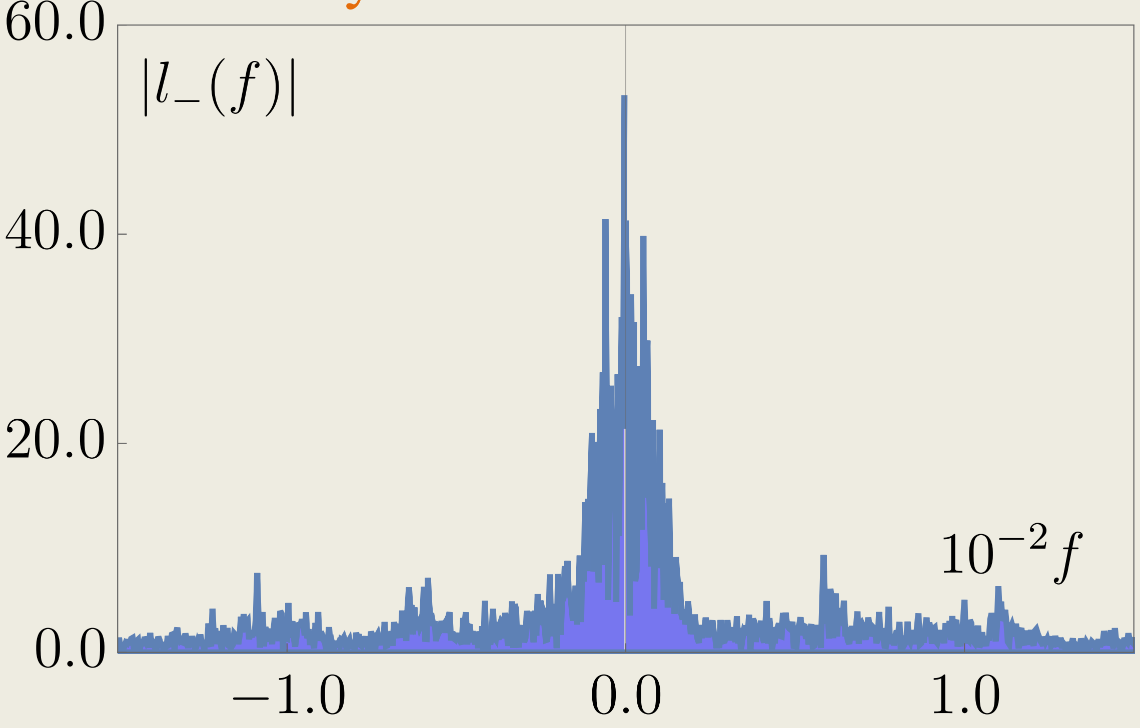
Observable: Power spectrum of radiated electric field. Measured with Michelson interferometry.

Proportional to $|l_-(f)|^2$

Synchronized chaos:



Ordinary chaos:



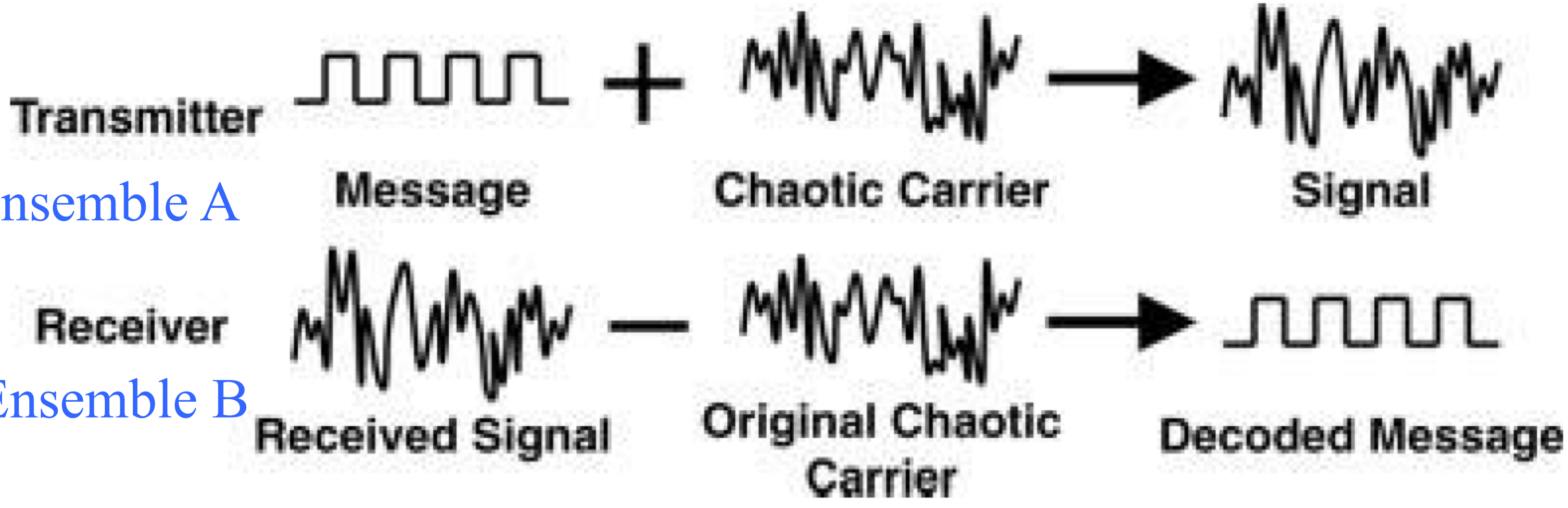
No reflection symmetry and peak at the origin for ordinary chaos

Chaotic synchronization: Applications - Steganography

The purpose of steganography is to hide the very existence of the message, not just its meaning as in cryptography.

1. Add message (small perturbation) to one of the chaotic temporal pattern from one ensemble

2. Subtract synchronized output from the transmitted signal. Retrieve the message



Ensemble A

Ensemble B

Picture from A. Uchida.

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- Phys. Rev. A **99**, 033802 (2019).
- Phys. Rev. A **100**, 023418 (2019).
- Ann. Phys. **417**, 168106 (2020).

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