Generalized Microcanonical Ensemble for Quantum Integrable Dynamics

Emil Yuzbashyan

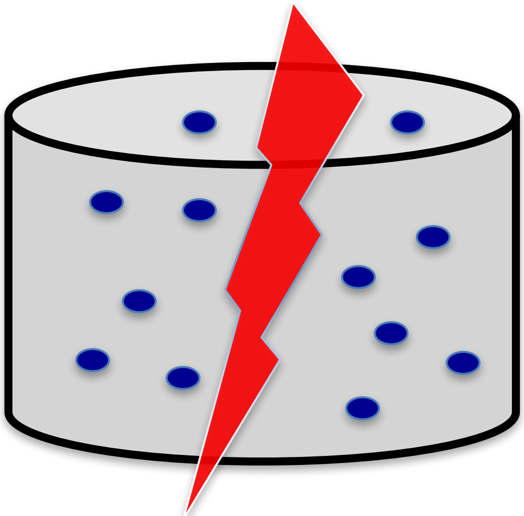
RUTGERS
THE STATE UNIVERSITY
OF NEW JERSEY

ICAM-I2CAM
Institute for Complex Adaptive Matter

Dynamics and Hydrodynamics of Certain Quantum Matter
March 20, 2017 – March 23, 2017

Quantum Quench

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$



1. Many-body system initially in equilibrium
2. Strong perturbation pulse drives the system far from equilibrium. Easy
3. But not too strong. No dissipation, unwanted interactions. The system evolves coherently with desired Hamiltonian for long time. Very difficult

Coherent Many-Body Dynamics:

$$i\frac{\partial|\psi\rangle}{\partial t} = \left(\hat{H}_0 + \hat{H}_{\text{int}}\right)|\psi\rangle$$

Quantum Quench: Coherent Many-Body Dynamics

Q: What happens to the system in time? Where does it end up as a result of unitary evolution? Does it equilibrate?

$$|\psi(t\to\infty)\rangle = ? \qquad \langle \hat{O}(t\to\infty)\rangle = ?$$

A: Depends on the system (on ***H***) and on the initial condition

a. Equilibration (thermalization) with some effective ***T***

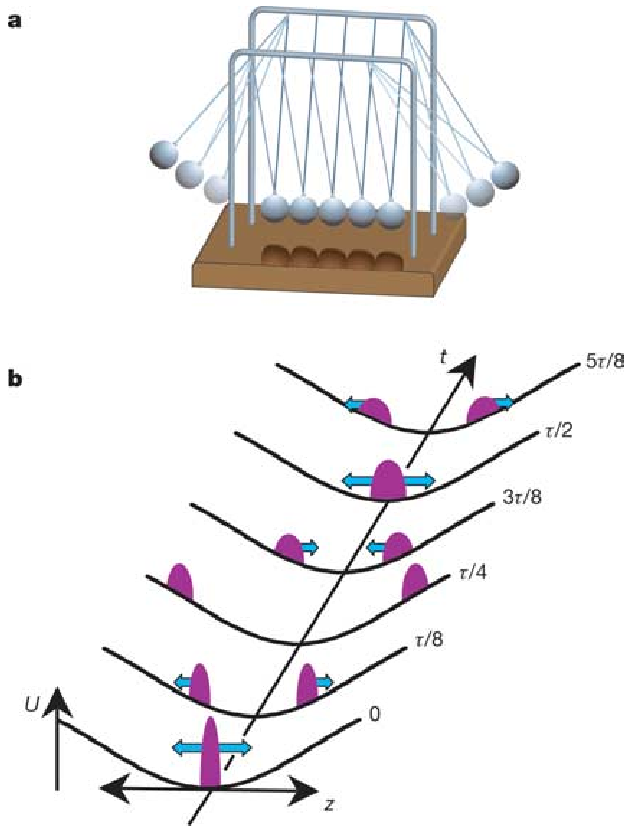
$$\langle \hat{O}(t\to\infty)\rangle = \mathrm{Tr}\,\hat{O}e^{-\hat{H}/T_{\mathrm{eff}}}$$

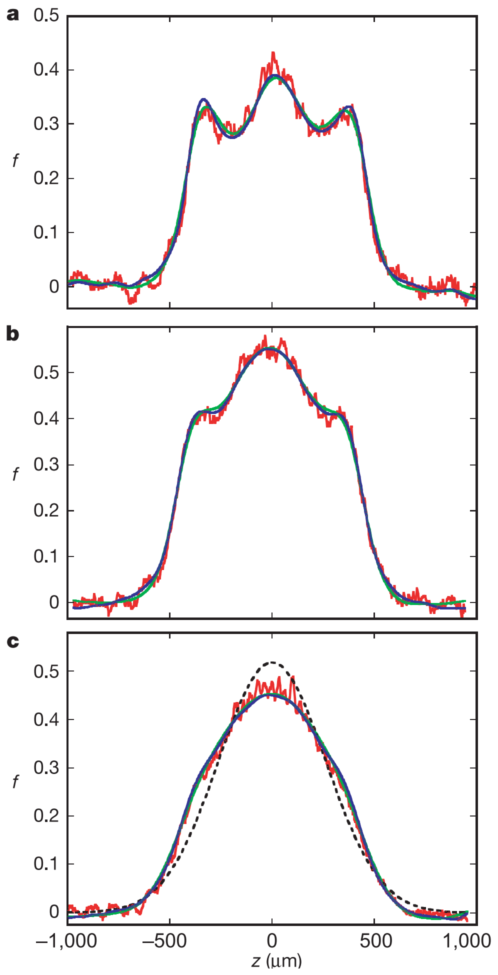
b. No thermalization – asymptotic state – nonequilibrium "phase" with new properties

$$\langle \hat{O}(t\to\infty)\rangle = ?$$

A quantum Newton's cradle

T. Kinoshita, T. Wenger, D. Weiss Nature (2006)



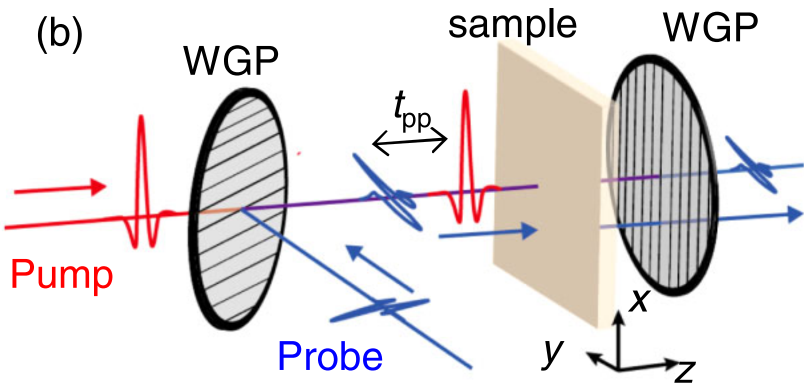


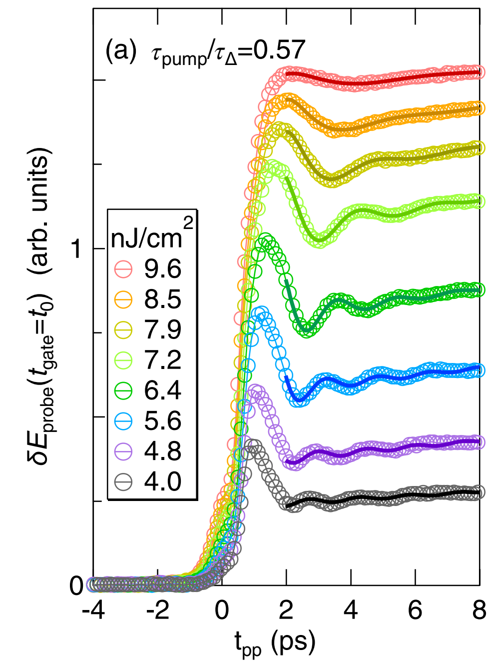
"[87]Rb atoms … do not noticeably equilibrate even after thousands of collisions. Our results are probably explainable by the well-known fact that a homogeneous 1D Bose gas with point-like collisional interactions is *integrable*."

PRL 111, 057002 (2013) PHYSICAL REVIEW LETTERS week ending 2 AUGUST 2013

Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

Ryusuke Matsunaga,[1] Yuki I. Hamada,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3] Hirotaka Terai,[2] Zhen Wang,[2] and Ryo Shimano[1]





$$|\psi(0)\rangle = |\textbf{noneq. state produced by the pulse}\rangle$$

$$\hat{H}_{\rm BCS} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} - g \sum_{\mathbf{k},\mathbf{p}} \hat{c}^\dagger_{\mathbf{k}\uparrow} \hat{c}^\dagger_{-\mathbf{k}\downarrow} \hat{c}_{-\mathbf{p}\downarrow} \hat{c}_{\mathbf{p}\uparrow}$$

$$i\frac{d|\psi\rangle}{dt} = \hat{H}_{\rm BCS}|\psi\rangle$$

Higgs mode
(order parameter)

$$\Delta(t) = g \sum_{\mathbf{p}} \langle \hat{c}_{-\mathbf{p}\downarrow} \hat{c}_{\mathbf{p}\uparrow} \rangle$$

Long time dynamics of a BCS superconductor in response to a sudden perturbation (quantum quench)

$$\hat{H}_{\text{BCS}} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} - g \sum_{\mathbf{k},\mathbf{p}} \hat{c}^{\dagger}_{\mathbf{k}\uparrow} \hat{c}^{\dagger}_{-\mathbf{k}\downarrow} \hat{c}_{-\mathbf{p}\downarrow} \hat{c}_{\mathbf{p}\uparrow}$$

$$i \frac{d|\psi\rangle}{dt} = \hat{H}_{\text{BCS}} |\psi\rangle$$

Higgs mode
(order parameter)

$$\Delta(t) = g \sum_{\mathbf{p}} \langle \hat{c}_{-\mathbf{p}\downarrow} \hat{c}_{\mathbf{p}\uparrow} \rangle$$

Q: $|\psi(t \to \infty)\rangle = ? \quad \Delta(t \to \infty) = ?$

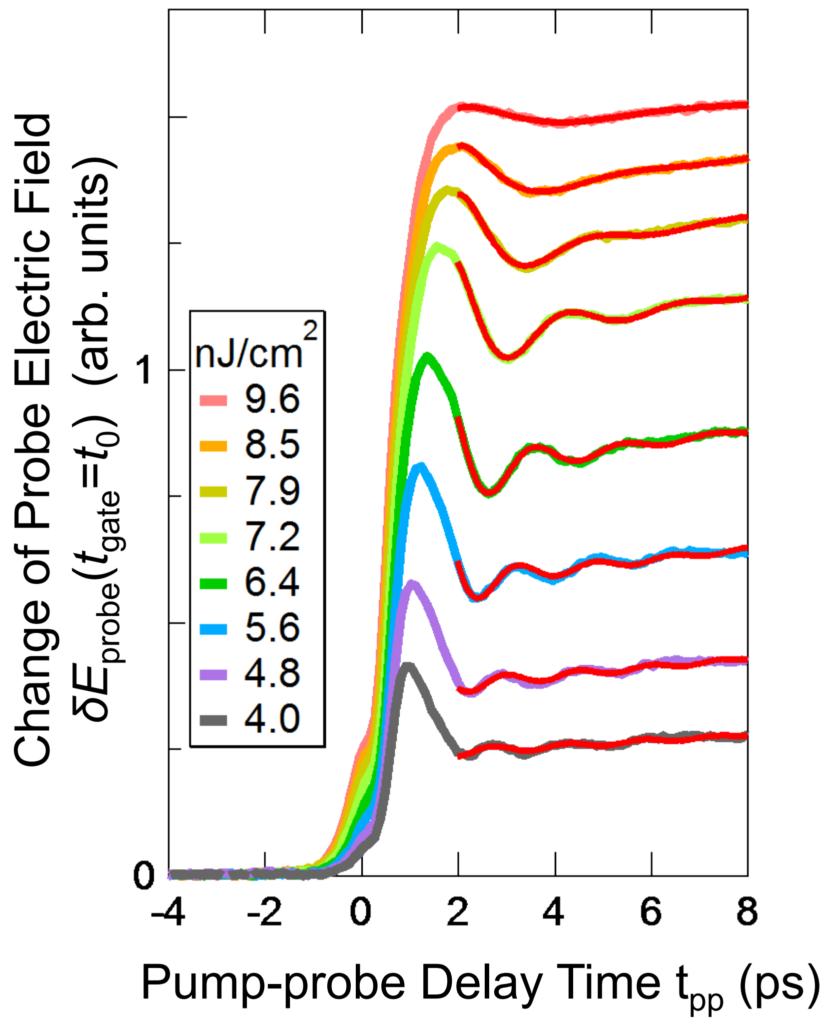
A: For moderate perturbation strength: $$|\Delta(t)| = \Delta_{\infty} + a \frac{\cos(2\Delta_{\infty} t + \alpha)}{\sqrt{\Delta_{\infty} t}}$$

Yuzbashyan, Tsyplyatyev, Altshuler, PRL (2006)

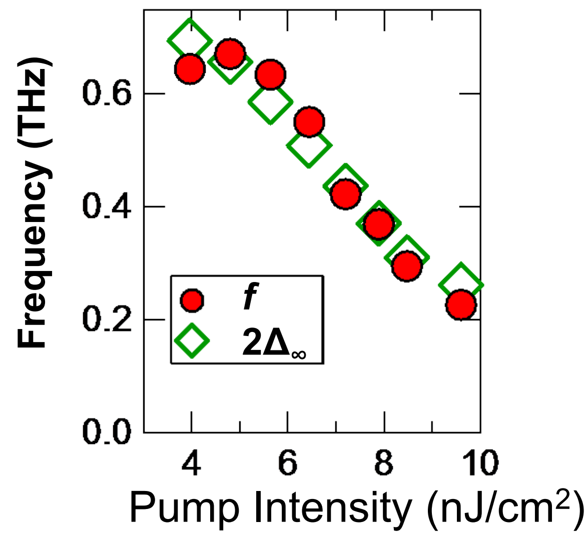
For stronger perturbations $|\Delta(t)|$ either vanishes or oscillates persistently at large times. In all cases the superconductor does NOT thermalize.

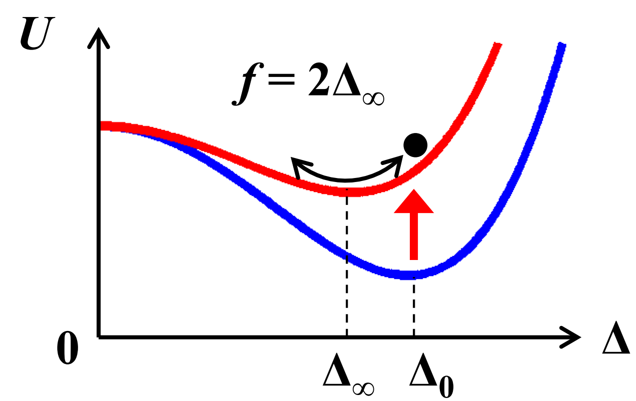
Order parameter dynamics

$$\delta\Delta(t_{pp}) = C_1 + C_2 t_{pp} + \frac{a}{(t_{pp})^b} \cos(2\pi f t_{pp} + \phi)$$



E. Yuzbashyan et al., PRL **96**, 230404 (2006).





R. Matsunaga et al., PRL**111**, 057002 (2013)

Slide from R. Shimano's talk at Rice conference "Interacting quantum systems driven out of equilibrium", May, 2016

Long time dynamics of a BCS superconductor in response to a sudden perturbation (quantum quench)

$$H_{\rm BCS} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

$$i\frac{d|\psi\rangle}{dt} = \hat{H}_{\rm BCS}|\psi\rangle$$

Higgs mode (order parameter) $\Delta(t) = g\sum_{\mathbf{p}}\langle s^-_{\mathbf{p}}(t)\rangle$

$\boldsymbol{H_{\rm BCS}}$ is integrable, Richardson (1964), Gaudin (1983)

Integrals of motion for $\boldsymbol{H_{\rm BCS}}$ – Gaudin magnets/ central spin models

$$H_{\mathbf{k}} = \sum_{\mathbf{p}} \frac{\vec{s}_{\mathbf{k}}\cdot\vec{s}_{\mathbf{p}}}{\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}}} - \frac{s^z_{\mathbf{k}}}{g}, \quad [H_{\mathbf{k}}, H_{\mathbf{p}}] = 0, \quad H_{\rm BCS} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} H_{\mathbf{k}} \Longrightarrow [H_{\rm BCS}, H_{\mathbf{k}}] = 0$$

$\#$ of integrals $=$ $\#$ of pseudospins $=$ $\#$ of pairs of states $(\mathbf{k}\uparrow, -\mathbf{k}\downarrow)$

Integrable systems do NOT thermalize
Do they follow Generalized Gibbs Ensemble (GGE)?

$$[H, H_i] = 0, \quad [H_i, H_j] = 0$$

$$\rho = C \exp\left(-\sum_i \beta_i H_i\right)$$

β_i are determined from:

$$\langle \psi(0)|H_i|\psi(0)\rangle = \mathrm{Tr}\,\rho H_i$$

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T \langle O(t)\rangle dt = \mathrm{Tr}\,\rho O$$

When does it work?

Not for finite size, long range interactions or global observables

For local interactions & observables and thermodynamic limit – sometimes **YES**, sometimes **NO –** depends on the set of available integrals (and also on H and the initial state)

- GGE fails for 1D Heisenberg spin chains
  Goldstein & Andrei, Phys. Rev. A (2014); Pozsgay et. al. PRL (2014)

- Does work for 1D Heisenberg spin chains if newly discovered integrals are added
  Ilievski et. al. PRL (2015)

Integrable systems do NOT thermalize

Do they follow Generalized Gibbs Ensemble (GGE)?

$$[H, H_i] = 0, \quad [H_i, H_j] = 0$$

$$\rho = C \exp\left(-\sum_i \beta_i H_i \right)$$

β_i are determined from:

$$\langle \psi(0) | H_i | \psi(0) \rangle = \mathrm{Tr}\, \rho H_i$$

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T \langle O(t) \rangle dt = \mathrm{Tr}\, \rho O$$

When does it work?

How do we determine if we have the "right" set of integrals and the criteria for the validity of GGE?

Problem: quantum integrability is NOT well-defined!

See e.g. Sutherland, *Beautiful Models* (2004), Caux &Mossel (2011), Yuzbashyan &Shastry (2013)

No natural notion of a nontrivial integral of motion, let alone of a complete set

For example, for any set of H_k such that $[H, H_k]=0$, can find H_0 so that:

$$H_k = \sum_n a_{kn} H_0^n$$

i.e. always only one functionally independent integral – H itself

Integrable systems do NOT thermalize

Do they follow Generalized Gibbs Ensemble (GGE)?

$$[H, H_i] = 0, \quad [H_i, H_j] = 0$$

$$\rho = C \exp\left(-\sum_i \beta_i H_i\right)$$

β_i are determined from:

$$\langle \psi(0) | H_i | \psi(0) \rangle = \mathrm{Tr}\, \rho H_i$$

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T \langle O(t) \rangle dt = \mathrm{Tr}\, \rho O$$

When does it work?

How do we determine if we have the “right” set of integrals and the criteria for the validity of GGE?

Problem: quantum integrability is NOT well-defined!

See e.g. Sutherland, *Beautiful Models* (2004), Caux &Mossel (2011), Yuzbashyan &Shastry (2013)

Without an independent notion of a complete set of nontrivial integrals of motion GGE is essentially unfalsifiable in Quantum Mechanics

Classical Integrability is well-defined

$H(p,q)$, where $q = (q_1, \ldots, q_n)$; $p = (p_1, \ldots, p_n)$; i.e. n degrees of freedom

Definition: $H(p,q)$ is integrable if it has n (maximum possible number) of functionally independent Poisson-commuting integrals

$$\{H_i(p,q), H_j(p,q)\} = 0, \quad i,j = 0, \ldots, n-1; \quad H_0(p,q) \equiv H(p,q)$$

$$\Downarrow$$

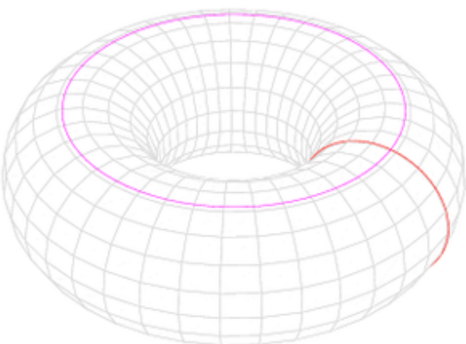
- ✓ *Unambiguous separation between integrable and not integrable*
- ✓ *Clear notion of a complete set of integrals to construct GGE*

Do Classical Mechanics before going Quantum?!

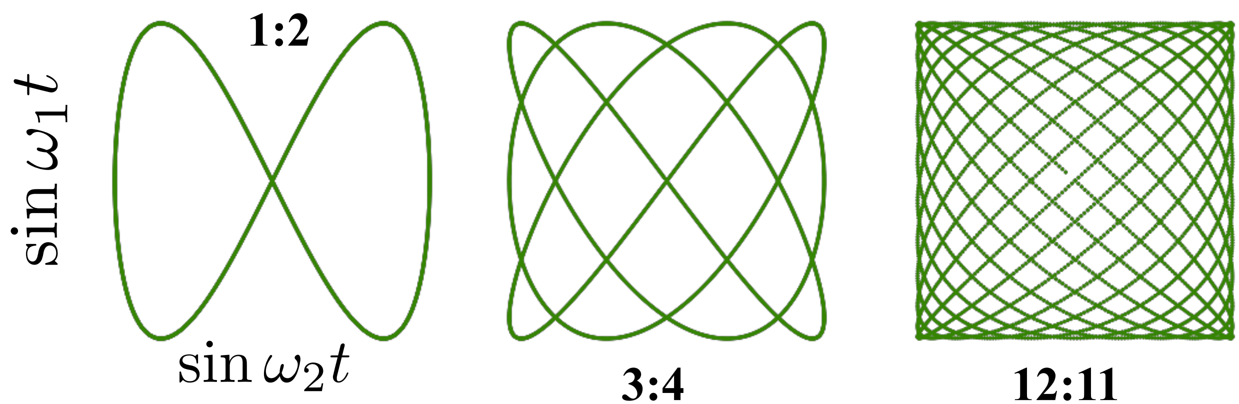
Generalized Gibbs Ensemble DeMystified in Classical Mechanics

Dynamics is on "invariant torus" – ***n***-dim portion of ***2n***-dim phase-space cut out by integrals of motion $H_1(p,q)=\mathrm{const}$, $H_2(p,q)=\mathrm{const}$, …, $H_n(p,q)=\mathrm{const}$

There are ***n*** typically incommensurate frequencies $\omega_1, \omega_2, \ldots, \omega_n$ (non-resonant torus)



Lissajous figures



Theorem about averages (Arnold, *Math. Methods of CM*):

For a non-resonant torus and any "reasonable" observable $O(p,q)$

time average = phase-space average over the torus

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T O\,(t)\,dt = \int O(\varphi)\,\frac{d\varphi}{(2\pi)^n}$$

Generalized Gibbs Ensemble DeMystified in Classical Mechanics

Theorem about averages (Arnold, *Math. Methods of CM*):

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T O(t)\,dt = \int O(\varphi)\frac{d\varphi}{(2\pi)^n}$$

Going back to the original variables ***p & q*** and using the fact that this is a canonical transform can prove **Generalized Microcanonical Ensemble (GME)**

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T O(t)\,dt = \int O(p,q)\rho(p,q)dpdq$$

E.Y., Ann. Phys. (2016)

$$\rho(p,q) = V^{-1}\prod_{k=1}^{n}\delta\left(H_k(p,q)-\alpha_k\right)$$

Works for any system size (any ***n***)
Exceptions: resonant tori

Additive integrals, thermodynamic limit

$H_k \propto n$

$n \to \infty$

See e.g. Ruelle, Stat. Mech.: Rigorous Results (1999)

Generalized (canonical) Gibbs

$$\rho(p,q) = C\exp\left[-\sum_k \beta_k H_k(p,q)\right]$$

Not always the case

Going Quantum

$$\rho(p,q) = V^{-1} \prod_{k=1}^{n} \delta\left(H_k(p,q) - \alpha_k\right)$$

Note: microcanonical ensemble doesn't work for finite ***n***

Works for any system size (any ***n***)

$\text{non-integrable CM} \neq V^{-1}\delta\left(H(p,q) - E\right)$

Q: What is Generalized Microcanonical Ensemble (GME) in the quantum case, i.e. a quantum analog of ***ρ (p, q)***?

Consider a system where we can gradually go from quantum to classical while maintaining integrability (e.g. Gaudin magnets)

- ✓ Is GME similarly exact in the quantum case for a finite system? If not, how does it improve as $\hbar \to 0$?
- ✓ How does GME compare to GGE?

Going Quantum

$$\rho(p,q) = V^{-1}\prod_{k=1}^{n}\delta\left(H_k(p,q)-\alpha_k\right)$$

Note: microcanonical ensemble doesn't work for finite ***n***

Works for any system size (any ***n***)

$$\text{non-integrable CM} \neq V^{-1}\delta\left(H(p,q)-E\right)$$

Q: What is Generalized Microcanonical Ensemble (GME) in the quantum case, i.e. a quantum analog of $\rho\,(p,\,q)$?

$$H_k(p,q) \to \hat{H}_k \text{ works for GGE, } \exp\left[-\sum_k \beta_k H_k(p,q)\right] \to \exp\left[-\sum_k \beta_k \hat{H}_k\right]$$

$$\text{Doesn't work for GME because } \langle \hat{H}_i \hat{H}_k \rangle \neq \langle \hat{H}_i \rangle \langle \hat{H}_k \rangle$$

Going Quantum

$$\rho(p,q) = V^{-1}\prod_{k=1}^{n}\delta\left(H_k(p,q)-\alpha_k\right)$$

Works for any system size (any ***n***)

Note: microcanonical ensemble doesn't work for finite ***n***

$$\text{non-integrable CM} \neq V^{-1}\delta\left(H(p,q)-E\right)$$

Q: What is Generalized Microcanonical Ensemble (GME) in the quantum case, i.e. a quantum analog of ***ρ (p, q)***?

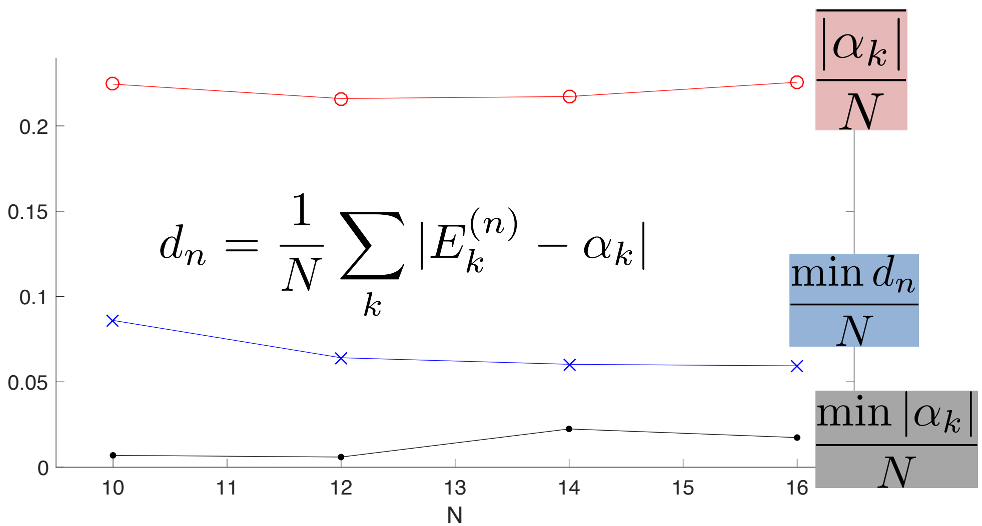
Need to broaden ***δ*** - functions.

"Windows", i.e. equal weight GME?

$$\alpha_k = \langle \hat{H}_k \rangle, \quad \hat{H}_k|\psi_n\rangle = E_k^{(n)}|\psi_n\rangle,$$

$$\mathcal{S} : |E_k^{(n)} - \alpha_k| < \delta_k, \quad \langle \hat{O} \rangle = \sum_{n\in\mathcal{S}} \langle \psi_n|\hat{O}|\psi_n\rangle$$

Doesn't work well for many integrals!



$$H_{\rm BCS} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s_{\mathbf{k}}^z - g\sum_{\mathbf{k},\mathbf{p}} s_{\mathbf{k}}^+ s_{\mathbf{p}}^-$$

α_k for quench $g_i = 0.5\delta \to g_f = 2\delta$

$$H_{\mathbf{k}} = \sum_{\mathbf{p}} \frac{\vec{s}_{\mathbf{k}}\cdot\vec{s}_{\mathbf{p}}}{\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}}} - \frac{s_{\mathbf{k}}^z}{g}$$

NO states sufficiently close to all ***α_k***

Going Quantum

$$\rho(p,q) = V^{-1} \prod_{k=1}^{n} \delta\left(H_k(p,q) - \alpha_k\right)$$

Note: microcanonical ensemble doesn't work for finite ***n***

Works for any system size (any ***n***)

non-integrable CM $\neq V^{-1}\delta\left(H(p,q) - E\right)$

Q: What is Generalized Microcanonical Ensemble (GME) in the quantum case, i.e. a quantum analog of ***ρ (p, q)***?

Need to broaden ***δ*** - functions.

"Windows", i.e. equal weight GME?

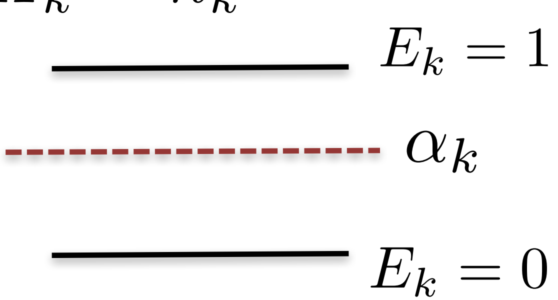
$$\alpha_k = \langle \hat{H}_k \rangle, \quad \hat{H}_k|\psi_n\rangle = E_k^{(n)}|\psi_n\rangle,$$

$$\mathcal{S} : |E_k^{(n)} - \alpha_k| < \delta_k, \quad \langle \hat{O} \rangle = \sum_{n\in\mathcal{S}} \langle \psi_n|\hat{O}|\psi_n\rangle$$

Doesn't work well for many integrals!

Or suppose integrals take discrete values, e.g. fermion occupation #s

$$H_k = \hat{n}_k$$



Unlike GGE or Classical Mechanics, NO viable generalization of the microcanonical ensemble for a quantum integrable system!

Forget about comparing it to GGE

See also Cassidy et. al. PRL (2011)

Going Quantum

$$\rho(p,q) = V^{-1}\prod_{k=1}^{n}\delta\left(H_k(p,q)-\alpha_k\right)$$

Works for any system size (any ***n***)

Note: microcanonical ensemble doesn't work for finite ***n***

$$\text{non-integrable CM} \neq V^{-1}\delta\left(H(p,q)-E\right)$$

Q: What is Generalized Microcanonical Ensemble (GME) in the quantum case, i.e. a quantum analog of ***ρ (p, q)***?

Functional broadening, e.g. Gaussian?

$$\hat{\rho} = C\exp\left[-\sum_{ik}(\hat{H}_i-\mu_i)M_{ik}(\hat{H}_k-\mu_k)\right]$$

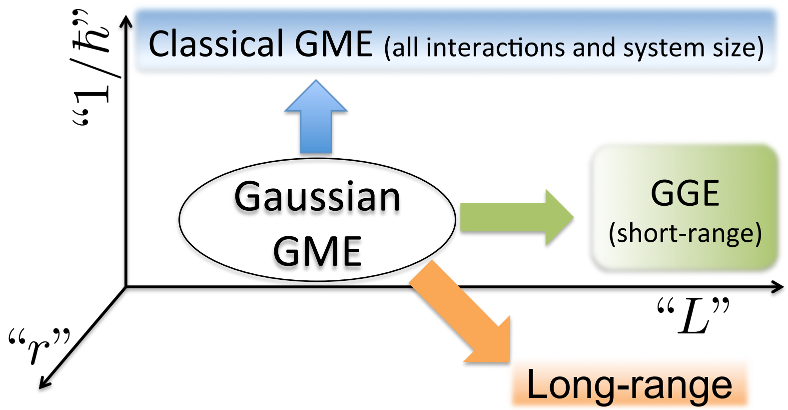
Classical limit:

$$\langle\hat{H}_i\hat{H}_k\rangle \to \langle\hat{H}_i\rangle\langle\hat{H}_k\rangle \implies \hat{\rho}\to\rho(p,q)$$

μ_i, M_{ik} are determined from:

$$\langle\hat{H}_i\rangle_0 = \mathrm{Tr}(\hat{\rho}\hat{H}_i) \text{ and } \langle\hat{H}_i\hat{H}_k\rangle_0 = \mathrm{Tr}(\hat{\rho}\hat{H}_i\hat{H}_k)$$

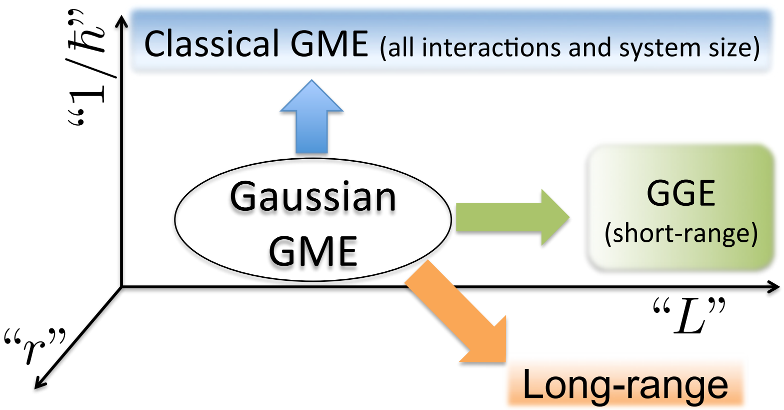
Quantum Generalized Microcanonical Ensemble = Gaussian GME



$$\hat{\rho} = C \exp\left[-\sum_{ik} (\hat{H}_i - \mu_i) M_{ik} (\hat{H}_k - \mu_k)\right]$$

μ_i, M_{ik} are determined from:
$\langle \hat{H}_i \rangle_0 = \mathrm{Tr}(\hat{\rho}\hat{H}_i)$ and $\langle \hat{H}_i \hat{H}_k \rangle_0 = \mathrm{Tr}(\hat{\rho}\hat{H}_i\hat{H}_k)$

1. Well defined and straightforward to implement for any system/size
2. Guaranteed exact in classical limit, $\hbar \to 0$ (any system size)
3. Captures leading quantum correction $(\propto \hbar)$ (any system size)
4. Works whenever GGE does and converges faster (**$1/L^2$**) than GGE (**$1/L$**) with system size, ***L***, i.e. captures leading finite size correction
5. Works well for systems with long-range interactions
6. Unlike GGE, also captures fluctuations of local & global observables



$$\hat{\rho} = C \exp\left[-\sum_{ik} (\hat{H}_i - \mu_i) M_{ik} (\hat{H}_k - \mu_k) \right]$$

μ_i, M_{ik} are determined from:
$\langle \hat{H}_i \rangle_0 = \mathrm{Tr}(\hat{\rho} \hat{H}_i)$ and $\langle \hat{H}_i \hat{H}_k \rangle_0 = \mathrm{Tr}(\hat{\rho} \hat{H}_i \hat{H}_k)$

Exact in classical limit and, moreover, captures the leading quantum correction

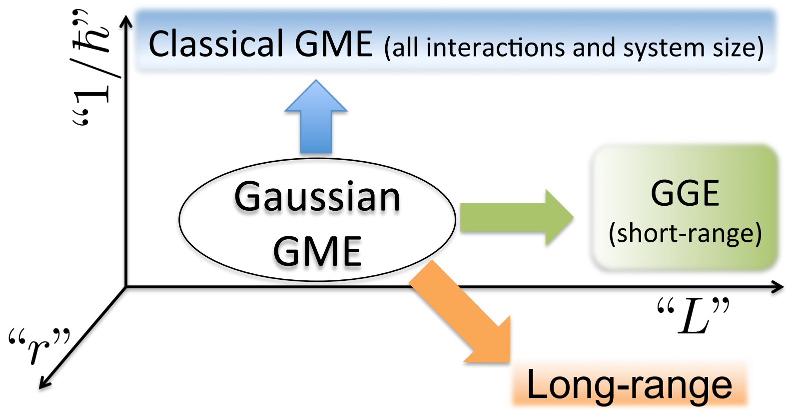
Ex 1: harmonic oscillator $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 \hat{q}^2}{2}$

$$|\psi(0)\rangle = |z\rangle = \text{coherent state}, \; a|z\rangle = z|z\rangle$$

$$\text{Classical limit: } \hbar \to 0, \; E_0 = \hbar\omega |z|^2 = \text{fixed}$$

$$\overline{\langle \hat{n}^k \rangle}_\infty \equiv \lim_{T\to\infty} \int_0^T \langle \hat{n}^k(t) \rangle dt = |z|^{2k} \left[1 + k(k-1)\frac{\hbar\omega}{E_0} + \dots \right], \quad (\hat{n} = a^\dagger a)$$

$$\langle \hat{n}^k \rangle_{\mathrm{GME}} \equiv \mathrm{Tr}(\hat{\rho} \hat{n}^k) = |z|^{2k} \left[1 + k(k-1)\frac{\hbar\omega}{E_0} + \dots \right]$$



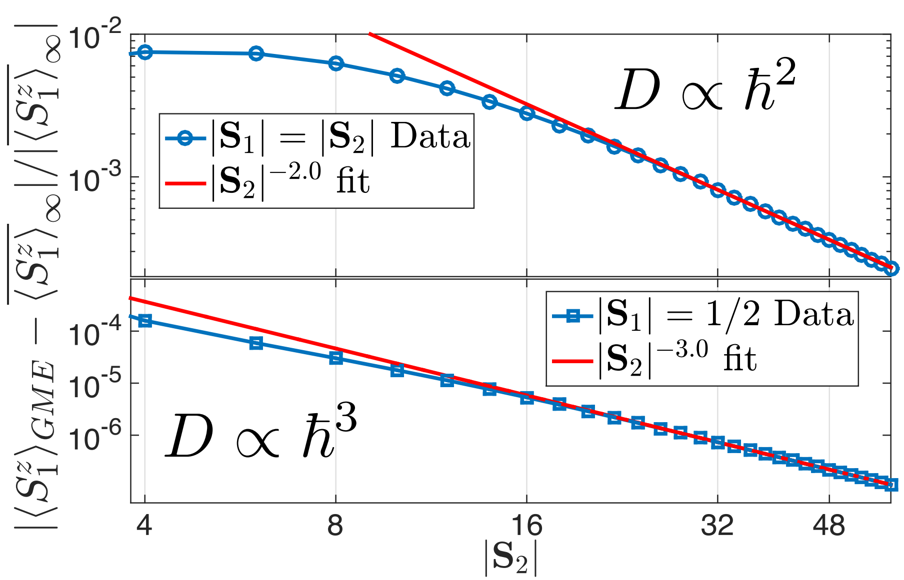
$$\hat{\rho} = C \exp\left[-\sum_{ik}(\hat{H}_i - \mu_i)M_{ik}(\hat{H}_k - \mu_k)\right]$$

μ_i, M_{ik} are determined from:
$\langle \hat{H}_i \rangle_0 = \mathrm{Tr}(\hat{\rho}\hat{H}_i)$ and $\langle \hat{H}_i \hat{H}_k \rangle_0 = \mathrm{Tr}(\hat{\rho}\hat{H}_i\hat{H}_k)$

Exact in classical limit and, moreover, captures the leading quantum correction

Ex 2: 2-spin Gaudin magnet

$$H_1 = BS_1^z + \gamma \vec{S}_1 \cdot \vec{S}_2, \quad H_2 = BS_2^z - \gamma \vec{S}_1 \cdot \vec{S}_2,$$
$$[H_1, H_2] = 0$$



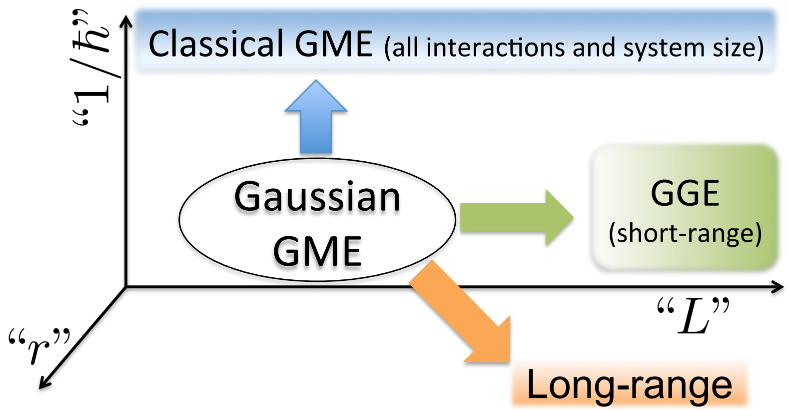
$$|\psi(0)\rangle = |\theta_1, \varphi_1\rangle \otimes |\theta_2, \varphi_2\rangle \text{ (coherent st.)}$$

Two scenarios: both spins classical or only S_2

(a) $\hbar \to 0,\ \hbar S_1 = \hbar S_2 = \text{fixed}$

(b) $\hbar \to 0,\ \hbar S_2 = \text{fixed},\ S_1 = 1/2$

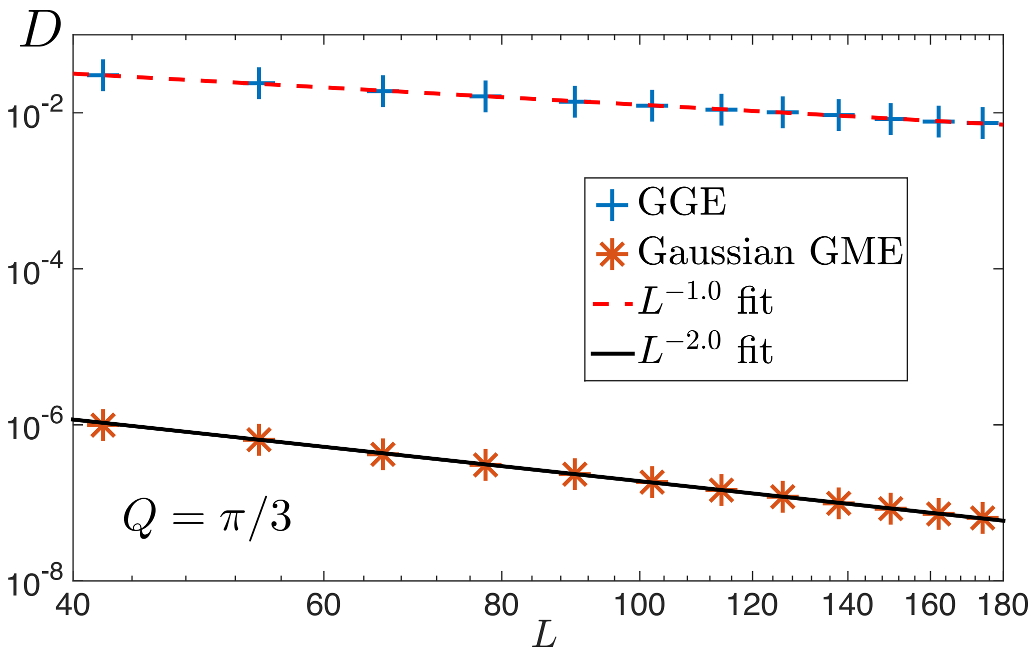
$$D = \frac{\langle S_1^z \rangle_{\mathrm{GME}} - \overline{\langle S_1^z \rangle}_\infty}{\overline{\langle S_1^z \rangle}_\infty}$$



$$\hat{\rho} = C \exp\left[-\sum_{ik} (\hat{H}_i - \mu_i) M_{ik} (\hat{H}_k - \mu_k) \right]$$

Works whenever GGE does and converges faster (**$1/L^2$**) than GGE (**$1/L$**) as system size, ***L***, grows

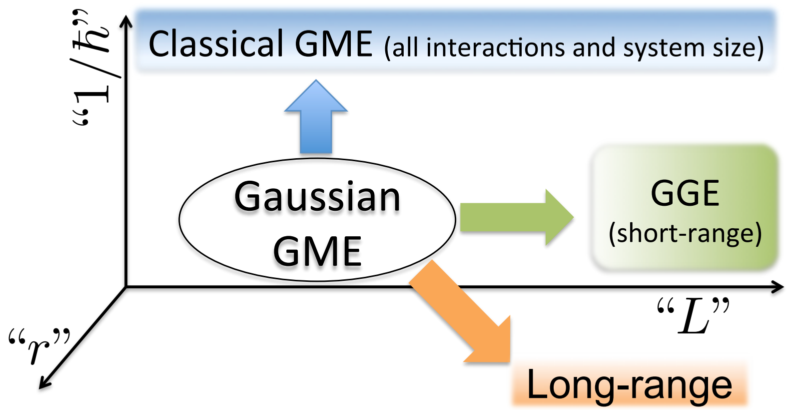
Example: $$H = -\sum_{j=1}^{L} (e^{i\phi} c_{j+1}^\dagger c_j + e^{-i\phi} c_j^\dagger c_{j+1}) + \sum_{j=1}^{L} [V_1 \cos(Qj) + V_2 \cos(2Qj)]\, n_j$$



GME exact for linear & bilinear combinations of occupation #s for any ***L***, so consider:

$$D = \frac{\langle \hat{n}_1 \hat{n}_2 \hat{n}_3 \rangle_{\mathrm{GE}} - \overline{\langle \hat{n}_1 \hat{n}_2 \hat{n}_3 \rangle_\infty}}{\langle \hat{n}_1 \rangle \langle \hat{n}_2 \rangle \langle \hat{n}_3 \rangle}$$

Quench from $\phi = V_1 = V_2 = 0$ to $\phi = 0.3, V_1 = 1.5, V_2 = 1.0$



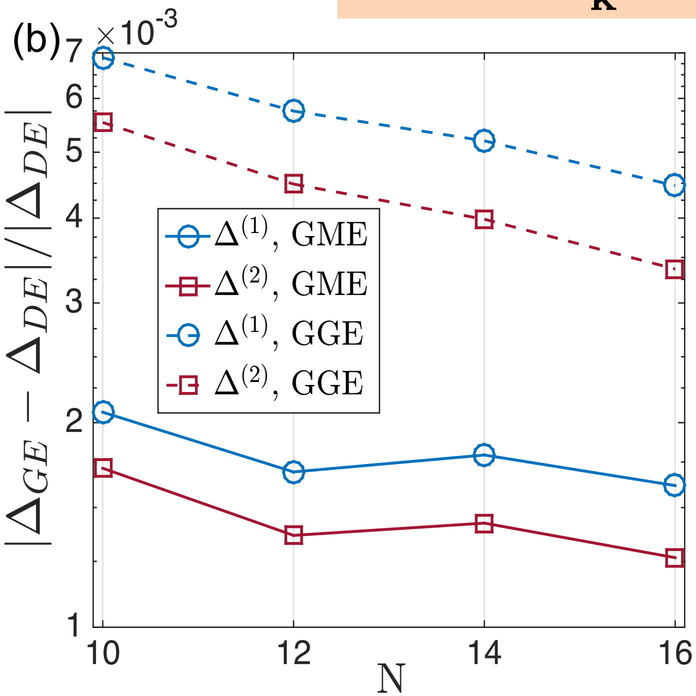
$$\hat{\rho} = C \exp\left[-\sum_{ik} (\hat{H}_i - \mu_i) M_{ik} (\hat{H}_k - \mu_k) \right]$$

Works well for systems with long-range interactions. Likely exact in $N \to \infty$ limit, since dynamics become effectively classical

Example:

$$H_{\mathrm{BCS}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

$$H_{\mathbf{k}} = \sum_{\mathbf{p}} \frac{\vec{s}_{\mathbf{k}} \cdot \vec{s}_{\mathbf{p}}}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{p}}} - \frac{s^z_{\mathbf{k}}}{g}$$



Canonical order parameters:

$$\Delta^{(1)} = g\sqrt{\sum_{\mathbf{k},\mathbf{p}} \langle s^+_{\mathbf{k}} s^-_{\mathbf{p}} \rangle - \frac{N}{2}},$$

$$\Delta^{(2)} = g \sum_{\mathbf{k}} \sqrt{\frac{1}{4} - \langle s^z_{\mathbf{k}} \rangle^2}$$

N - # of spins, $s_{\mathbf{k}} = 1/2$, total $S^z = 0$

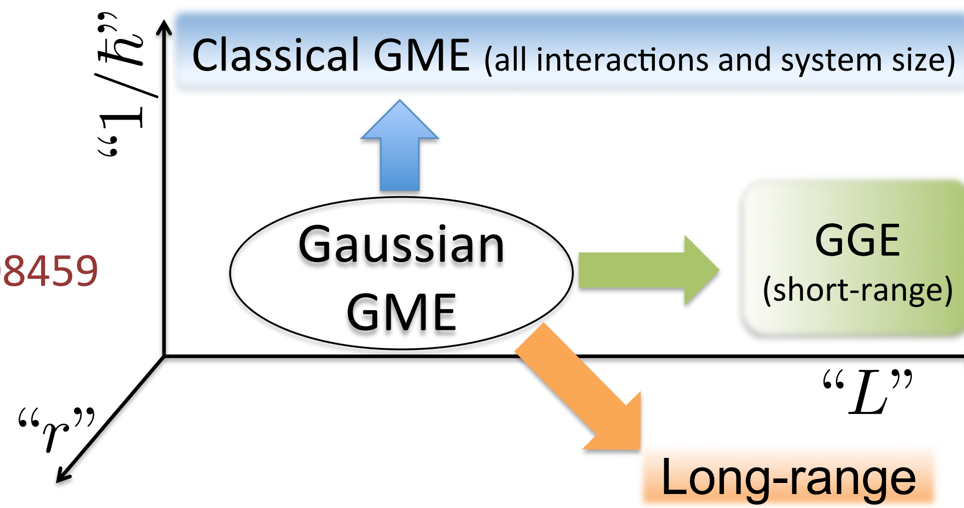
Quench $g_i = 0.5\delta \to g_f = 2.0\delta$, where δ = spacing between $\epsilon_{\mathbf{k}}$

Anatoli Polkovnikov
Boston University

Hyungwon Kim
Rutgers University

Yuzbashyan, Ann. Phys. 367, 288 (2016)

Kim, Polkovnikov, Yuzbashyan, arXiv:1606.08459



the David & Lucile Packard FOUNDATION

National Science Foundation
WHERE DISCOVERIES BEGIN

Kavli Institute for Theoretical Physics