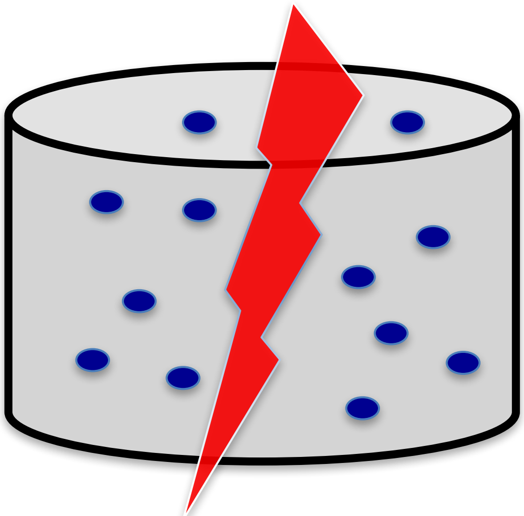
Far from equilibrium phases of superfluid matter

Emil Yuzbashyan

RUTGERS
THE STATE UNIVERSITY
OF NEW JERSEY

Coherent Many-Body Dynamics

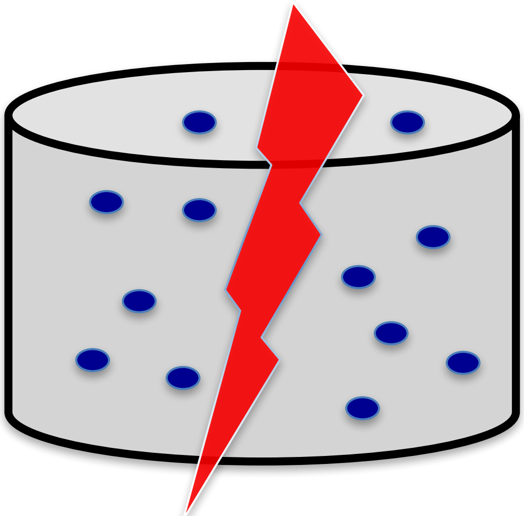
$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$



1. Interacting system initially in equilibrium
2. Strong perturbation pulse drives the system far from equilibrium. Easy

Coherent Many-Body Dynamics

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$



Quantum Quench

1. Interacting system initially in equilibrium
2. Strong perturbation pulse drives the system far from equilibrium. Easy
3. But not too strong. No dissipation, decoherence, controlled interactions. The system evolves coherently with desired Hamiltonian for long time. Very difficult

$$i\frac{\partial|\psi\rangle}{\partial t} = \left(\hat{H}_0 + \hat{H}_{\text{int}}\right)|\psi\rangle$$

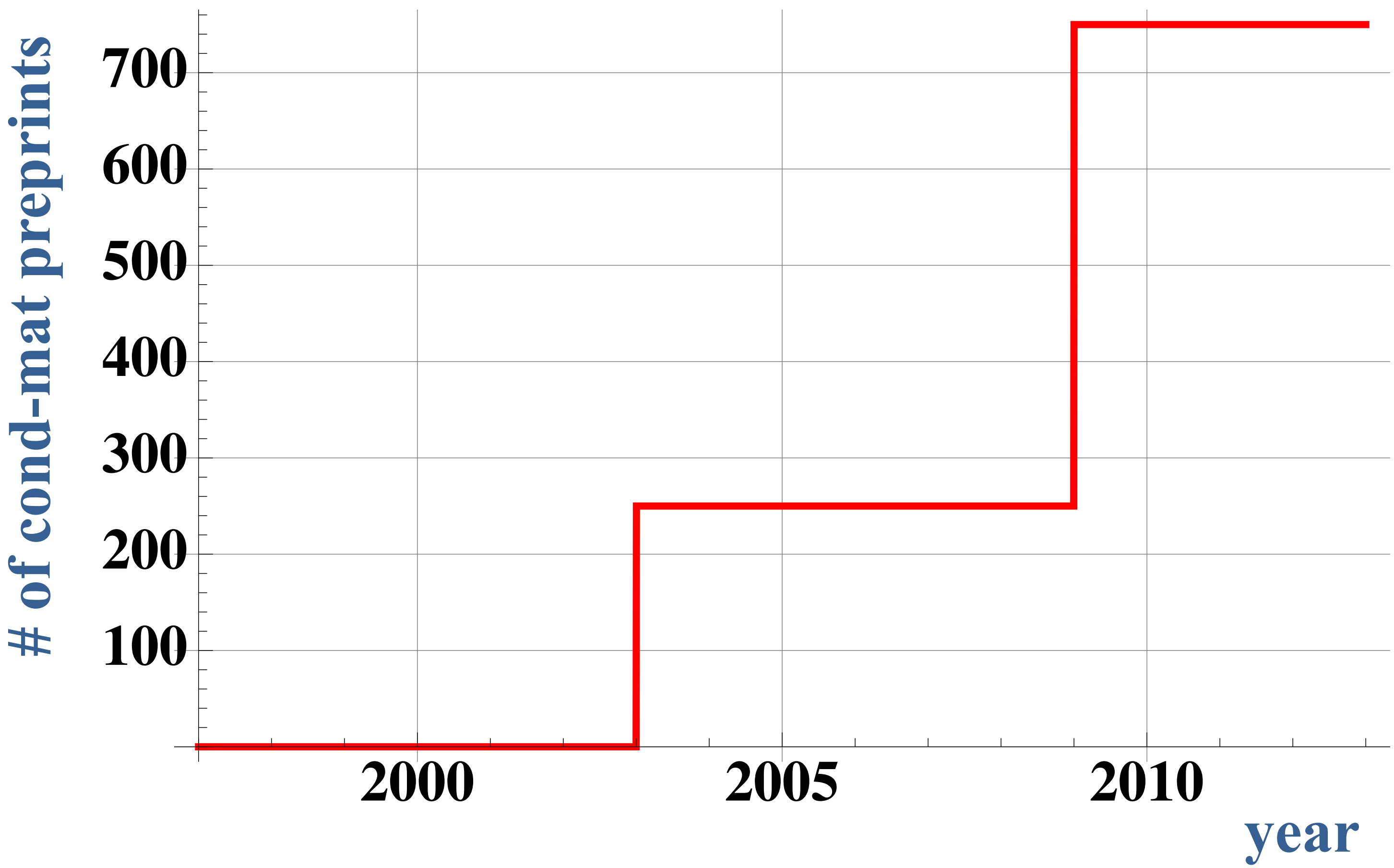
Coherent Many-Body Dynamics

Discovery Channel

A Man Just Tight Rope Walked Across A Gorge Near The Grand Canyon With No Safety Net For 23 Minutes And Survived

Coherent *Many-Body Dynamics*

Experimental access only recently ≈ 2004



Phase transition in the # of publications on the subject

Coherent Many-Body Dynamics: *Quantum Quench*

Q: **What happens to the system in time? Where does it end up as a result of unitary evolution? Does it equilibrate?**

$$|\psi(t \to \infty)\rangle = ? \quad \langle \hat{O}(t \to \infty)\rangle = ?$$

A: **Depends on the system (on *H*)**

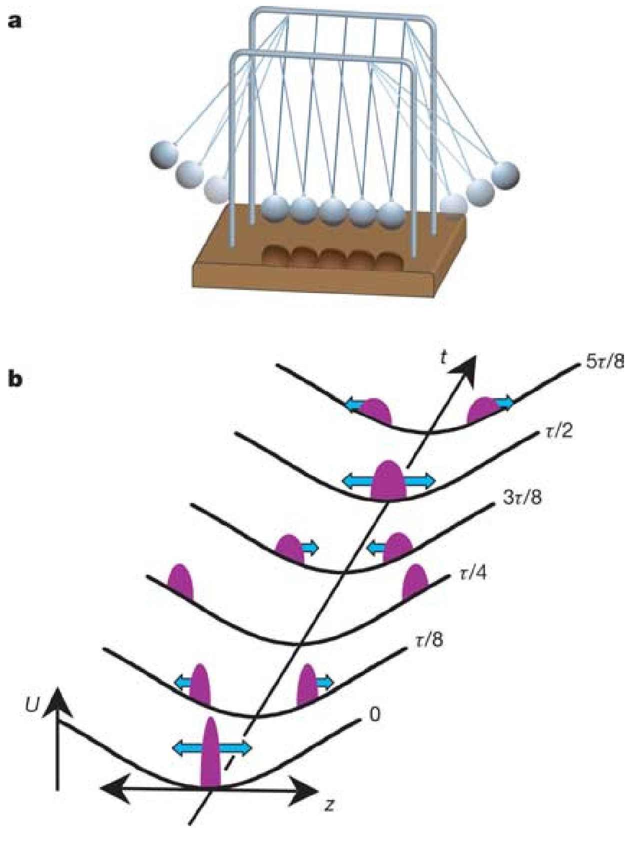
a. **Equilibration (thermalization) with some effective *T***

$$\langle \hat{O}(t \to \infty)\rangle = \mathrm{Tr}\, \hat{O} e^{-\hat{H}/T_{\mathrm{eff}}}$$

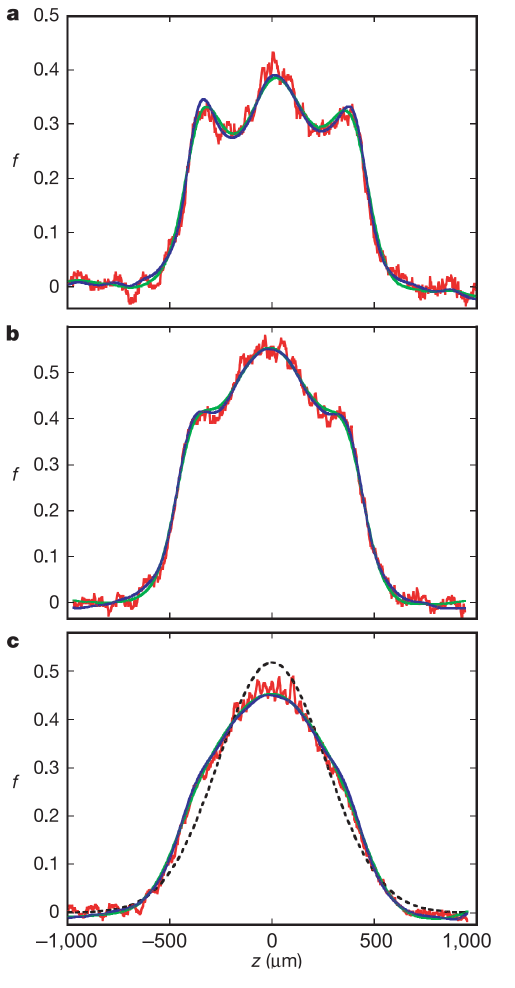
b. ***No equilibration – asymptotic state – nonequilibrium "phase" with properties distinct from equilibrium phases***

$$\langle \hat{O}(t \to \infty)\rangle = ?$$

A quantum Newton's cradle

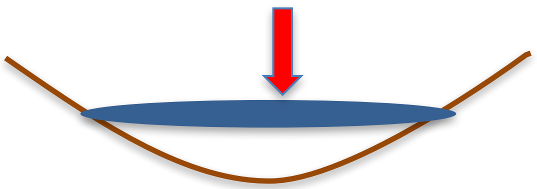


T. Kinoshita, T. Wenger, D. Weiss
Nature (2006)



"^{87}Rb atoms … *do not noticeably equilibrate* even after thousands of collisions. Our results are probably explainable by the well-known fact that a homogeneous 1D Bose gas with point-like collisional interactions is integrable."

Free expansion of interacting 1D Bose gas out of a trap

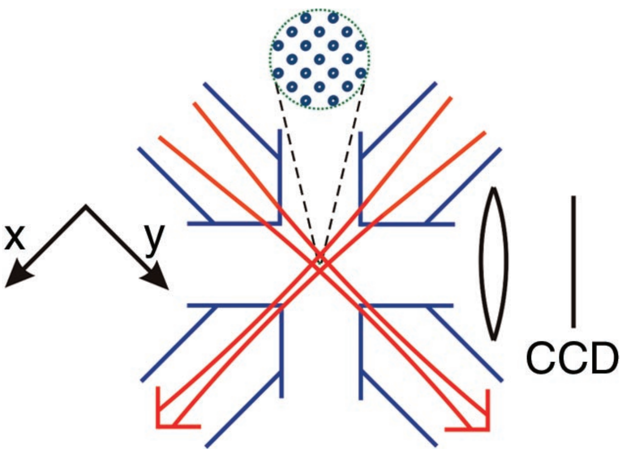


$$H_i = \sum_\alpha \left[\frac{p_\alpha^2}{2} + \frac{\omega^2 x_\alpha^2}{2} \right] + g \sum_{\alpha\beta} \delta(x_\alpha - x_\beta)$$

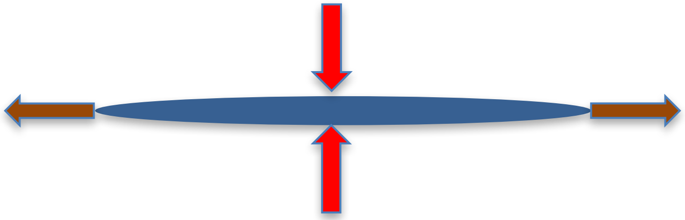
^{87}Rb atoms in a 1D harmonic trap

Kinoshita et. al. Science (2004)

$$|\psi_i\rangle = |\text{ground state of } H_i\rangle$$



Free expansion of interacting 1D Bose gas out of a trap

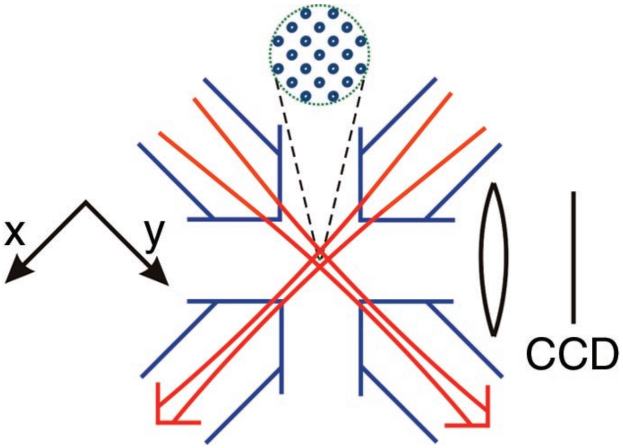


$$H_i = \sum_\alpha \left[\frac{p_\alpha^2}{2} + \frac{\omega^2 x_\alpha^2}{2} \right] + g \sum_{\alpha\beta} \delta(x_\alpha - x_\beta)$$

[87]Rb atoms in a 1D harmonic trap

Kinoshita et. al. Science (2004)

$$|\psi_i\rangle = |\text{ground state of } H_i\rangle$$



At $t = 0$ the trap is removed and the gas expands in 1D governed by:

$$H_f = \sum_\alpha \frac{p_\alpha^2}{2} + g \sum_{\alpha\beta} \delta(x_\alpha - x_\beta)$$

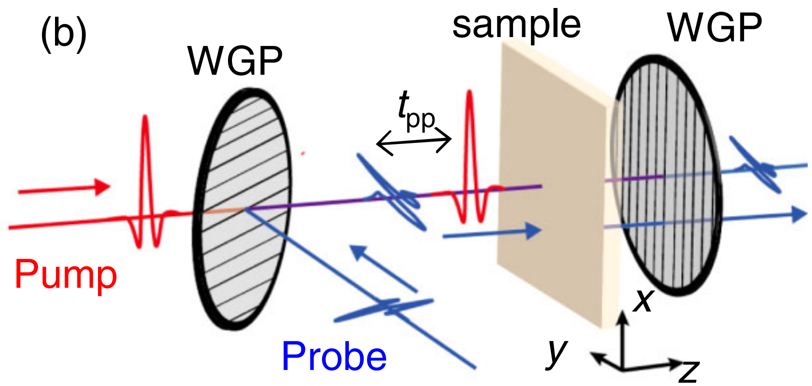
Q: What happens to the system in time? Where does it end up as a result of unitary evolution? Does it equilibrate?

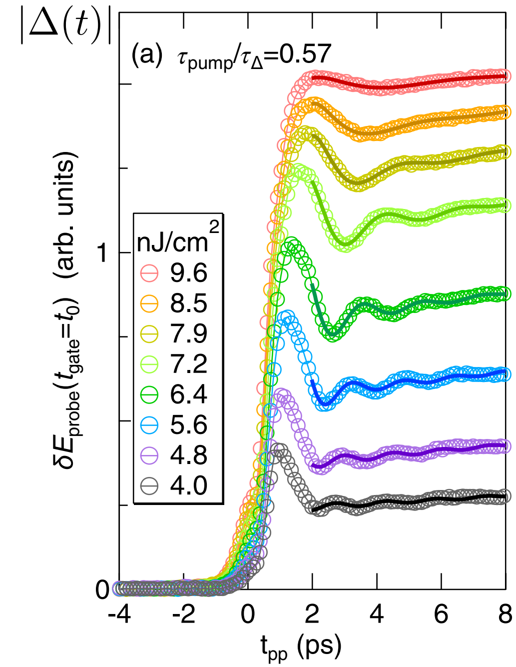
A: Bosons fermionize, momentum distribution approaches Fermi-Dirac, the system does NOT equilibrate (thermalize).

PRL **111,** 057002 (2013) PHYSICAL REVIEW LETTERS week ending 2 AUGUST 2013

Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

Ryusuke Matsunaga,[1] Yuki I. Hamada,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3] Hirotaka Terai,[2] Zhen Wang,[2] and Ryo Shimano[1]





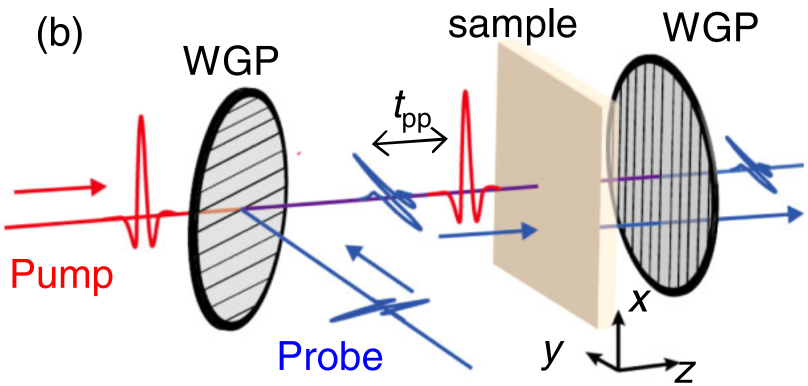
$\tau_\Delta = \hbar/\Delta_0 \approx 3\text{ps}$ – **timescale on which $|\Delta(t)|$ evolves**

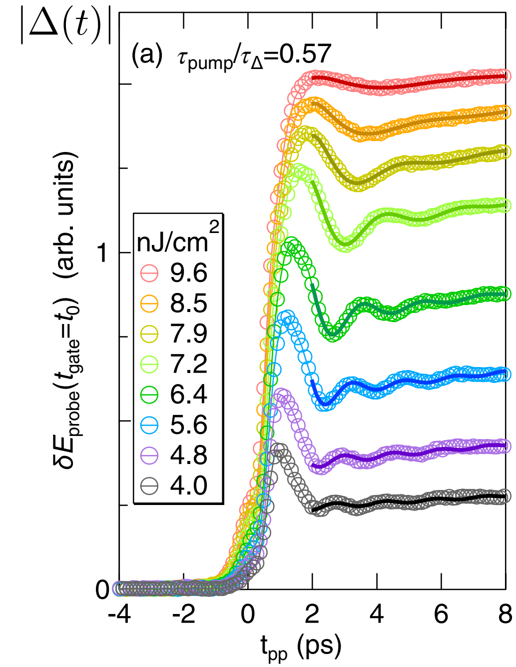
Difficulty: need $\tau_{\text{quench}} \equiv \tau_{\text{pump}} \sim \tau_\Delta$

PRL **111,** 057002 (2013) PHYSICAL REVIEW LETTERS week ending 2 AUGUST 2013

Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

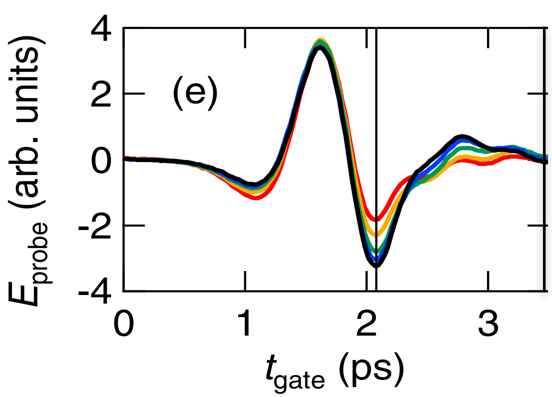
Ryusuke Matsunaga,[1] Yuki I. Hamada,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3]
Hirotaka Terai,[2] Zhen Wang,[2] and Ryo Shimano[1]





$\tau_\Delta = \hbar/\Delta_0 \approx 3\text{ps}$ – **timescale on which** $|\Delta(t)|$ **evolves**

Difficulty: need $\tau_{\text{quench}} \equiv \tau_{\text{pump}} \sim \tau_\Delta$

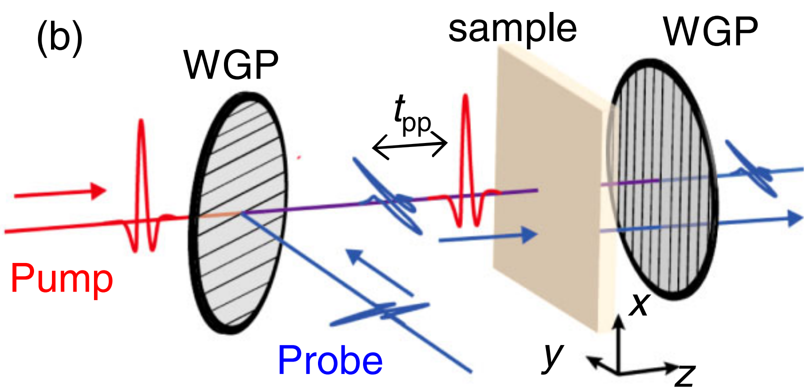


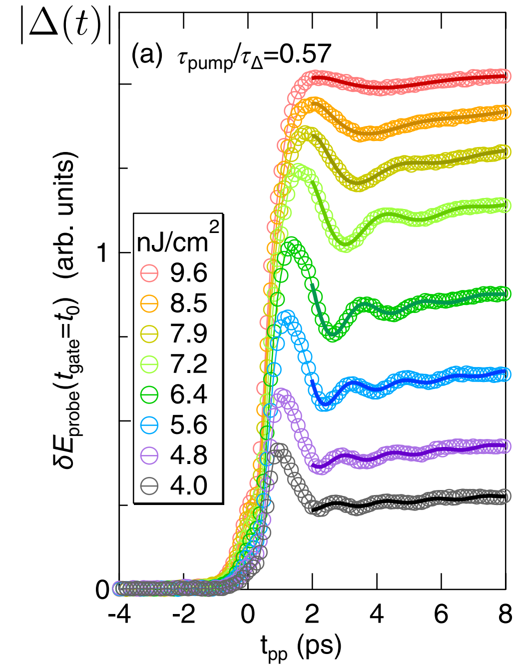
“With the recent development of THz technology, such an intense and monocyclelike THz pulse has become available.”

PRL **111,** 057002 (2013) PHYSICAL REVIEW LETTERS week ending 2 AUGUST 2013

Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

Ryusuke Matsunaga,[1] Yuki I. Hamada,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3]
Hirotaka Terai,[2] Zhen Wang,[2] and Ryo Shimano[1]





$\tau_\Delta = \hbar/\Delta_0 \approx 3\text{ps}$ – **timescale on which $|\Delta(t)|$ evolves**

$$|\psi(0)\rangle = |\text{noneq. state produced by the pulse}\rangle$$

Condensate Hamiltonian

$$\hat{H}_{\rm BCS} \equiv \hat{H}_{\rm 1ch} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{k}\sigma}\hat{c}_{\mathbf{k}\sigma} - g\sum_{\mathbf{k},\mathbf{p}} \hat{c}^\dagger_{\mathbf{k}\uparrow}\hat{c}^\dagger_{-\mathbf{k}\downarrow}\hat{c}_{-\mathbf{p}\downarrow}\hat{c}_{\mathbf{p}\uparrow}$$

$$0 < t \ll \tau_{\rm pb}$$

$$i\frac{d|\psi\rangle}{dt} = \hat{H}_{\rm BCS}|\psi\rangle$$

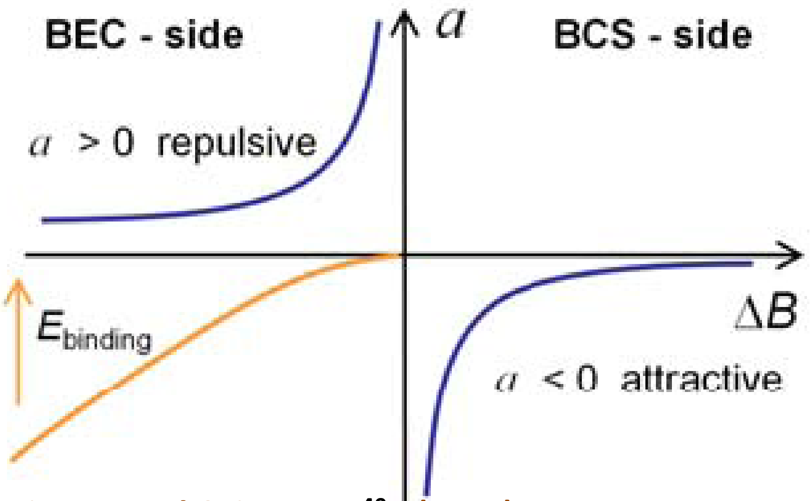
Q: $|\psi(t\to\infty)\rangle = ?\quad \Delta(t\to\infty) = ?$

$t\to\infty$ **means** $\tau_\Delta \ll t \ll \tau_{\rm pb}$

Collisionless (nonadiabatic) regime

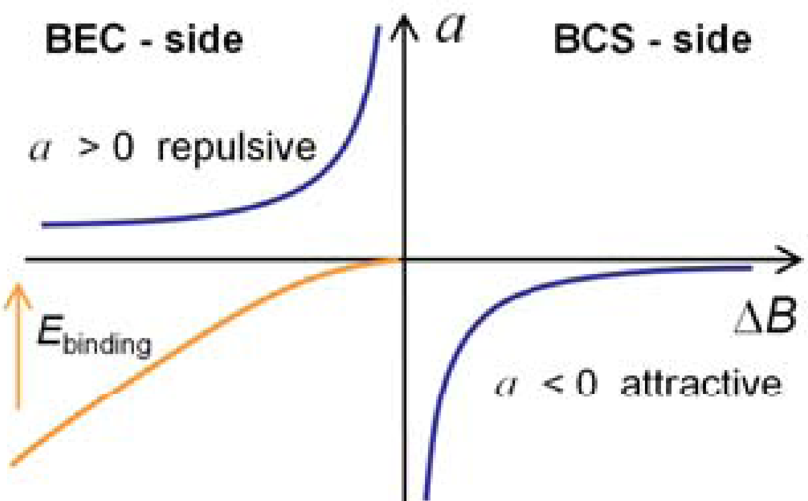
$\tau_{\rm pb}$ – **pair-breaking time. Clean sample, weak coupling:** $\tau_{\rm pb} \gg \tau_\Delta$

Atomic s-wave superfluid – ultracold fermions (^{40}K, ^{6}Li)



Greiner, Regal &Jin, JILA, ^{40}K (2004)

Atomic s-wave superfluid – ultracold fermions (^{40}K, ^{6}Li)

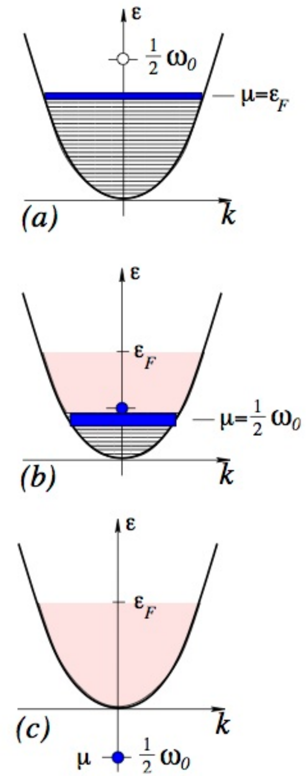


Greiner, Regal &Jin, JILA, ^{40}K (2004)

Detuning: $\omega \approx 2\mu_B(B - B_0)$

Gap: $\Delta = -g\langle b \rangle$

Resonance width: $\gamma = \dfrac{g^2\nu_F}{\epsilon_F}$



Away from unitary point **OR** for narrow resonance the BCS-BEC condensate is described by the 2-channel model

$$\hat{H}_{2\mathrm{ch}} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + \omega b^{\dagger} b + g \sum_{\mathbf{k}} \left(b^{\dagger} \hat{c}_{-\mathbf{k}\downarrow} \hat{c}_{\mathbf{k}\uparrow} + b \hat{c}^{\dagger}_{\mathbf{k}\uparrow} \hat{c}^{\dagger}_{-\mathbf{k}\downarrow} \right)$$

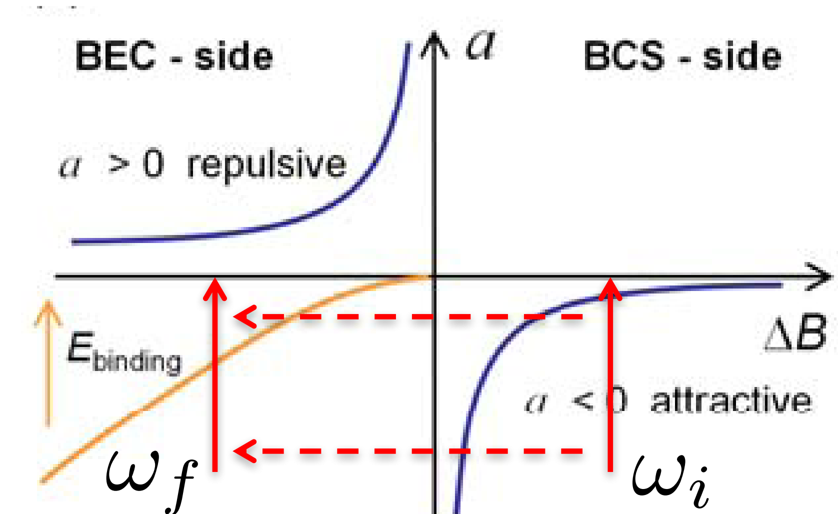
atoms molecules

$(a)\; \omega \gg 2\epsilon_F$ BCS of atoms – no molecules, weakly paired Fermi sea

$(c)\; \omega \ll -\epsilon_F$ BEC of molecules – no atoms, everything condensed into a single mode ***b***

Broad resonance limit $\gamma \to \infty$: $\hat{H}_{2\mathrm{ch}} \to \hat{H}_{1\mathrm{ch}}$, i.e. the BCS model as in THz pulse setup

Atomic s-wave superfluid – ultracold fermions (^{40}K, ^{6}Li)

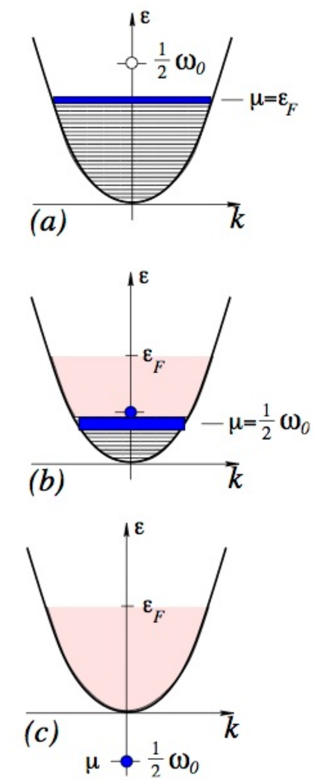


Greiner, Regal &Jin, JILA, ^{40}K (2004)

Detuning: $\omega \approx 2\mu_B(B - B_0)$

Gap: $\Delta = -g\langle b\rangle$

Resonance width: $\gamma = \dfrac{g^2\nu_F}{\epsilon_F}$



Away from unitary point **OR** for narrow resonance the BCS-BEC condensate is described by the 2-channel model

$$\hat{H}_{2\text{ch}} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{k}\sigma}\hat{c}_{\mathbf{k}\sigma} + \omega b^\dagger b + g\sum_{\mathbf{k}}\left(b^\dagger \hat{c}_{-\mathbf{k}\downarrow}\hat{c}_{\mathbf{k}\uparrow} + b\hat{c}^\dagger_{\mathbf{k}\uparrow}\hat{c}^\dagger_{-\mathbf{k}\downarrow}\right)$$

Quantum quench: sudden change of detuning: $\omega_i \to \omega_f$ at $t = 0$

$$|\psi(0)\rangle = |\text{ground state for } \omega_i\rangle$$

$$i\frac{d|\psi\rangle}{dt} = \hat{H}_{2\text{ch}}|\psi\rangle \quad 0 < t \ll \tau_{\text{pb}}$$

Q: $|\psi(t\to\infty)\rangle = ? \quad \Delta(t\to\infty) = ?$

$t \to \infty$ means $\tau_\Delta \ll t \ll \tau_{\text{pb}}$

Away from unitary pt. or $\gamma \to 0$: $\tau_{\text{pb}} \gg \tau_\Delta$

How to address condensate dynamics?

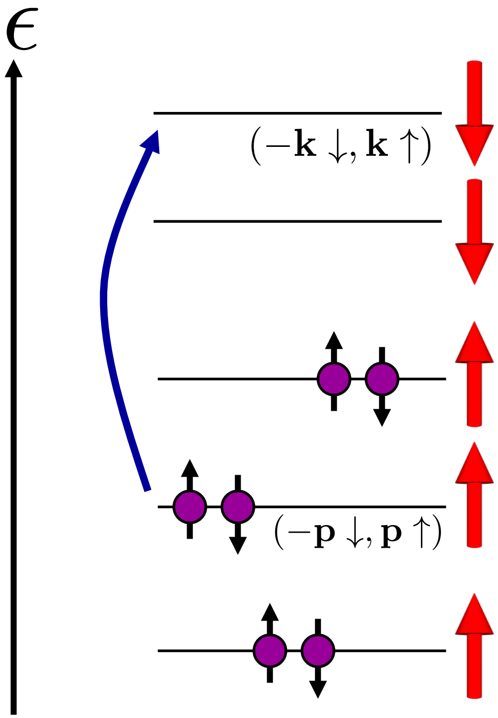
P. W. Anderson

Anderson pseudospins

$$\hat{H}_{\mathrm{BCS}} \equiv \hat{H}_{1\mathrm{ch}} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} - g \sum_{\mathbf{k},\mathbf{p}} \hat{c}^{\dagger}_{\mathbf{k}\uparrow} \hat{c}^{\dagger}_{-\mathbf{k}\downarrow} \hat{c}_{-\mathbf{p}\downarrow} \hat{c}_{\mathbf{p}\uparrow}$$

$$s^z_{\mathbf{k}} = \frac{\hat{n}_{\mathbf{k}} - 1}{2} \qquad s^-_{\mathbf{k}} = \hat{c}_{-\mathbf{k}\downarrow} \hat{c}_{\mathbf{k}\uparrow}, \quad s^+_{\mathbf{k}} = \hat{c}^{\dagger}_{\mathbf{k}\uparrow} \hat{c}^{\dagger}_{-\mathbf{k}\downarrow}$$

$$H_{\mathrm{BCS}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$



P. W. Anderson, Phys. Rev. 112, 1900 (1958)

How to address condensate dynamics?

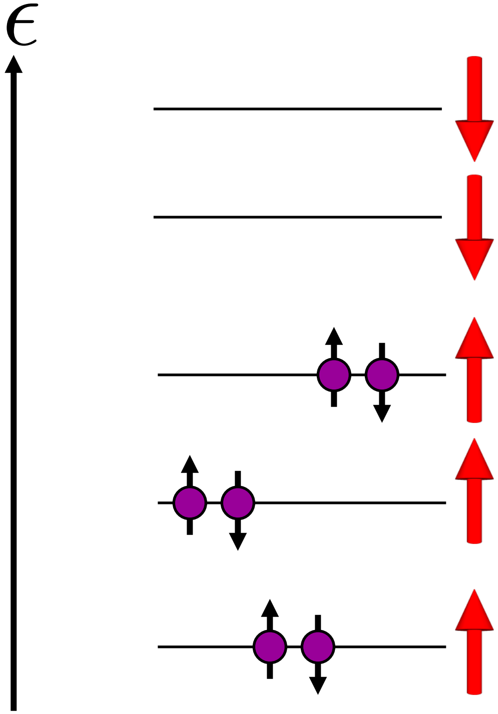
P. W. Anderson

Anderson pseudospins

$$H_{\mathrm{BCS}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

Bloch eqs.

$$i\frac{d|\psi\rangle}{dt} = \hat{H}_{\mathrm{BCS}}|\psi\rangle \Rightarrow \dot{\vec{s}}_{\mathbf{k}} = (-2\vec{\Delta} + 2\epsilon_{\mathbf{k}}\hat{z}) \times \vec{s}_{\mathbf{k}}$$



Order parameter: $$\Delta = g\sum_{\mathbf{k}} s^-_{\mathbf{k}}$$

Complex/vector representation: $$\Delta = \Delta_x - i\Delta_y, \ \vec{\Delta} = \Delta_x\hat{x} + \Delta_y\hat{y}$$

P. W. Anderson, **Phys. Rev. 112, 1900 (1958)**

How to address condensate dynamics?

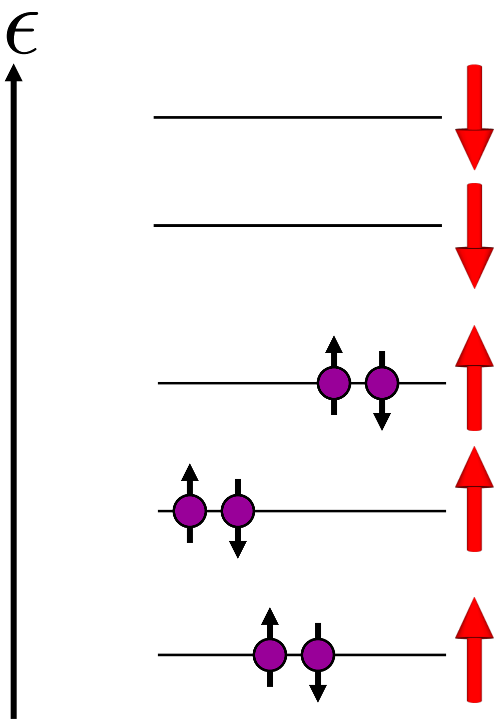
P. W. Anderson

Anderson pseudospins

$$H_{\rm BCS} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

Bloch eqs.

$$i\frac{d|\psi\rangle}{dt} = \hat{H}_{\rm BCS}|\psi\rangle \quad \Rightarrow \quad \dot{\vec{s}}_{\mathbf{k}} = (-2\vec{\Delta} + 2\epsilon_{\mathbf{k}}\hat{z}) \times \vec{s}_{\mathbf{k}}$$



Order parameter: $$\Delta = g\sum_{\mathbf{k}} s^-_{\mathbf{k}}$$

Complex/vector representation: $$\Delta = \Delta_x - i\Delta_y, \quad \vec{\Delta} = \Delta_x\hat{x} + \Delta_y\hat{y}$$

Mean field exact in thermodynamic limit due to the infinite range of interactions. Can replace quantum spins with classical spins (vectors)!

Equilibrium: Anderson (1958); Richardson (1977), etc.

Dynamics: Anderson (1958); Volkov, Kogan (1973); Galaiko (1972), etc.

Quench dynamics: Faribault, Calabrese, Caux (2009)

How to address condensate dynamics?

P. W. Anderson

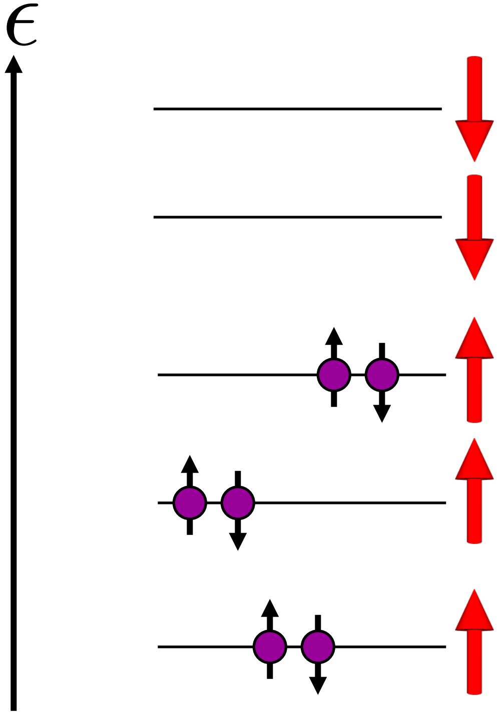
Anderson pseudospins

Inhomogeneous Dicke or Tavis-Cummings model

$$H_{2\mathrm{ch}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} + \omega b^\dagger b + g \sum_{\mathbf{k}} \left(b^\dagger s^-_{\mathbf{k}} + b s^+_{\mathbf{k}}\right)$$

Bloch eqs.

$$i\frac{d|\psi\rangle}{dt} = \hat{H}_{2\mathrm{ch}}|\psi\rangle \Rightarrow \dot{\vec{s}}_{\mathbf{k}} = \left(-2\vec{\Delta} + 2\epsilon_{\mathbf{k}}\hat{z}\right) \times \vec{s}_{\mathbf{k}}$$



Order parameter:

$$\dot{\Delta} = -i\omega\Delta - ig^2 \sum_{\mathbf{k}} s^-_{\mathbf{k}}$$

Complex/vector representation:

$$\Delta = \Delta_x - i\Delta_y, \quad \vec{\Delta} = \Delta_x\hat{x} + \Delta_y\hat{y}$$

$$\Delta = -g\langle b\rangle$$

Mean field exact in thermodynamic limit when *b* is macroscopically occupied. Can replace quantum spins/oscillator with classical spins/oscillator!

Equilibrium: Richardson (1977), Gaudin (1983) etc.
Quench dynamics: Strater, Tsyplyatyev, Faribault (2012)

2D p-wave topological superconductor

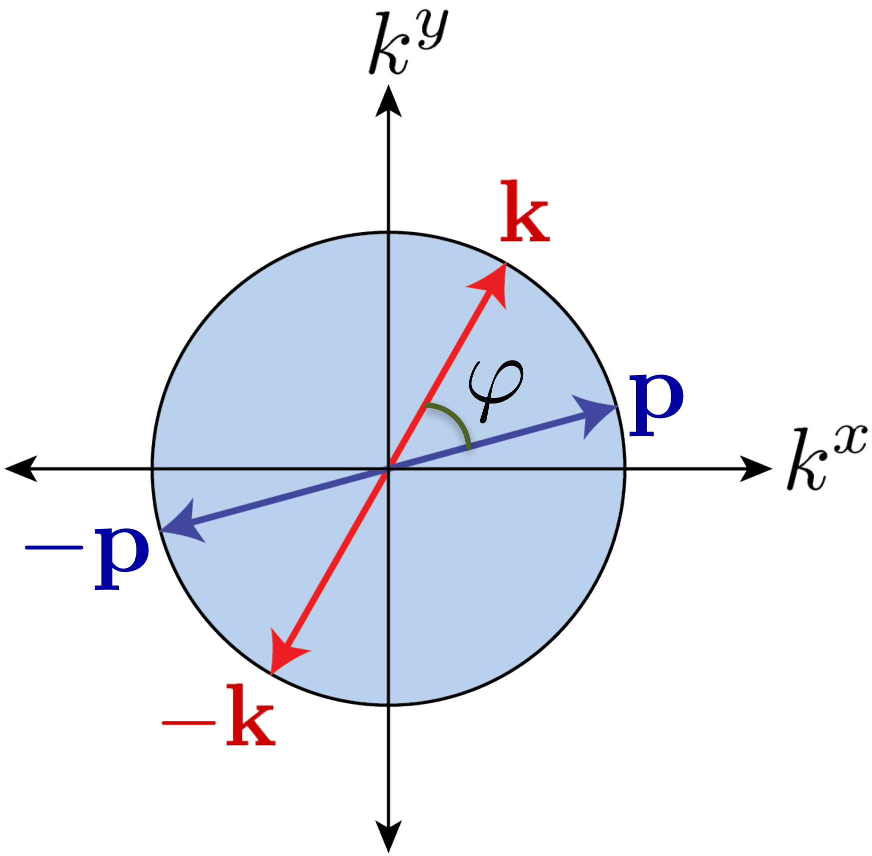
Spinless (or spin-polarized) fermions in 2D: p-wave BCS Hamiltonian

$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k} \cdot \mathbf{p} \, \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}^{\dagger}_{-\mathbf{k}} \hat{c}_{-\mathbf{p}} \hat{c}_{\mathbf{p}}$$

$$V(\varphi) = \sum_{n=-\infty}^{\infty} e^{in\varphi}$$

$n = 0$ – **s-wave**

$n = \pm 1$ – **p-wave**



2D p-wave topological superconductor

Spinless (or spin-polarized) fermions in 2D: p-wave BCS Hamiltonian

$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k} \cdot \mathbf{p} \, \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}^{\dagger}_{-\mathbf{k}} \hat{c}_{-\mathbf{p}} \hat{c}_{\mathbf{p}}$$

<u>"p + i p" superconducting ground state</u>:

$$\Delta(\mathbf{k}) \equiv g \sum_{\mathbf{k}} \mathbf{p} \cdot \mathbf{k} \langle c_{-\mathbf{k}} c_{\mathbf{k}} \rangle = \Delta_0 (k^x - i k^y)$$
$$= \Delta_0 k \exp(-i\phi_k)$$

$$E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + k^2 \Delta_0^2}$$

Fully gapped, non s-wave

2D p-wave topological superconductor

Spinless (or spin-polarized) fermions in 2D: p-wave BCS Hamiltonian

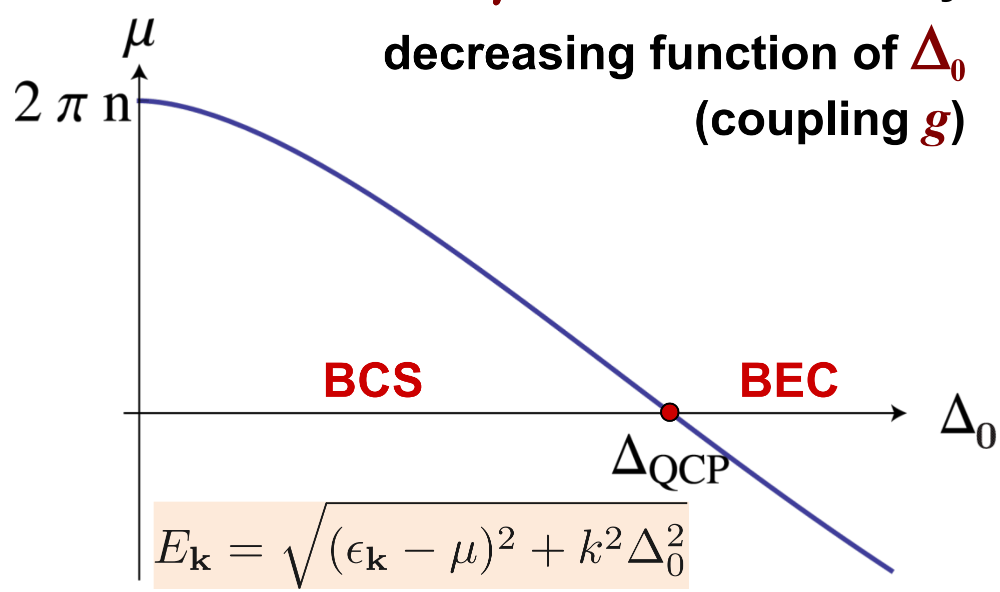
$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k} \cdot \mathbf{p}\, \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}^{\dagger}_{-\mathbf{k}} \hat{c}_{-\mathbf{p}} \hat{c}_{\mathbf{p}}$$

At fixed density n:

- μ **is a monotonically decreasing function of Δ_0 (coupling g)**

"p + i p" superconducting ground state:

$$\Delta(\mathbf{k}) \equiv g \sum_{\mathbf{k}} \mathbf{p} \cdot \mathbf{k} \langle c_{-\mathbf{k}} c_{\mathbf{k}} \rangle = \Delta_0 (k^x - i k^y)$$
$$= \Delta_0 k \exp(-i\phi_k)$$



$$E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + k^2 \Delta_0^2}$$

2D p-wave topological superconductor

Spinless (or spin-polarized) fermions in 2D: p-wave BCS Hamiltonian

$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k}\cdot\mathbf{p}\, \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}^{\dagger}_{-\mathbf{k}} \hat{c}_{-\mathbf{p}} \hat{c}_{\mathbf{p}}$$

Pseudospin representation:

$$H = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^{z}_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k}\cdot\mathbf{p}\, s^{+}_{\mathbf{k}} s^{-}_{\mathbf{p}}$$

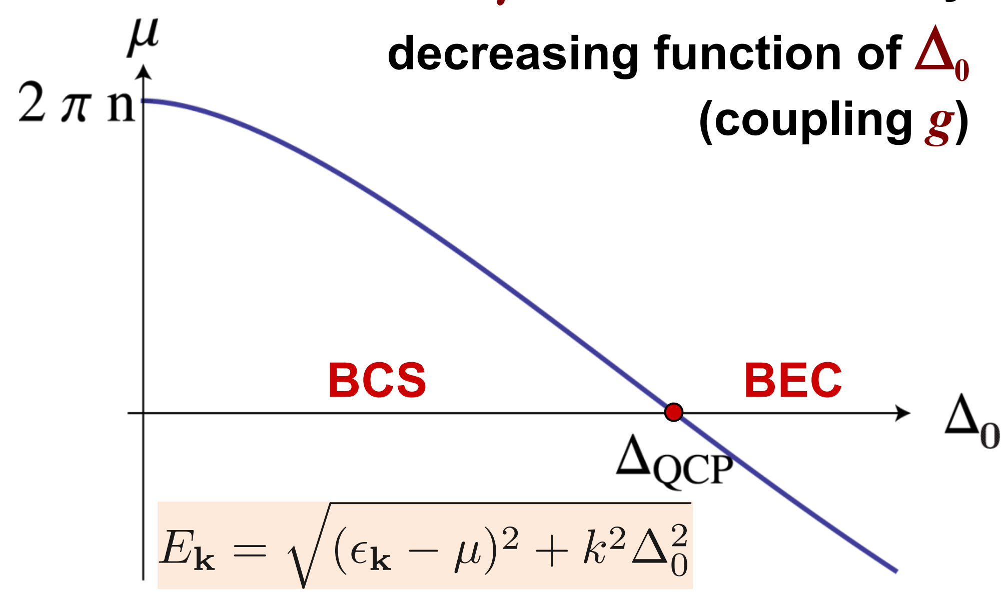
"p + i p" superconducting ground state:

$$\Delta(\mathbf{k}) \equiv g \sum_{\mathbf{k}} \mathbf{p}\cdot\mathbf{k} \langle c_{-\mathbf{k}} c_{\mathbf{k}} \rangle = \Delta_0 (k^x - i k^y)$$
$$= \Delta_0 k \exp(-i\phi_k)$$

$$\Delta(\mathbf{k}) = g \sum_{\mathbf{k}} \mathbf{p}\cdot\mathbf{k} s^{-}_{\mathbf{k}}$$

At fixed density n:

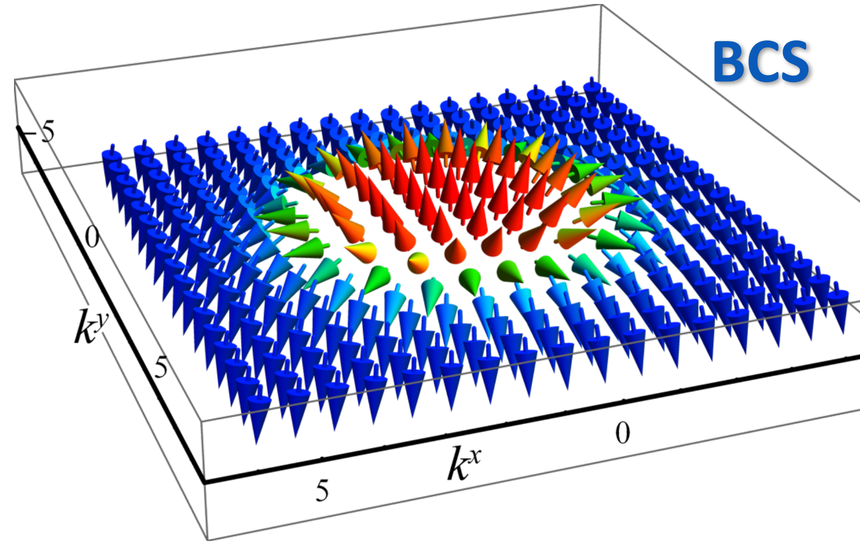
- μ is a monotonically decreasing function of Δ_0 (coupling g)



$$E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + k^2 \Delta_0^2}$$

2D p-wave topological superconductor

$$H = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k}\cdot\mathbf{p}\, s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

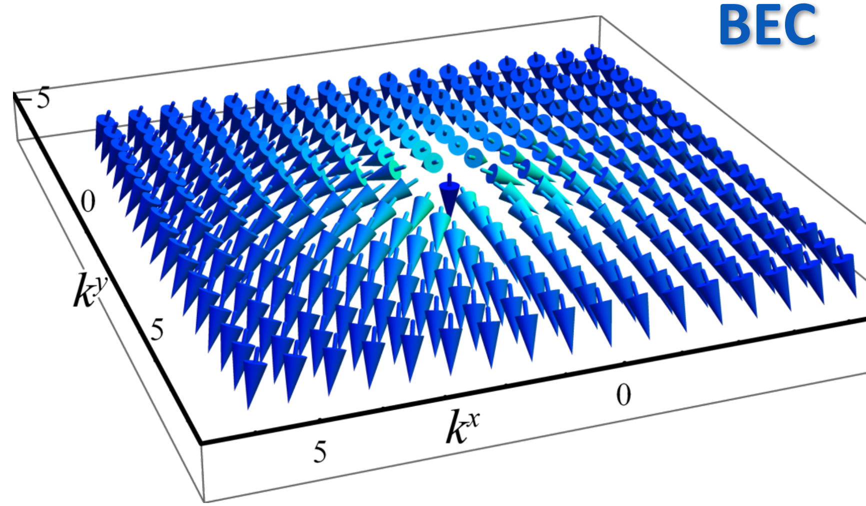


Pseudospin winding number *Q* :

$$Q = \begin{cases} 1, & \mu > 0\,(\Delta_0 < \Delta_{\text{QCP}}) \quad \text{BCS} \\ 0, & \mu < 0\,(\Delta_0 > \Delta_{\text{QCP}}) \quad \text{BEC} \end{cases}$$

Volovik (1988); Read & Green (2000)

- ***Weak-pairing*** **BCS state topologically non-trivial**
- ***Strong-pairing*** **BEC state topologically trivial**



Retarded GF winding number *W* :

$$W \equiv \frac{\pi\epsilon^{\alpha\beta\gamma}}{3}\text{Tr}\int_{\omega,\mathbf{k}} \left(\hat{G}^{-1}\partial_{k^\alpha}\hat{G}\right)\left(\hat{G}^{-1}\partial_{k^\beta}\hat{G}\right)\left(\hat{G}^{-1}\partial_{k^\gamma}\hat{G}\right)$$

- ✓ **Same as pseudospin winding *Q* in ground state**
- ✓ **Signals presence of chiral edge states**

Far from equilibrium topological superconductivity?

2D weak-pairing BCS p+ip superconductor: Fully-gapped, "strong" topological state (class D)

$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{c}^\dagger_{\mathbf{k}} \hat{c}_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k} \cdot \mathbf{p} \, \hat{c}^\dagger_{\mathbf{k}} \hat{c}^\dagger_{-\mathbf{k}} \hat{c}_{-\mathbf{p}} \hat{c}_{\mathbf{p}}$$

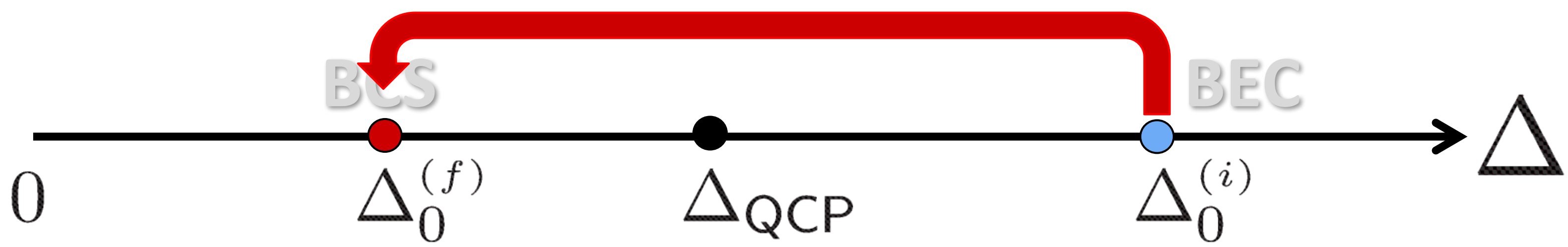
Quantum quench: sudden change of interaction strength: $g_i \rightarrow g_f$ **at** $t = 0$

$$|\psi(0)\rangle = |\ (p+ip) \text{ ground state for } g_i \rangle$$

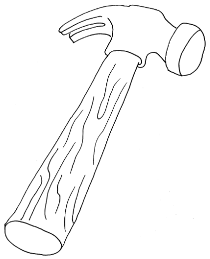
$$i \frac{d|\psi\rangle}{dt} = \hat{H}|\psi\rangle$$

Q: $|\psi(t \rightarrow \infty)\rangle = ? \quad \Delta(t \rightarrow \infty) = ?$

Is topological order robust against hard nonequilibrium driving???



How to address condensate dynamics?



Superconductor

Δ_0

?

Quantum Quench

$$H_{2ch} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} + \omega b^\dagger b + g \sum_{\mathbf{k}} (b^\dagger s^-_{\mathbf{k}} + b s^+_{\mathbf{k}})$$

$$H_{BCS} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

$$H = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k}\cdot\mathbf{p}\, s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

p-wave

Need to solve full (infinitely) many classical spin evolution:

$$\dot{\vec{s}}_{\mathbf{k}} = (-2\vec{\Delta} + 2\epsilon_{\mathbf{k}}\hat{z}) \times \vec{s}_{\mathbf{k}}$$

2-channel

$$\dot{\Delta} = -i\omega\Delta - ig^2 \sum_{\mathbf{k}} s^-_{\mathbf{k}}$$

1-channel = BCS

$$\Delta = g \sum_{\mathbf{k}} s^-_{\mathbf{k}}$$

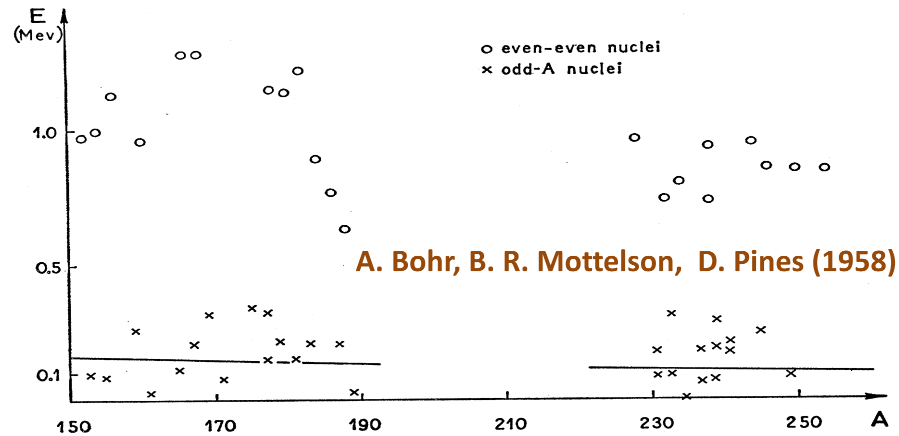
p-wave

$$\Delta(\mathbf{k}) = g \sum_{\mathbf{k}} \mathbf{p}\cdot\mathbf{k}\, s^-_{\mathbf{k}}$$

Initial state, $\vec{s}_{\mathbf{k}}(t=0) = \ldots,$ determined by quench (perturbation) details

Nonlinear, many-body, far from equilibrium – normally would be intractable analytically

But it turns out all these pairing models are integrable…



A. Bohr, B. R. Mottelson, D. Pines (1958)

BCS = 1-channel model

(nuclear superconductivity)

Richardson & Sherman (1964)

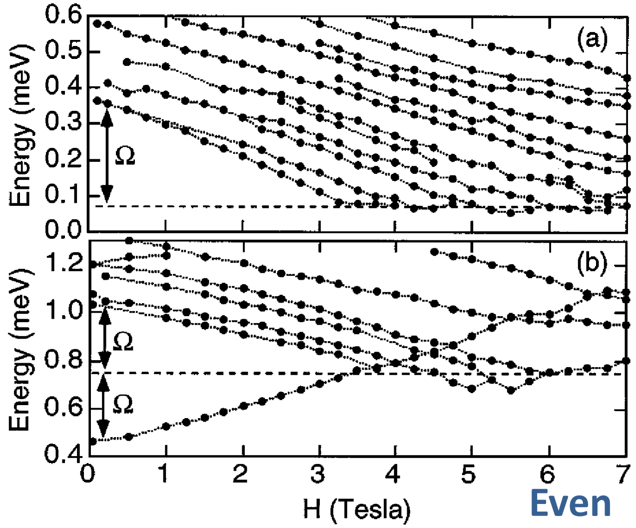
"Exact eigenstates of the pairing-force Hamiltonian"

Inhomogeneous Dicke = 2-channel model

Gaudin (1983)

"La fonction d'onde de Bethe"

Black, Ralph, Tinkham (1996)



Topological (p+ip) superconductor

Richardson (2002)

"New Class of Solvable and Integrable Many-Body Models"

Dunning, Ibanez, Links, Sierra & Zhao (2010)

"Exact solution of the p+ip pairing Hamiltonian…"

Rombouts, Dukelsky, and Ortiz (2010)

"…integrable p+ip fermionic superfluid"

Applications to superconducting qubits (finite size corrections to the BCS theory): Von Delft (2001); Dukelsky & Sierra (1999); Schechter et. al. (2001) …

Integrals of motion for $H_{2\mathrm{ch}}$ – Gaudin magnets

$$H_{\mathbf{p}} = (2\epsilon_{\mathbf{p}} - \omega)s^z_{\mathbf{p}} + g\left(b^\dagger s^-_{\mathbf{p}} + b s^+_{\mathbf{p}}\right) + g^2 \sum_{\mathbf{q}\neq\mathbf{p}} \frac{\vec{s}_{\mathbf{p}}\cdot\vec{s}_{\mathbf{q}}}{\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{q}}}, \qquad \hat{N} = b^\dagger b + \sum_{\mathbf{p}} s^z_{\mathbf{p}}$$

$$H_{2\mathrm{ch}} = \omega\hat{N} + \sum_{\mathbf{p}} H_{\mathbf{p}}$$

Richardson-Gaudin integrability:
quantum/equilibrium/finite size – Bethe Ansatz like solution for the spectrum, Richardson (1964); Gaudin (1983).

Integrals of motion for $H_{2\mathrm{ch}}$ – Gaudin magnets

$$H_{\mathbf{p}} = (2\epsilon_{\mathbf{p}} - \omega)s^z_{\mathbf{p}} + g\left(b^\dagger s^-_{\mathbf{p}} + b s^+_{\mathbf{p}}\right) + g^2 \sum_{\mathbf{q}\neq\mathbf{p}} \frac{\vec{s}_{\mathbf{p}}\cdot\vec{s}_{\mathbf{q}}}{\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{q}}}, \qquad \hat{N} = b^\dagger b + \sum_{\mathbf{p}} s^z_{\mathbf{p}}$$

$$H_{2\mathrm{ch}} = \omega\hat{N} + \sum_{\mathbf{p}} H_{\mathbf{p}}$$

Richardson-Gaudin integrability: quantum/equilibrium/finite size – Bethe Ansatz like solution for the spectrum, Richardson (1964); Gaudin (1983).

Condensate dynamics: nonequilibrium/thermodynamic (continuum) limit/classical

Need to solve full (infinitely) many classical spin evolution:

$$\dot{\vec{s}}_{\mathbf{k}} = (-2\vec{\Delta} + 2\epsilon_{\mathbf{k}}\hat{z}) \times \vec{s}_{\mathbf{k}} \qquad \dot{\Delta} = -i\omega\Delta - ig^2\sum_{\mathbf{k}} s^-_{\mathbf{k}}$$

Nonlinear integrable PDE, cf. Korteweg–de Vries, nonlinear Schrodinger, Landau-Lifshitz, sine-Gordon etc. Difference – nonlocal (integro-differential), no translational invariance. Requires a nonstandard approach.

Integrals of motion for $H_{2\text{ch}}$ – Gaudin magnets

$$H_{\mathbf{p}} = (2\epsilon_{\mathbf{p}} - \omega)s^z_{\mathbf{p}} + g\left(b^\dagger s^-_{\mathbf{p}} + b s^+_{\mathbf{p}}\right) + g^2 \sum_{\mathbf{q}\neq\mathbf{p}} \frac{\vec{s}_{\mathbf{p}}\cdot\vec{s}_{\mathbf{q}}}{\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{q}}}, \qquad \hat{N} = b^\dagger b + \sum_{\mathbf{p}} s^z_{\mathbf{p}}$$

$$H_{2\text{ch}} = \omega\hat{N} + \sum_{\mathbf{p}} H_{\mathbf{p}}$$

Advanced approach to "Richardson-Gaudin" integrability (secret life of Gaudin models):

Sklyanin (1987) *"Separation of variables in the Gaudin model"*
Kuznetsov (1992) *"Quadrics on real Riemannian spaces … connection with Gaudin magnet"*
Takasaki (1998) *"Gaudin Model, KZ Equation, and Isomonodromic Problem on Torus"*
Frenkel (2004) *"Gaudin model and opers"*
⋮

Exact solution for condensate dynamics:

BCS s-wave

E.Y., Altshuler, Kuznetsov, Enolskii, J. Phys. A (2005)
E.Y., Altshuler, Tsyplyatyev, PRL (2006)

p-wave

Foster, Dzero, Gurarie, E.Y., PRB (2014)
Foster, Gurarie, Dzero, E.Y., PRL (2014)

2-channel model

E.Y., Dzero, Gurarie, Foster, PRA (2015)

Q: **Can we explicitly determine the large time dynamics after a quench from the exact solution?** $|\psi(t \to \infty)\rangle = ? \quad \Delta(t \to \infty) = ?$

A: **For realistic (e.g. quench) initial data the exact solution is too complicated to be directly useful**

But… there is a remarkable # of degrees of freedom reduction mechanism. In thermodynamic limit the system "flows in time" to a small number m of new "renormalized" spins. "RG in time" in exact solution (new technique in the theory of classical integrability)

Q: **Can we explicitly determine the large time dynamics after a quench from the exact solution?** $|\psi(t\to\infty)\rangle = ?\quad \Delta(t\to\infty) = ?$

A: **For realistic (e.g. quench) initial data the exact solution is too complicated to be directly useful**

But… there is a remarkable # of degrees of freedom reduction mechanism. In thermodynamic limit the system "flows in time" to a small number m of new "renormalized" spins. "RG in time" in exact solution (new technique in the theory of classical integrability)

At $t\to\infty$ the # of spins effectively drops from $n=\infty$ to m. For a quench in any of the above pairing models $m=0,1$ or 2 depending on the strength of the quench.

$$\Delta(t\to\infty) \text{ after quench } = \Delta(t) \text{ for } H^{\text{red}}$$

H^{red} = same Hamiltonian, but for $m=0,1,2$ new spins

Example: $H_{2\mathrm{ch}}$ $m=2$

$$H_{2\mathrm{ch}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} + \omega b^\dagger b + g \sum_{\mathbf{k}} \left(b^\dagger s^-_{\mathbf{k}} + b s^+_{\mathbf{k}}\right)$$

$$\Delta(t) = -g b(t)$$

$$\Downarrow \; t \to \infty$$

$$b = b_x - i b_y,$$
$$\vec{b} = b_x \hat{x} + b_y \hat{y}$$

$$H^{\mathrm{red}}_{2\mathrm{ch}} = 2\widetilde{\epsilon} S^z + \widetilde{\omega} b^\dagger b + g\left(b^\dagger S^- + b S^+\right)$$

$$\dot{\vec{S}} = (-2\vec{\Delta} + 2\widetilde{\epsilon}\hat{z}) \times \vec{S}, \quad \dot{\Delta} = -i\widetilde{\omega}\Delta - i g^2 S^-$$

$$\vec{s}_{\mathbf{k}}(t) = \alpha_{\mathbf{k}} \vec{S}(t) + \beta_{\mathbf{k}} \vec{b}(t) + \gamma_{\mathbf{k}} \hat{z}$$

- **This is a particular solution of original eqs. of motion**
- **This $\Delta(t)$ is realized at large times for certain quenches**
- **α_k, β_k, γ_k,... are determined by the integrals of motion**

$$\Delta(t \to \infty) \text{ after quench} = \Delta(t) \text{ for } H^{\mathrm{red}}$$

$$H^{\mathrm{red}} = \text{same Hamiltonian, but for } m = 0, 1, 2 \text{ new spins}$$

Example: $H_{2\mathrm{ch}}$ $m=2$

$$H_{2\mathrm{ch}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} + \omega b^\dagger b + g \sum_{\mathbf{k}} \left(b^\dagger s^-_{\mathbf{k}} + b s^+_{\mathbf{k}}\right)$$

$$\Delta(t) = -g b(t)$$

$$b = b_x - i b_y,$$
$$\vec{b} = b_x \hat{x} + b_y \hat{y}$$

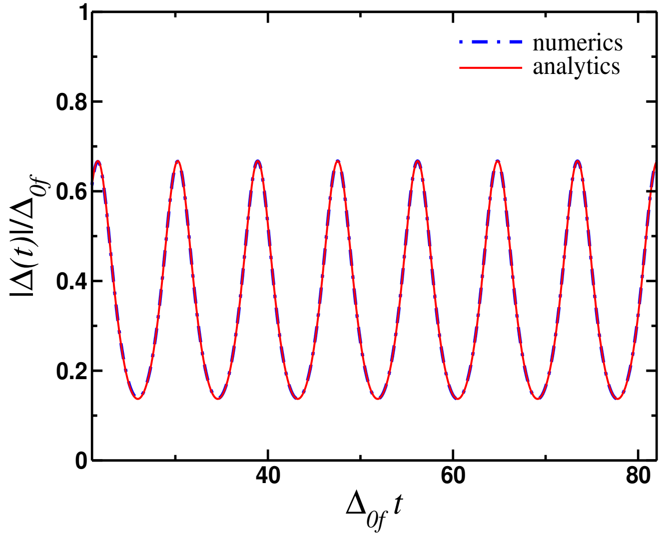
$$\Downarrow \; t \to \infty$$

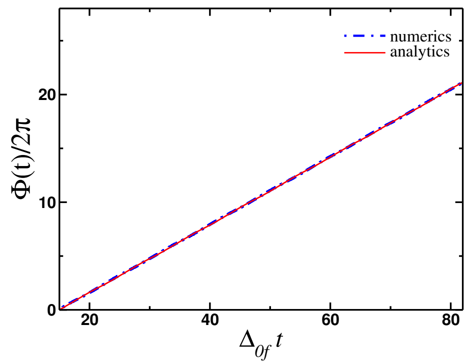
$$H^{\mathrm{red}}_{2\mathrm{ch}} = 2\widetilde{\epsilon} S^z + \widetilde{\omega} b^\dagger b + g\left(b^\dagger S^- + b S^+\right)$$

$$\dot{\vec{S}} = \left(-2\vec{\Delta} + 2\widetilde{\epsilon}\hat{z}\right) \times \vec{S}, \qquad \dot{\Delta} = -i\widetilde{\omega}\Delta - i g^2 S^-$$

Long time asymptote of the order parameter:

$$|\Delta(t)| = \sqrt{a + b^2 \mathrm{dn}^2[bt]},$$
$$\Phi(t) = \dots$$





$$\Delta(t \to \infty) \text{ after quench} = \Delta(t) \text{ for } H^{\mathrm{red}}$$

$$H^{\mathrm{red}} = \text{same Hamiltonian, but for } m = 0, 1, 2 \text{ new spins}$$

Example: $H_{2\text{ch}}$ $m=1$

$$H_{2\text{ch}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} + \omega b^\dagger b + g \sum_{\mathbf{k}} (b^\dagger s^-_{\mathbf{k}} + b s^+_{\mathbf{k}})$$

$$\Delta(t) = -g b(t)$$

$$\Big\Downarrow \; t \to \infty$$

$$b = b_x - i b_y,$$
$$\vec{b} = b_x \hat{x} + b_y \hat{y}$$

$$H^{\text{red}}_{2\text{ch}} = \widetilde{\omega} b^\dagger b$$

Redefine:

$$\dot{\Delta} = -i\widetilde{\omega}\Delta \qquad \widetilde{\omega} \equiv 2\mu_\infty$$

Long time asymptote of the order parameter:

$$\Delta(t\to\infty) = \Delta_\infty e^{-2i\mu_\infty t}$$

Original spins:

$$\vec{s}_{\mathbf{k}}(t) = \beta_{\mathbf{k}} b(t) + \gamma_{\mathbf{k}} \hat{z}$$

- This is a particular solution of original eqs. of motion
- This $\Delta(t)$ is realized **at large times** for certain quenches
- β_k, γ_k,… are determined by the integrals of motion

$$\Delta(t\to\infty) \text{ after quench} = \Delta(t) \text{ for } H^{\text{red}}$$

$$H^{\text{red}} = \text{same Hamiltonian, but for } m = 0, 1, 2 \text{ new spins}$$

Example: H_{2ch} $m=0$

$$H_{2\mathrm{ch}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} + \omega b^\dagger b + g \sum_{\mathbf{k}} \left(b^\dagger s^-_{\mathbf{k}} + b s^+_{\mathbf{k}}\right)$$

$$\Delta(t) = -g b(t)$$

$$b = b_x - i b_y,$$
$$\vec{b} = b_x \hat{x} + b_y \hat{y}$$

$$\Downarrow \; t \to \infty$$

$$H^{\mathrm{red}}_{2\mathrm{ch}} = 0$$

$$\Delta = 0$$

Original spins:

$$\vec{s}_{\mathbf{k}} = \gamma_{\mathbf{k}} \hat{z}$$

Long time asymptote of the order parameter: $\Delta(t \to \infty) = 0$

- **This is a particular solution of original eqs. of motion**
- **This $\Delta(t)$ is realized at large times for certain quenches**
- **γ_k,… are determined by the integrals of motion**

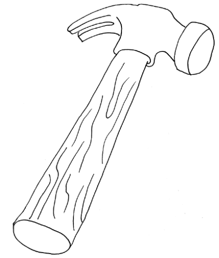
$$\dot{\vec{s}}_{\mathbf{k}} = (-2\vec{\Delta} + 2\epsilon_{\mathbf{k}} \hat{z}) \times \vec{s}_{\mathbf{k}}$$

$$\dot{\Delta} = -i\omega\Delta - i g^2 \sum_{\mathbf{k}} s^-_{\mathbf{k}}$$

$$\Delta(t \to \infty) \text{ after quench} = \Delta(t) \text{ for } H^{\mathrm{red}}$$

$$H^{\mathrm{red}} = \text{same Hamiltonian, but for } m = 0, 1, 2 \text{ new spins}$$

Q: What happens to the system in time? Where does it end up as a result of unitary evolution? Does it equilibrate?



Superconductor

Δ_0

?

A:

I. No equilibration (thermalization) at all

II. System goes into an asymptotic state with properties quite distinct from equilibrium (new "phase" of superfluid matter).

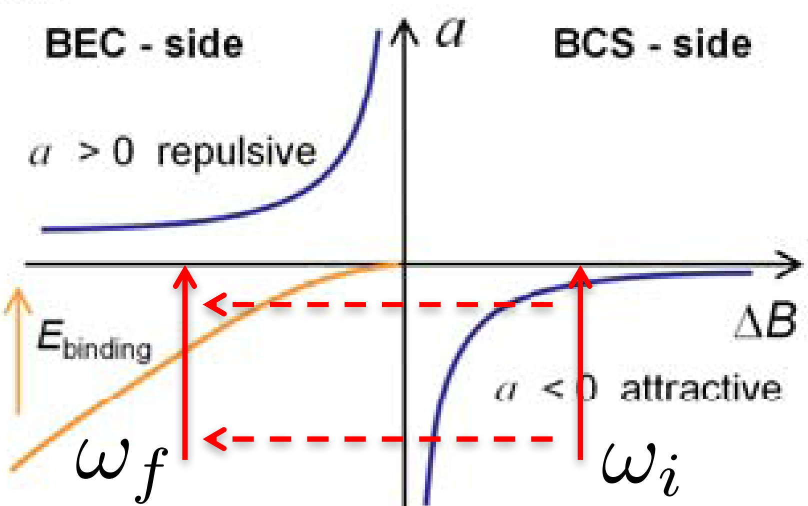
III. Three main far from equilibrium "phases" (as opposed to only one in equilibrium at $T=0$) common to all our models

IV. Which "phase" is realized depends on the strength of the quench

V. Not specific to integrable models. More general mechanism at work. Consider e.g. [Scaramazza, E.Y. (2018)]

$$H=\sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s_{\mathbf{k}}^z+\omega b^\dagger b+\sum_{\mathbf{k}} g_{\mathbf{k}}\left(b^\dagger s_{\mathbf{k}}^- + b s_{\mathbf{k}}^+\right), \quad g_{\mathbf{k}} \text{ – any momentum-dependent coupling}$$

Atomic s-wave superfluid – ultracold fermions (^{40}K, ^{6}Li)



Greiner, Regal &Jin, JILA, ^{40}K (2004)

Detuning: $\omega \approx 2\mu_B(B - B_0)$

Gap: $\Delta(t) = -gb(t)$

Resonance width: $\gamma = \dfrac{g^2\nu_F}{\epsilon_F}$

Away from unitary point OR for narrow resonance

$$H_{2\mathrm{ch}} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} + \omega b^\dagger b + g \sum_{\mathbf{k}} \left(b^\dagger s^-_{\mathbf{k}} + b s^+_{\mathbf{k}}\right)$$

Quantum quench: sudden change of detuning: $\omega_i \to \omega_f$ at $t = 0$

Each quench is characterized by three parameters: $\omega_i, \omega_f, \gamma$

Equivalently can choose: $\Delta_{0i}, \Delta_{0f}, \gamma$

Δ_{0i}, Δ_{0f} – *ground state* **gaps for** ω_i, ω_f

Exact quench phase diagram: 2-channel model in 3d

Phases II and II':

Order parameter amplitude goes to a constant

$$\Delta(t) \to \Delta_\infty e^{-2i\mu_\infty t},$$
$$\Delta_\infty \neq \Delta_{0f}$$

Phase I:

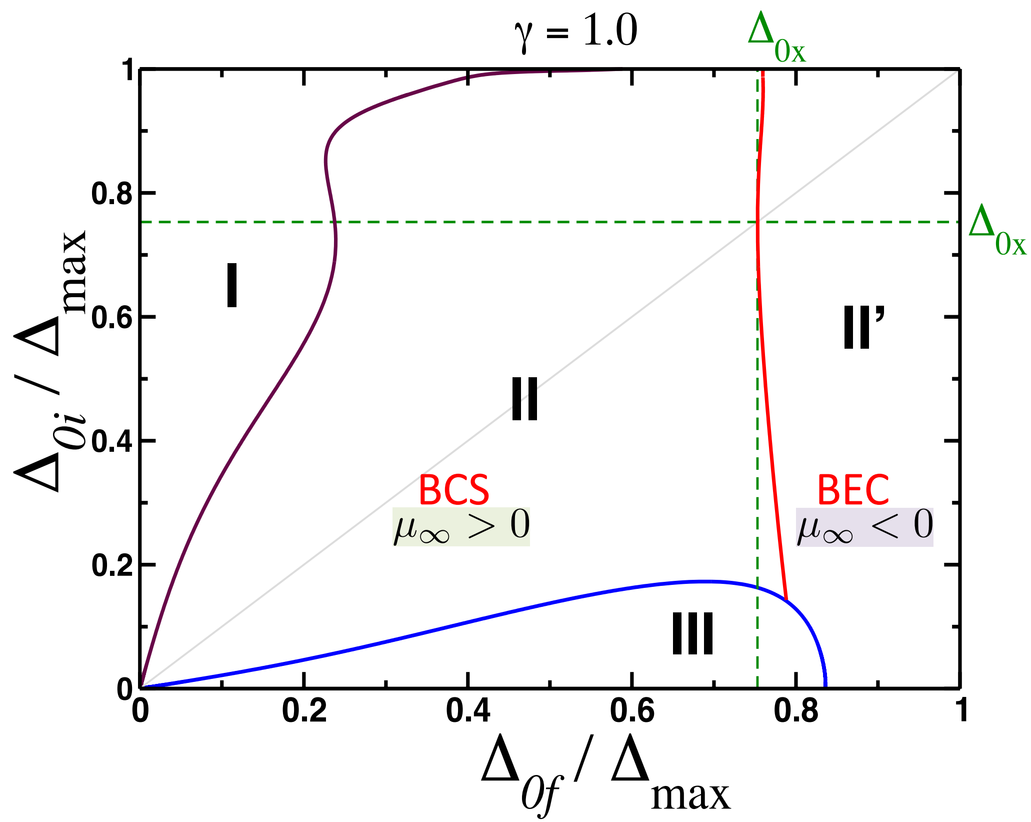
Order parameter vanishes, but nonzero superfluid stiffness (gapless superconductivity)

$$\Delta(t) \to 0,\ n_s = n/2$$

Phase III:

Order parameter amplitude oscillates periodically

$$|\Delta(t)| \to \sqrt{a + b^2 \mathrm{dn}^2\left[bt, k'\right]}$$



Asymptotic states of 2-channel dynamics

Δ_{0i}, Δ_{0f} – ***ground state* gaps for** ω_i, ω_f

$\mu_\infty = 0$ line

$\omega_i \to \omega_f$ **at** $t = 0$

Exact quench phase diagram: 2-channel model in 3d

Phases II and II':

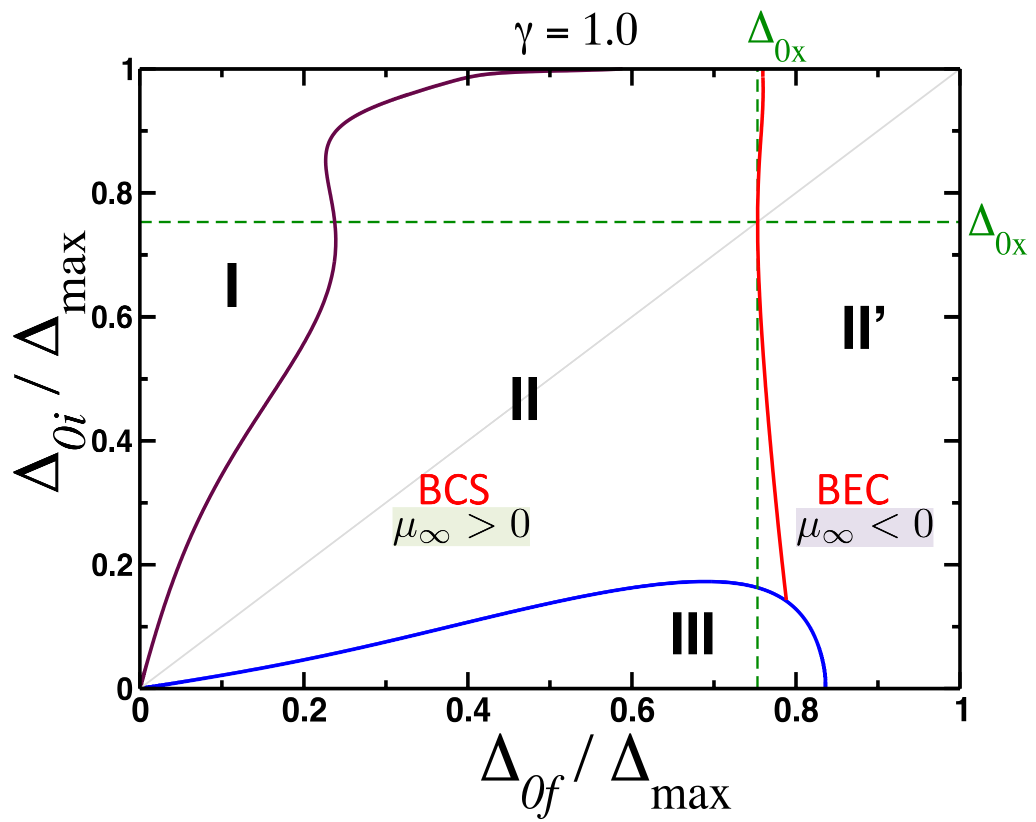
Order parameter amplitude goes to a constant

$$\Delta(t) \to \Delta_\infty e^{-2i\mu_\infty t},$$
$$\Delta_\infty \neq \Delta_{0f}$$

$$\mu_\infty > 0$$
$$|\Delta(t)| = \Delta_\infty + a\frac{\cos(2\Delta_\infty t + \alpha)}{\sqrt{\Delta_\infty t}}$$

$$\mu_\infty < 0$$
$$|\Delta(t)| = \Delta_\infty + b\frac{\cos(2\omega_{\min} t + \alpha)}{(\Delta_\infty t)^{3/2}}$$
$$\omega_{\min} = \sqrt{\mu_\infty^2 + \Delta_\infty^2}$$

E.Y., Dzero, Gurarie, Foster, PRB (2015)



Asymptotic states of 2-channel dynamics

Δ_{0i}, Δ_{0f} – *ground state* gaps for ω_i, ω_f

$\mu_\infty = 0$ line

$\omega_i \to \omega_f$ at $t = 0$

Exact quench phase diagram: s-wave BCS in 3d

Phases II and II':

Order parameter amplitude goes to a constant

$$\Delta(t) \to \Delta_\infty e^{-2i\mu_\infty t}, \quad \Delta_\infty \neq \Delta_{0f}$$

$$\mu_\infty > 0$$

$$|\Delta(t)| = \Delta_\infty + a\frac{\cos(2\Delta_\infty t + \alpha)}{\sqrt{\Delta_\infty t}}$$

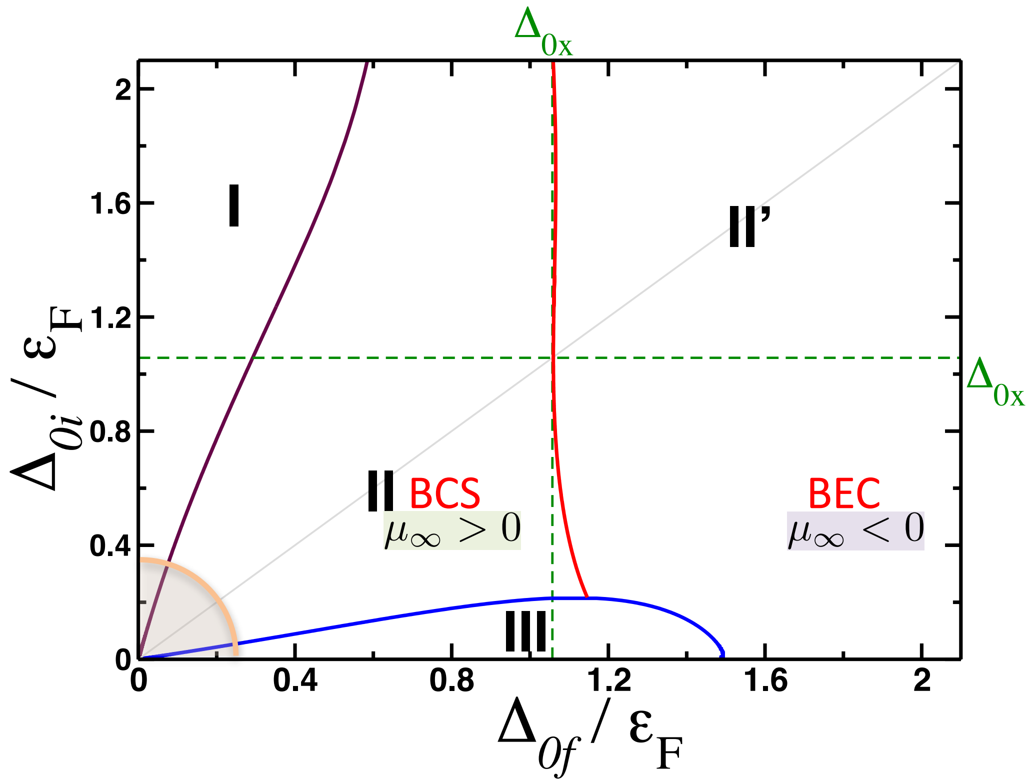
E.Y., Tsyplyatyev, Altshuler, PRL (2006)

$$\mu_\infty < 0$$

$$|\Delta(t)| = \Delta_\infty + b\frac{\cos(2\omega_{\min} t + \alpha)}{(\Delta_\infty t)^{3/2}}$$

$$\omega_{\min} = \sqrt{\mu_\infty^2 + \Delta_\infty^2}$$

E.Y., Dzero, Gurarie, Foster, PRB (2015)



Asymptotic states of 2-channel dynamics

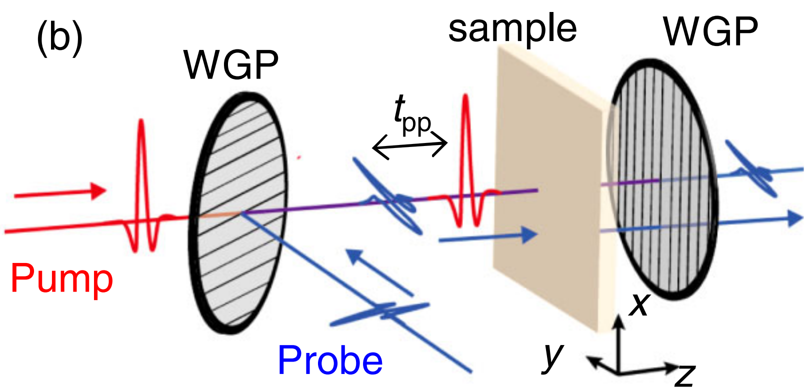
Δ_{0i}, Δ_{0f} – ***ground state* gaps for** ω_i, ω_f

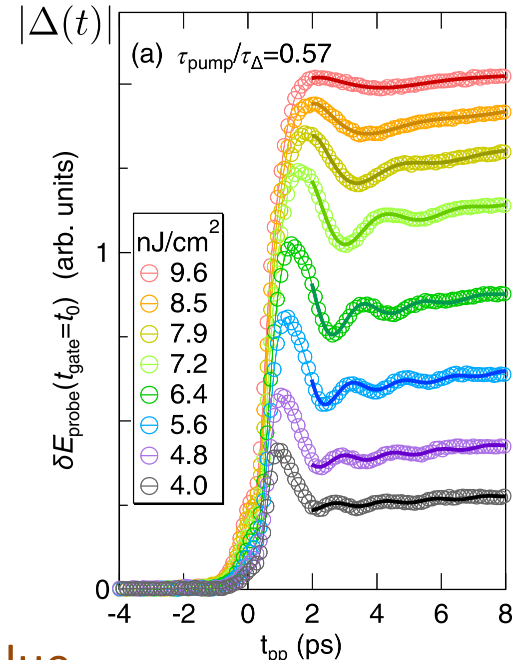
$\mu_\infty = 0$ line

$\omega_i \to \omega_f$ **at** $t = 0$

Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

Ryusuke Matsunaga,[1] Yuki I. Hamada,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3] Hirotaka Terai,[2] Zhen Wang,[2] and Ryo Shimano[1]





"…oscillation frequency is in excellent accordance with the value of the asymptotic gap energy… The results are well accounted for by the theoretically anticipated BCS order parameter oscillation"

$$\mu_\infty > 0$$

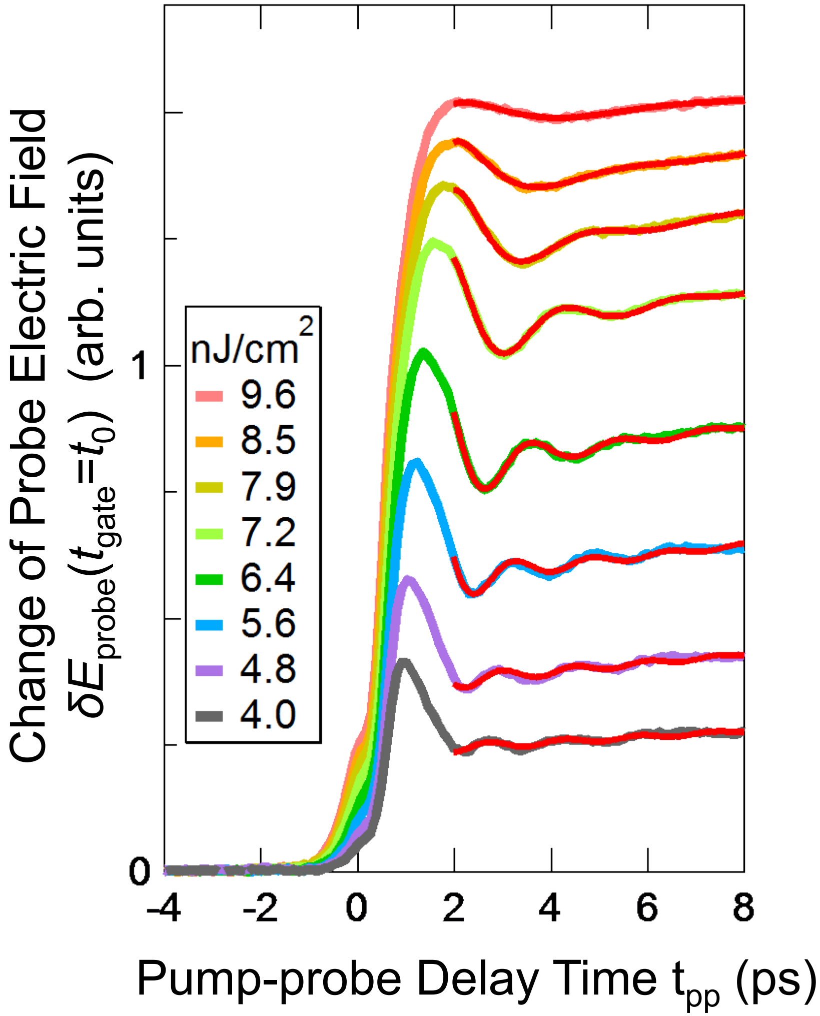
$$|\Delta(t)| = \Delta_\infty + a\frac{\cos(2\Delta_\infty t + \alpha)}{\sqrt{\Delta_\infty t}}$$

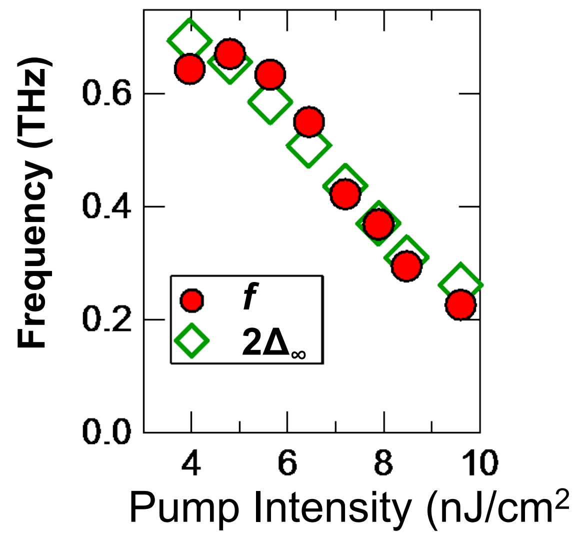
E.Y., Tsyplyatyev, Altshuler, PRL (2006)

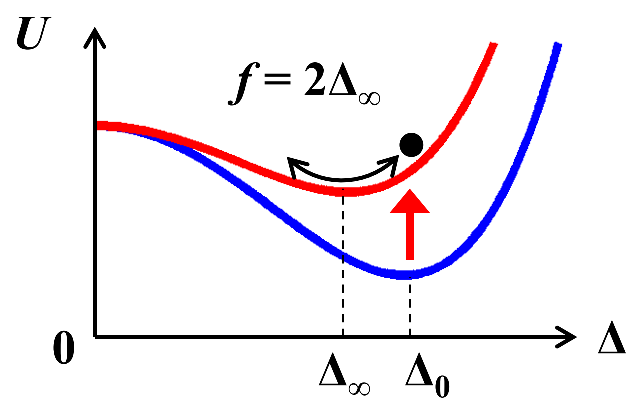
Order parameter dynamics

$$\delta\Delta(t_{pp}) = C_1 + C_2 t_{pp} + \frac{a}{(t_{pp})^b}\cos(2\pi f t_{pp} + \phi)$$

E. Yuzbashyan et al., PRL **96**, 230404 (2006).







R. Matsunaga et al., PRL**111**, 057002 (2013)

Slide from R. Shimano's talk at Rice conference "Interacting quantum systems driven out of equilibrium", May, 2016

Exact quench phase diagram: 2-channel model in 2d

Phases II and II':

Order parameter amplitude goes to a constant

$$\Delta(t) \to \Delta_\infty e^{-2i\mu_\infty t}, \quad \Delta_\infty \neq \Delta_{0f}$$

Phase I:

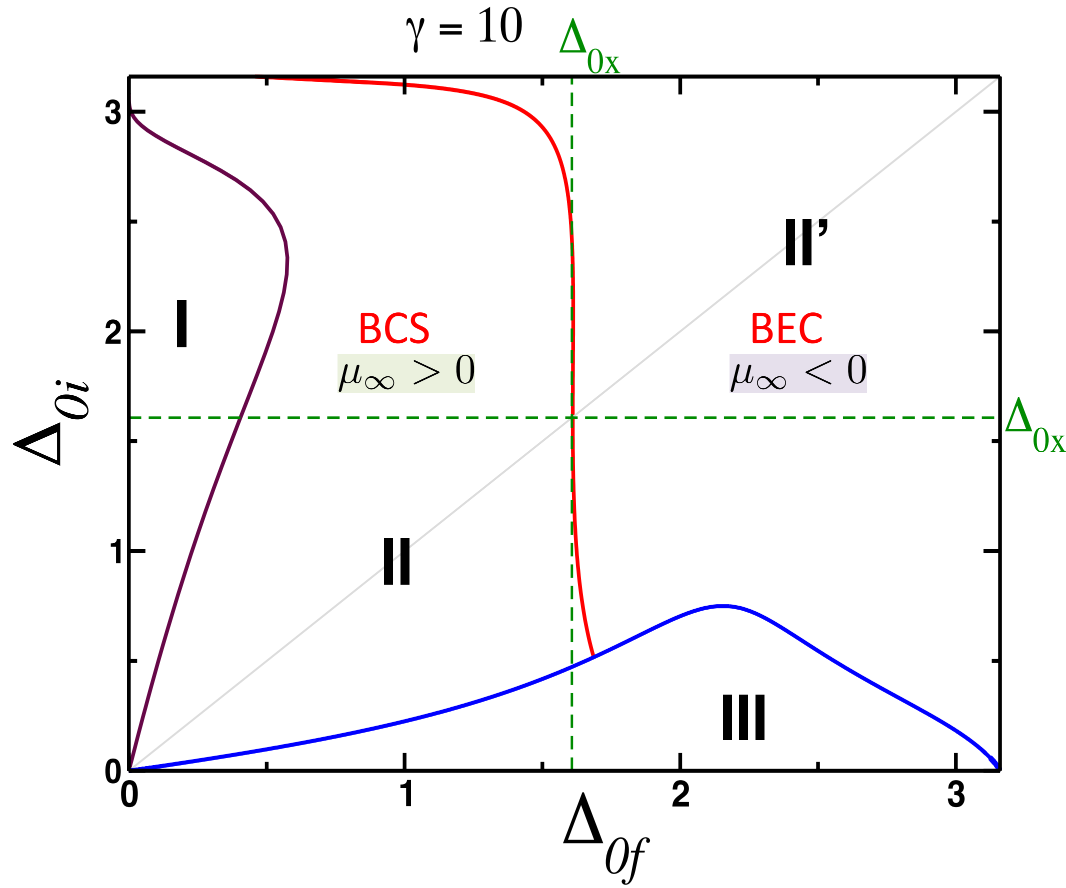
Order parameter vanishes, but nonzero superfluid stiffness (gapless superconductivity)

$$\Delta(t) \to 0, \; n_s = n/2$$

Phase III:

Order parameter amplitude oscillates periodically

$$|\Delta(t)| \to \sqrt{a + b^2 \mathrm{dn}^2\left[bt, k'\right]}$$



Asymptotic states of 2-channel dynamics

Δ_{0i}, Δ_{0f} – *ground state* gaps for ω_i, ω_f

$\mu_\infty = 0$ line

$\omega_i \to \omega_f$ at $t = 0$

Exact quench phase diagram: 2-channel model in 2d

Phases II and II':

Order parameter amplitude goes to a constant

$$\Delta(t) \to \Delta_\infty e^{-2i\mu_\infty t}, \quad \Delta_\infty \neq \Delta_{0f}$$

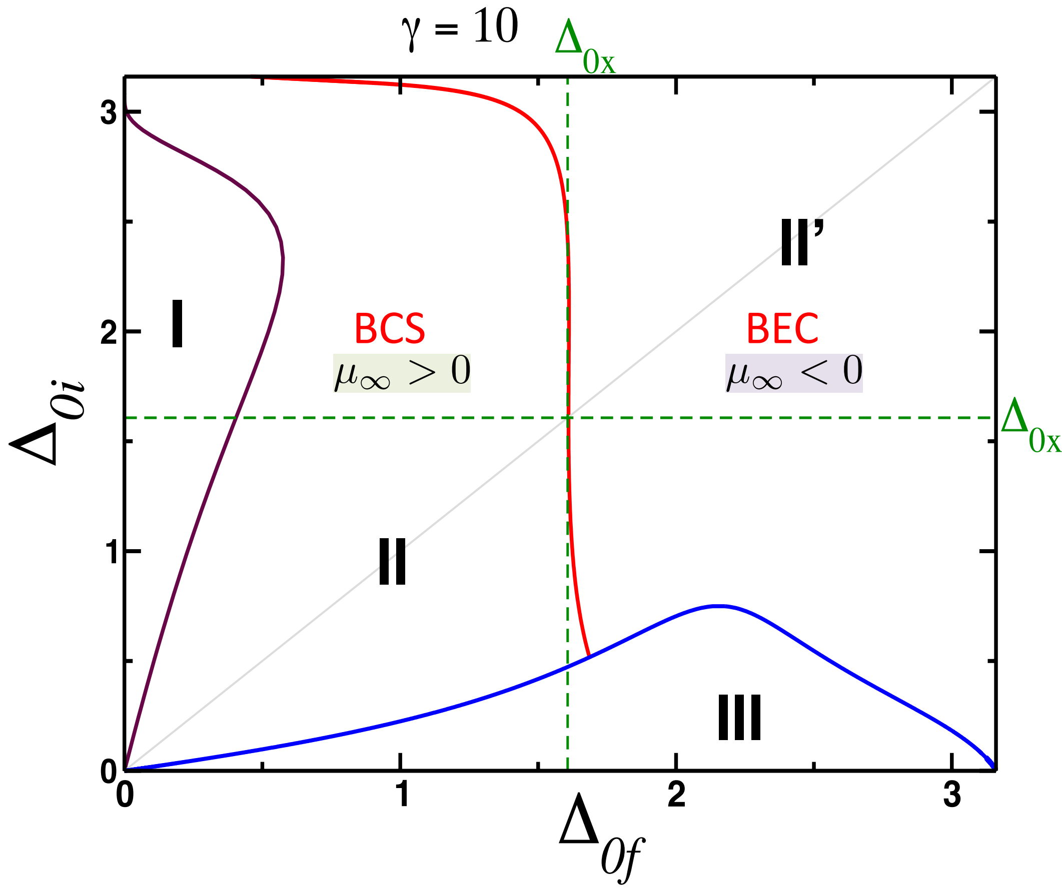
$\mu_\infty > 0$

$$|\Delta(t)| = \Delta_\infty + a\frac{\cos(2\Delta_\infty t + \alpha)}{\sqrt{\Delta_\infty t}}$$

$\mu_\infty < 0$

$$|\Delta(t)| = \Delta_\infty - b\frac{\sin(2\omega_{\min} t)}{t\ln^2(\epsilon_F t)}$$

$$\omega_{\min} = \sqrt{\mu_\infty^2 + \Delta_\infty^2}$$



Asymptotic states of 2-channel dynamics

Δ_{0i}, Δ_{0f} – ***ground state* gaps for** ω_i, ω_f

$\mu_\infty = 0$ line

$\omega_i \to \omega_f$ **at** $t = 0$

Far from equilibrium topological superconductivity?

2D weak-pairing BCS p+ip superconductor: Fully-gapped, "strong" topological state (class D)

$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k} \cdot \mathbf{p} \, \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}^{\dagger}_{-\mathbf{k}} \hat{c}_{-\mathbf{p}} \hat{c}_{\mathbf{p}}$$

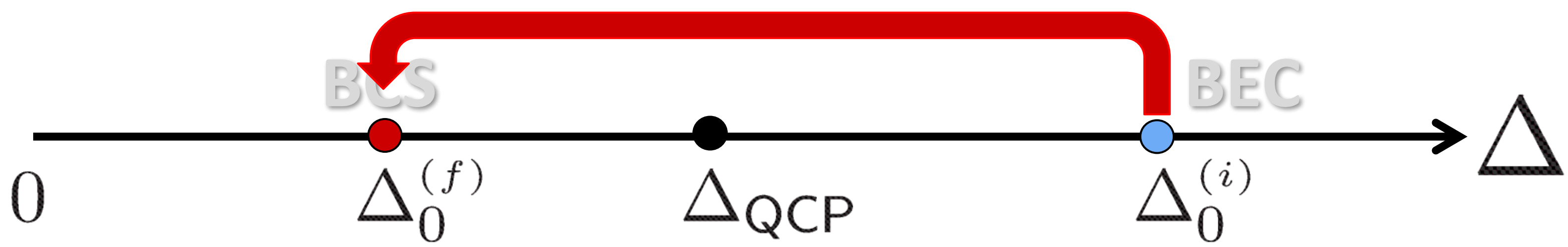
Quantum quench: sudden change of interaction strength: $g_i \rightarrow g_f$ **at** $t = 0$

$$|\psi(0)\rangle = |\ (p + ip)\ \text{ground state for}\ g_i\rangle$$

$$i \frac{d|\psi\rangle}{dt} = \hat{H}|\psi\rangle$$

Q: $|\psi(t \rightarrow \infty)\rangle = ? \quad \Delta(t \rightarrow \infty) = ?$

Is topological order robust against hard nonequilibrium driving???



2D p-wave topological superconductor

$$H = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k}\cdot\mathbf{p}\, s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

Pseudospin winding number *Q* :

$$Q = \begin{cases} 1, & \mu > 0\,(\Delta_0 < \Delta_{\mathrm{QCP}}) \quad \mathrm{BCS} \\ 0, & \mu < 0\,(\Delta_0 > \Delta_{\mathrm{QCP}}) \quad \mathrm{BEC} \end{cases}$$

Volovik (1988); Read & Green (2000)

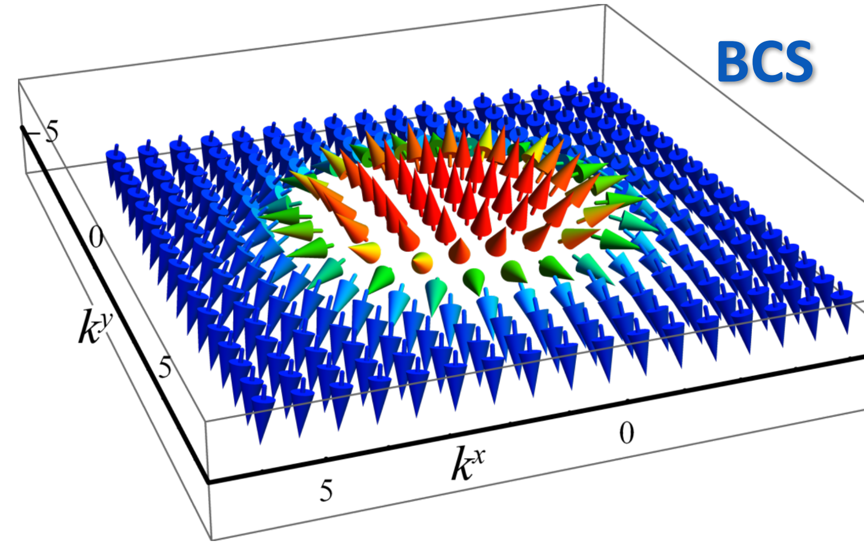
- ***Weak-pairing*** **BCS state topologically non-trivial**
- ***Strong-pairing*** **BEC state topologically trivial**

Retarded GF winding number *W* :

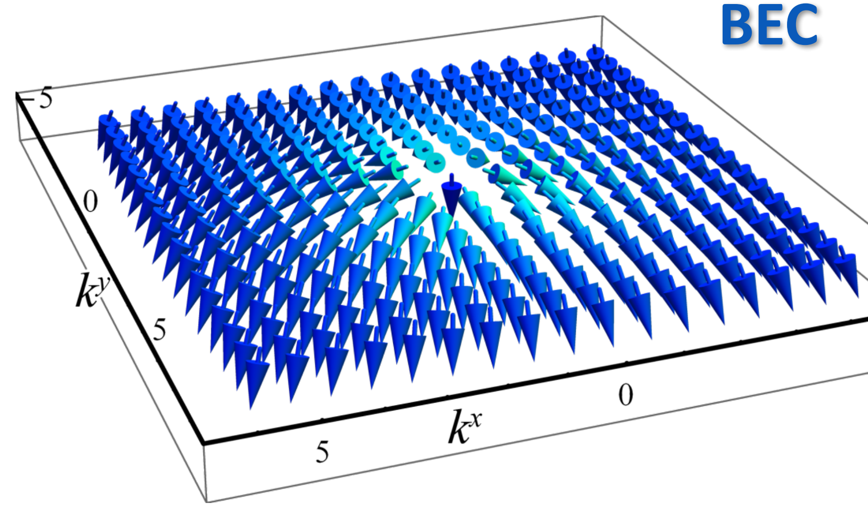
$$W \equiv \frac{\pi\epsilon^{\alpha\beta\gamma}}{3}\mathrm{Tr}\int_{\omega,\mathbf{k}} \left(\hat{G}^{-1}\partial_{k^\alpha}\hat{G}\right)\left(\hat{G}^{-1}\partial_{k^\beta}\hat{G}\right)\left(\hat{G}^{-1}\partial_{k^\gamma}\hat{G}\right)$$

- ✓ **Same as pseudospin winding *Q* in ground state**
- ✓ **Signals presence of chiral edge states**

BCS



BEC



Far from equilibrium topological superconductivity

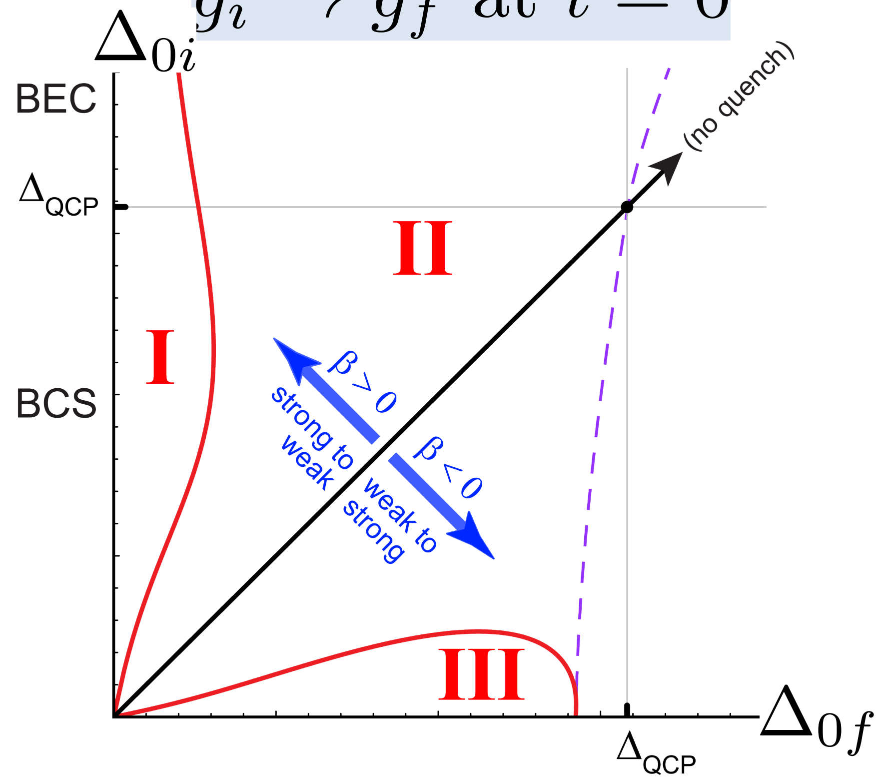
2D p-wave BCS Hamiltonian

$$H = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s_{\mathbf{k}}^z - g \sum_{\mathbf{k},\mathbf{p}} \mathbf{k}\cdot\mathbf{p}\, s_{\mathbf{k}}^+ s_{\mathbf{p}}^-$$

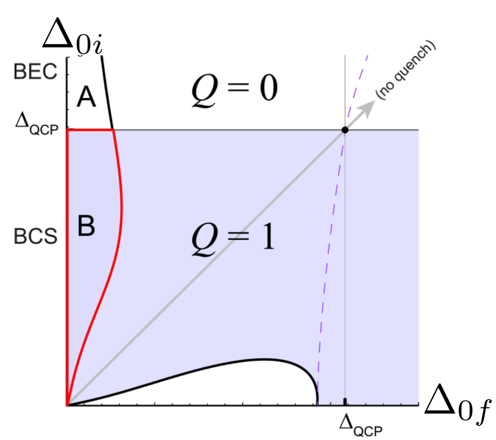
I. Same main three far from equilibrium phases

II. Richer topological features as compared to equilibrium, e.g. 2 winding #s instead of 1

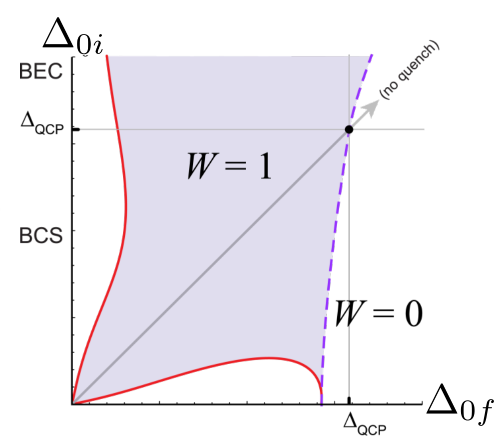
$g_i \to g_f$ at $t = 0$



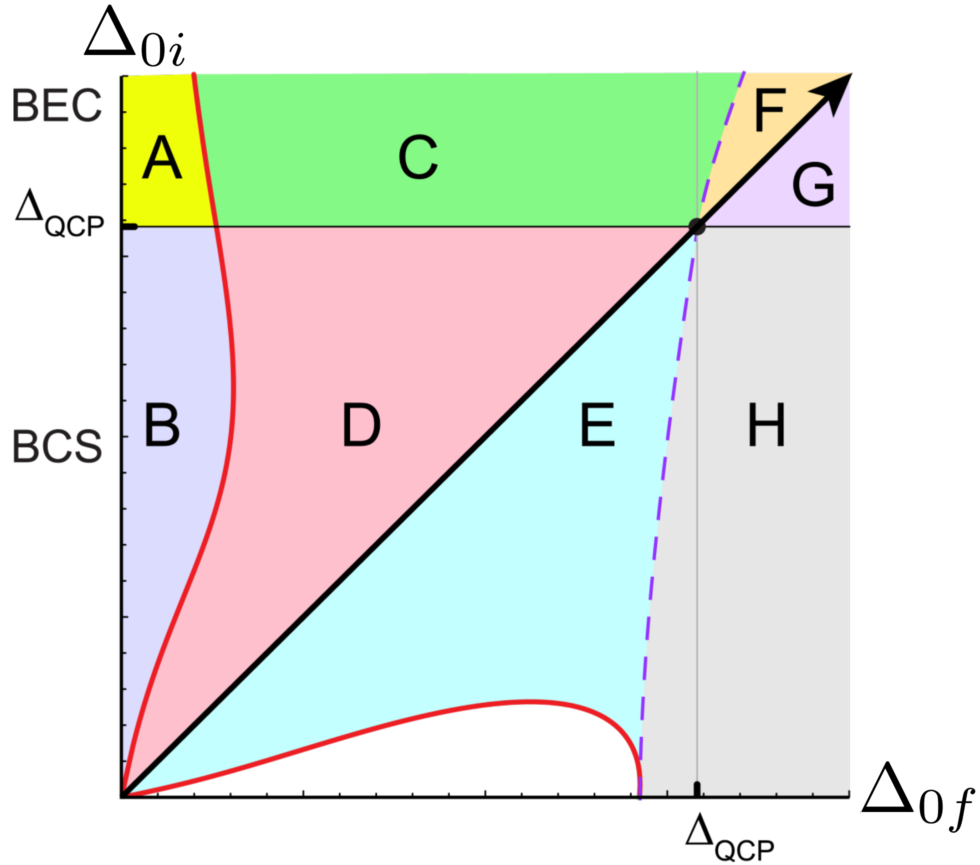
Pseudospin winding Q



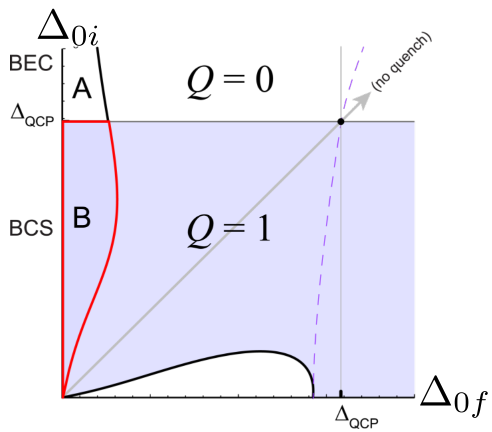
Retarded GF winding W



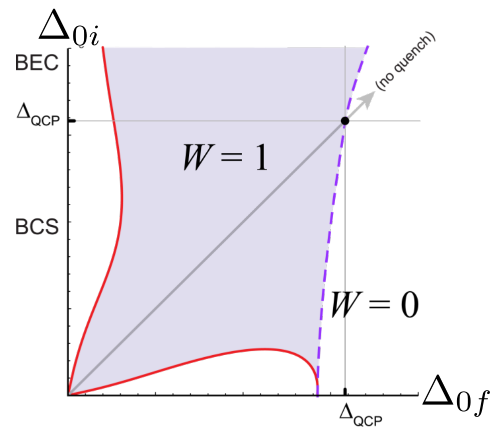
Post-quench topology: Regions



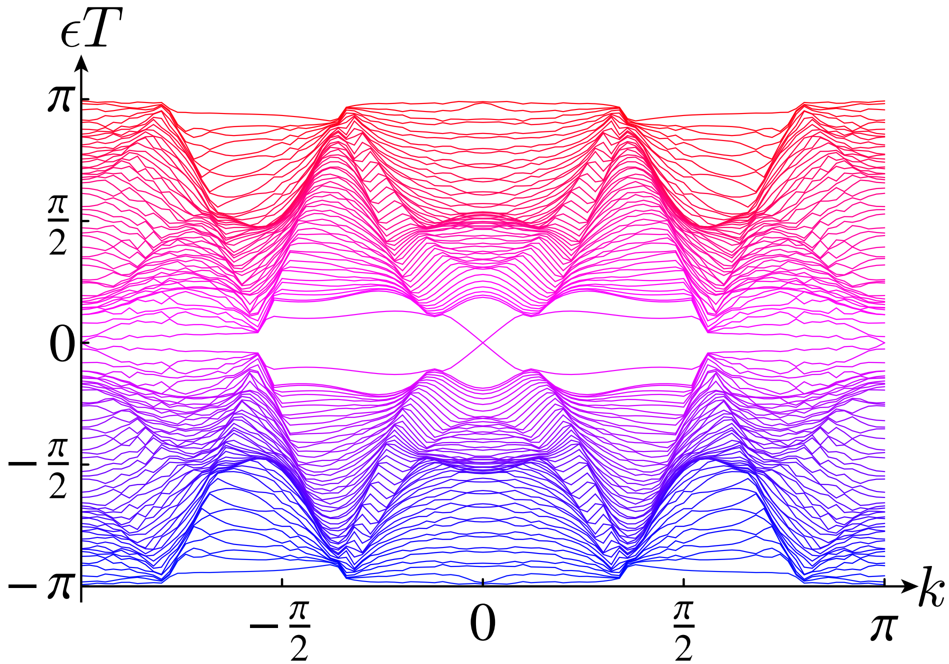
Pseudospin winding Q

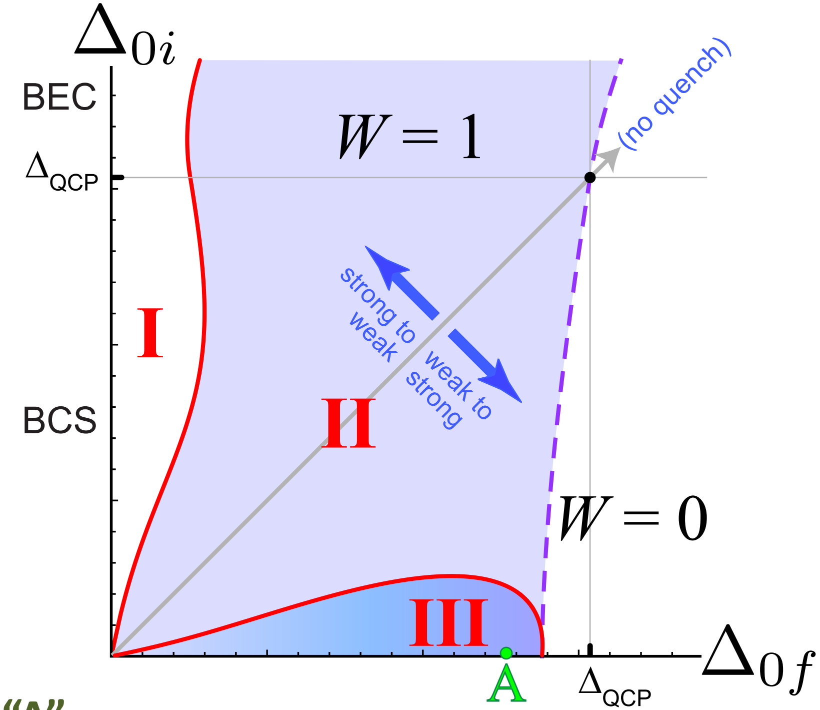


Retarded GF winding W



Quench-induced Floquet topological p-wave superfluids





Floquet spectrum for a quench in Region III, point "A". Majorana edge-modes for a time-dependent state of p-wave superfluidity are xing in the center.

No external drive – quench-induced!

$$g_i \to g_f \text{ at } t = 0$$

Foster, Gurarie, Dzero, E.Y., PRL (2014)

All this happens in time. What about space?

$|\psi(0)\rangle = |\text{gr. state for } g_i\rangle$ – homogeneous in space

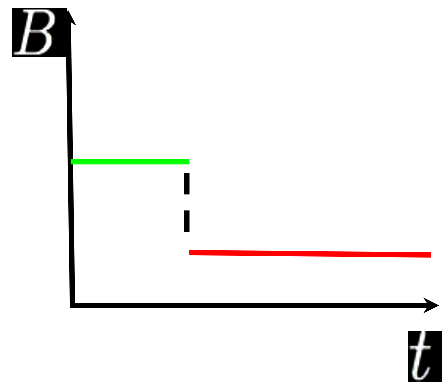
$g_i \to g_f \text{ at } t = 0$ – spatially uniform quench

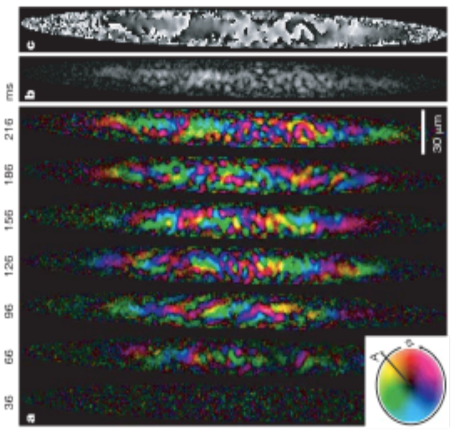
$\Delta(t)$ – homogeneous in space

Can spatial inhomogeneities be induced by a uniform quench?

Pattern formation: cosmology in a lab

Parameter (coupling) quench – “Big Bang”





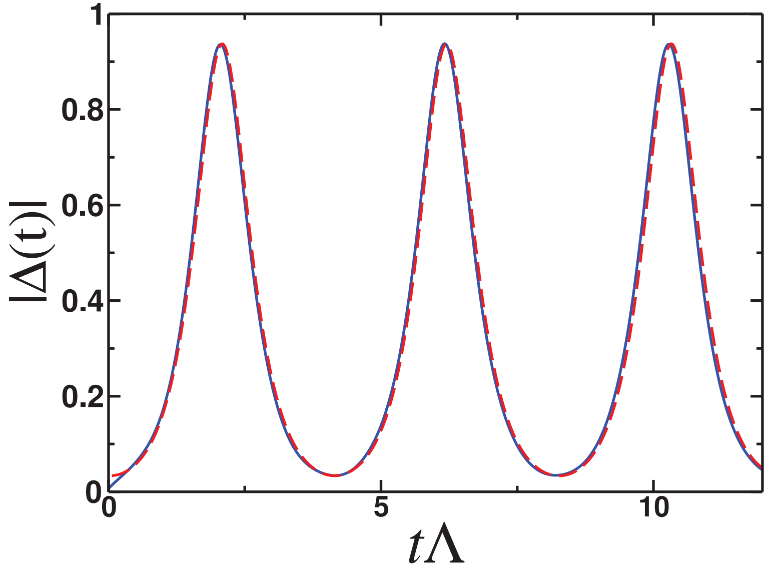
magnetic domain formation in ferromagnetic BEC following a sudden quench of the applied magnetic field, Sadler et al., Nature (London), 2006

Quench-induced parametric resonance??

Phase III:
Order parameter oscillates periodically

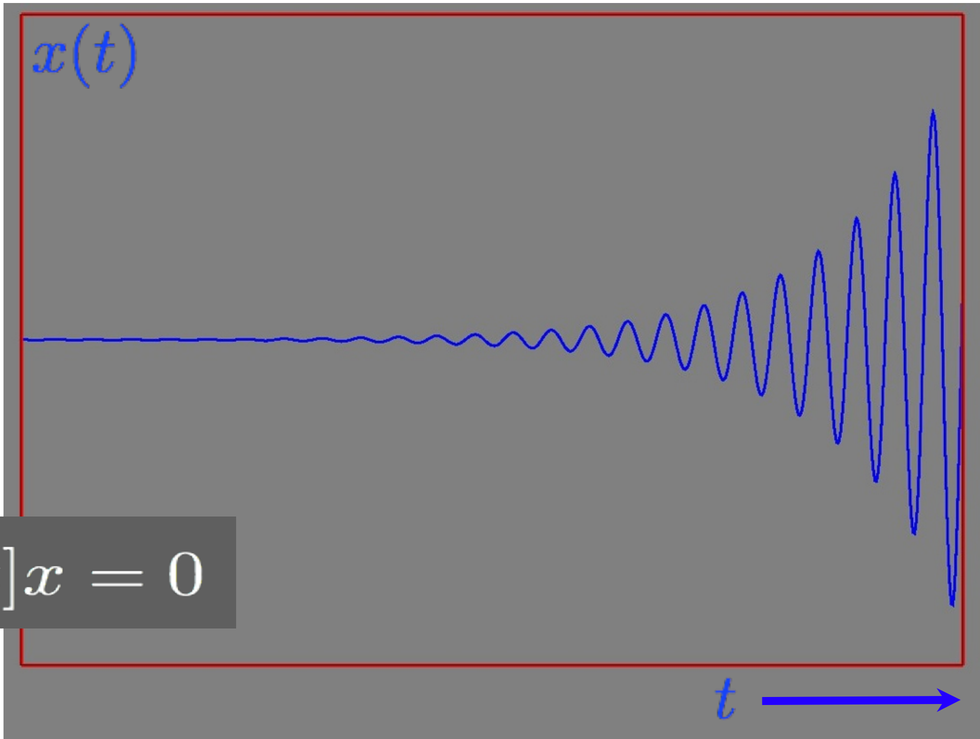
$$\frac{d\vec{s}_{\mathbf{k}}}{dt} = (-2\vec{\Delta} + 2\epsilon_{\mathbf{k}}\hat{z}) \times \vec{s}_{\mathbf{k}}$$

$$|\vec{b}_{\mathbf{k}}| = \sqrt{\epsilon_{\mathbf{k}}^2 + |\vec{\Delta}|^2}$$

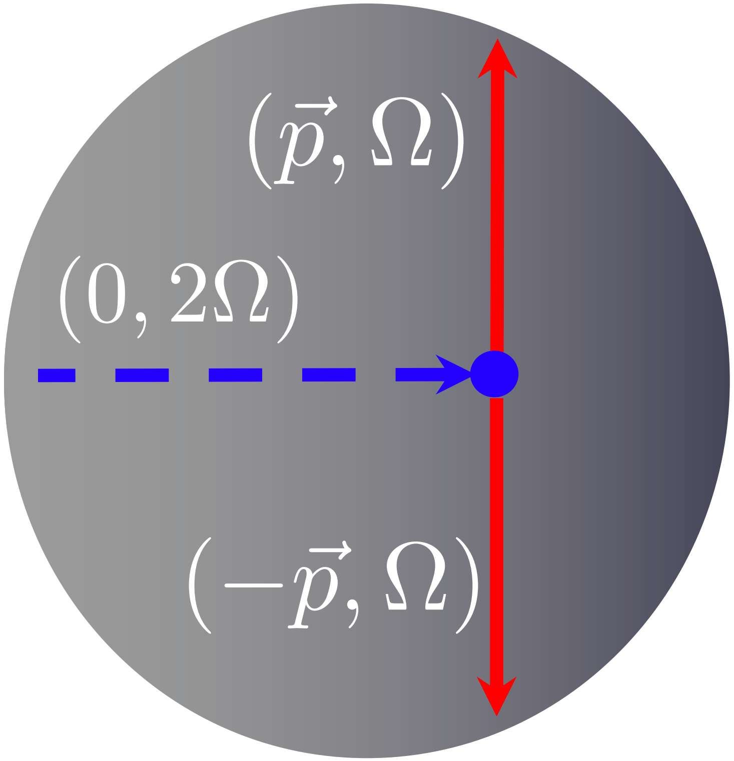


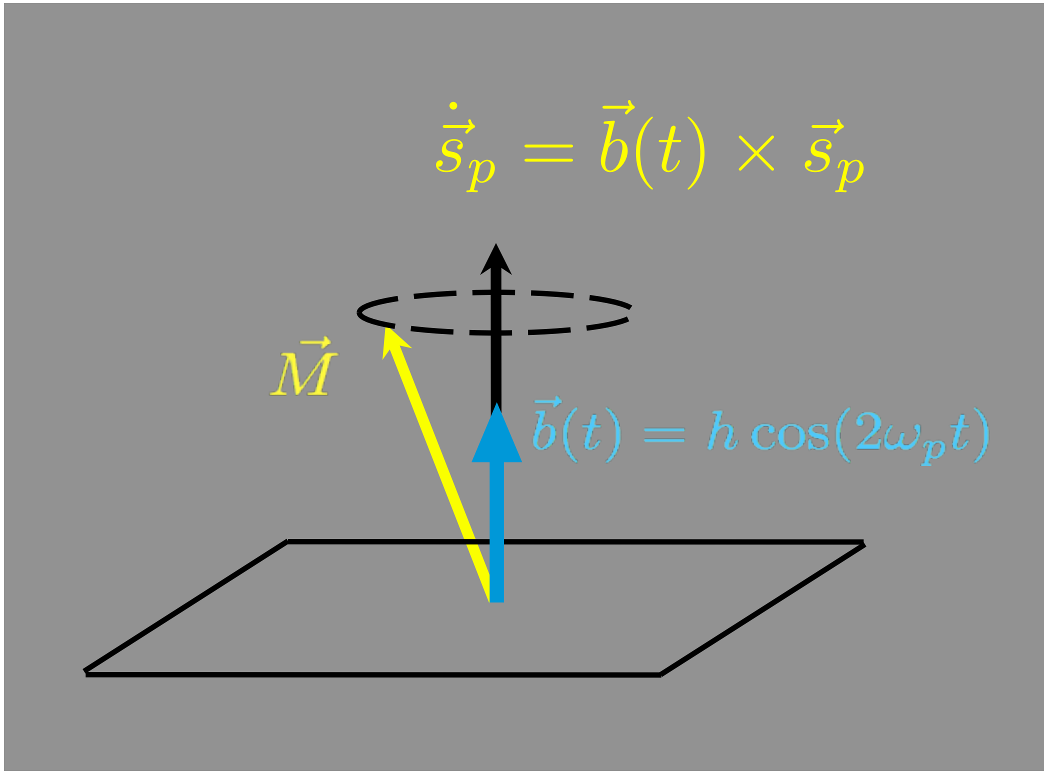
Parametric resonance in continuous media?

$$\ddot{x} + \omega_0^2[1 + h\cos(2\omega_0 + \varepsilon)t]x = 0$$



Spin wave turbulence





dielectric ferromagnet in a uniaxial field (YIG)

microscopic theory of spin wave turbulence

Zakharov, L'vov & Starobinets, 1974

Cooper pair turbulence

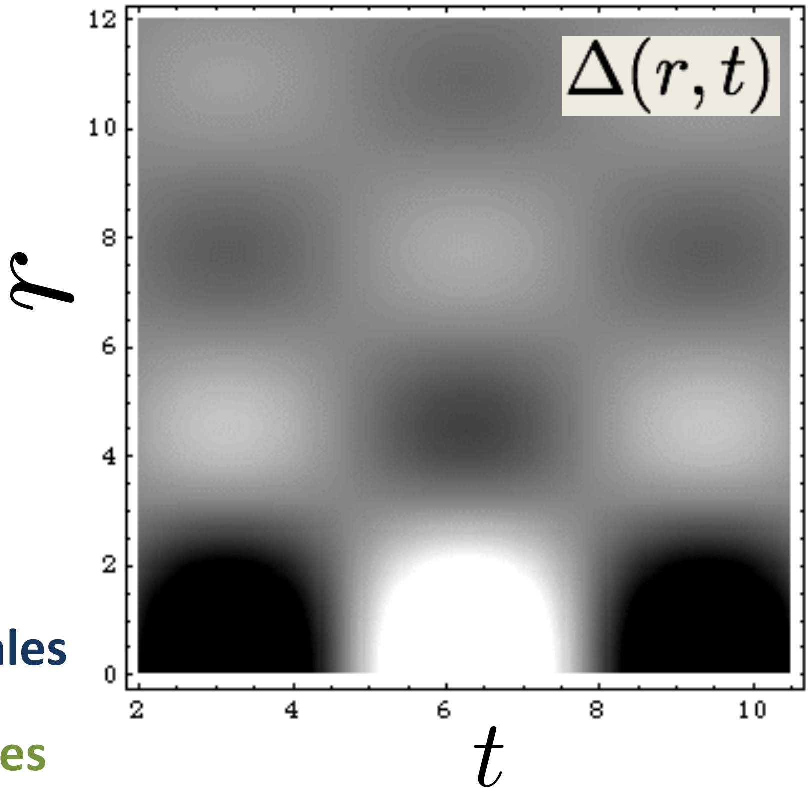
$$\delta\Delta(\vec{r},t) = \frac{\sqrt{q}\Delta_s c_s \sin(k_s|\vec{r}-\vec{r}_0|)}{k_s|\vec{r}-\vec{r}_0|}A(t)$$

$$L \gg \xi$$

"bubble" of a superfluid

$\vec{r}_0$ – position

$A(t)$ – periodic with random amplitude



Flow of energy to progressively smaller length scales

Typical situation – random superposition of bubbles

Spontaneous eruption of spatial inhomogeneities confirmed recently in numerical simulations of 2D attractive Hubbard model, Chern& Barros, arXiv:1803.04118v2; see also Dzero, E.Y., Altshuler, arXiv:1806.03474

Amplitude dynamics in nonadiabatic regime

Collisionless relaxation of the energy gap in superconductors

A. F. Volkov and Sh. M. Kogan

Institute of Radio and Electronics, USSR Academy of Sciences
(Submitted June 15, 1973)
Zh. Eksp. Teor. Fiz. **65**, 2038–2046 (November 1973)

See also:
Galaiko, JETP 34, 203 (1972)
Ivlev, JETP Lett. 15, 313 (1972)
Galperin, Kozub, Spivak, JETP 54, 1126 (1981)
Littlewood, Varma, Phys. Rev. B 26 4883 (1982)

Nonadiabatic regime: $t_{\rm pert} \le \tau_\Delta \ll \tau_\varepsilon$

$$\dot{\vec{s}}_{\mathbf{k}} = (-2\vec{\Delta} + 2\epsilon_{\mathbf{k}}\hat{z}) \times \vec{s}_{\mathbf{k}} + \vec{I}_{\rm coll}(\mathbf{k})$$

$$H_{\rm BCS} = \sum_{\mathbf{k}} 2\epsilon_{\mathbf{k}} s^z_{\mathbf{k}} - g \sum_{\mathbf{k},\mathbf{p}} s^+_{\mathbf{k}} s^-_{\mathbf{p}}$$

$$\vec{I}_{\rm coll}(\mathbf{k}) \sim \delta\vec{s}_{\mathbf{k}}/\tau_\varepsilon, \quad \dot{\vec{s}}_{\mathbf{k}} \sim \delta\vec{s}_{\mathbf{k}}/\tau_\Delta$$

$\vec{s}_{\mathbf{k}}$ – **classical spins (vectors)**, $|\vec{s}_{\mathbf{k}}| = 1$

H_{2ch} + non-condensed modes: pair-breaking rates in the long time steady state

$$\hat{V}(t) = g \sum_{\mathbf{p_1},\mathbf{p_2}} \left[\hat{b}^\dagger_{\mathbf{p}_1+\mathbf{p}_2}(t)\hat{c}_{\mathbf{p}_1\uparrow}(t)\hat{c}_{\mathbf{p}_2\downarrow}(t) + \hat{b}_{\mathbf{p}_1+\mathbf{p}_2}(t)\hat{c}^\dagger_{\mathbf{p}_2\downarrow}(t)\hat{c}^\dagger_{\mathbf{p}_1\uparrow}(t) \right]$$

- **Molecular production rate**
  - **Much slower for quenches to the far BCS side**
  - **For quenches to the far BEC side (in the steady state)**

$$\tau^{-1}_{\mathrm{mol}} \sim \frac{\gamma \Delta^2_\infty \Delta_{0i}}{\epsilon_F |\mu_\infty|} \to 0$$

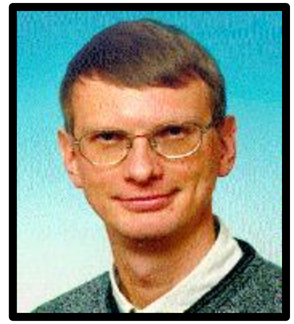
$$\tau^{-1}_{\mathrm{dyn}} \sim |\mu_\infty|, \quad \tau_{\mathrm{dyn}} \ll \tau_{\mathrm{mol}}$$

- **Two-particle collisions**
  - **For quenches to far BCS**

$$\tau^{-1}_{\mathrm{in}} \sim \left(\frac{g^2 \nu_F}{\omega_f}\right)^2 \frac{\Delta^2_\infty}{\epsilon_F} = \gamma^2 \epsilon_F \left(\frac{\Delta_\infty}{\omega_f}\right)^2, \quad \tau^{-1}_{\mathrm{dyn}} \sim |\Delta_\infty|, \quad \tau_{\mathrm{dyn}} \ll \tau_{\mathrm{mol}}$$

  - **For quenches to far BEC: similar result**

Boris Altshuler
Columbia University



Vadim Kuznetsov
Leeds University

Sasha Tsyplyatyev
University of Sheffield

Maxim Dzero
Kent State University

Matt Foster
Rice University

Victor Gurarie
University of Colorado, Boulder

National Science Foundation
WHERE DISCOVERIES BEGIN

the David & Lucile Packard FOUNDATION