Dynamical Vanishing of the Order Parameter in a Fermionic Condensate

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(Received 17 March 2006; published 16 June 2006)

We analyze the dynamics of a condensate of ultracold atomic fermions following an abrupt change of the pairing strength. At long times, the system goes to a nonstationary steady state, which we determine exactly. The superfluid order parameter asymptotes to a constant value. We show that the order parameter *vanishes* when the pairing strength is decreased below a certain critical value. In this case, the steady state of the system combines properties of normal and superfluid states—the gap and the condensate fraction vanish, while the superfluid density is nonzero.

PACS numbers: 03.75.Kk, 74.20.Fg

Recently, several remarkable experiments have demonstrated Cooper pairing in cold atomic Fermi gases [1–4]. Key signatures of a paired state—condensation of Cooper pairs [1,2] and the pairing gap [4] have been observed. In addition, trapped gases provide a unique tool to explore aspects of fermion pairing normally inaccessible in superconductors. One of the most exciting prospects is a study of far from equilibrium coherent dynamics of fermionic condensates [5–9], made possible due to the precise experimental control over interactions between atoms [10,11]. The dynamics can be initiated by quickly changing the pairing strength with external magnetic field.

In the present Letter, we determine the time evolution of a fermionic condensate in response to a sudden change of interaction strength. Initially, the gas is in equilibrium at zero temperature on the BCS side of the Feshbach resonance with a coupling constant $g_i > 0$. At $t = 0$ the coupling is suddenly changed to a smaller value $g_f > 0$ on the same side of the resonance, $g_i \rightarrow g_f$, Fig. 1 (inset). Ground states of the system at the old, g_i, and new, g_f, values of the coupling are characterized by corresponding BCS gaps, Δ_i and Δ_f, respectively. We consider the case $\Delta_i \geq \Delta_f$. It has been shown previously that following the change of coupling, the time-dependent order parameter $\Delta(t)$ asymptotes to a constant value [9], $|\Delta(t)| \rightarrow \Delta_\infty$ on a time scale $\tau_\Delta = 1/\Delta_i$. Here we evaluate Δ_∞ in terms of Δ_i and Δ_f.

We show that when the coupling is decreased below a certain critical value, Δ_∞ *vanishes*, Fig. 1. On a τ_Δ time scale the system goes to a steady *nonstationary* state that combines properties of normal and superfluid states in a peculiar way. For example, the gap vanishes, while the superfluid density remains finite. Provided the system is continuously cooled, the BCS ground state with a gap Δ_f is reached on the energy relaxation time scale τ_ϵ, which is typically much larger than τ_Δ. Experimental signatures of the novel state include the absence of the gap in rf absorption spectrum, and zero condensate fraction after a fast projection onto the Bose-Einstein condensation (BEC) side (see below).

At times $t \ll \tau_\epsilon$, dynamics of the condensate in the weak coupling regime can be described by the BCS model. Here we are interested in the thermodynamic limit, in which case one can use the BCS mean-field approach [12]. Using Anderson's pseudospin representation [12], one can describe the mean-field evolution by a *classical* spin Hamiltonian [5,6,12]

$$H = \sum_j 2\epsilon_j s_j^z - g \sum_{j,k} s_j^+ s_k^-, \qquad (1)$$

where ϵ_j are single-particle energies relative to the Fermi level and $s_j^\pm = s_j^x \pm i s_j^y$. The summation in Eq. (1) is over $|\epsilon_j| < E_F$, where E_F is the Fermi energy. Dynamical variables $\mathbf{s}_j$ are vectors of fixed length, $|\mathbf{s}_j| = 1/2$. The BCS order parameter is $\Delta(t) = \Delta_x - i\Delta_y = g \sum_j s_j^-$. Equations of motion for classical spins $\mathbf{s}_j$ are
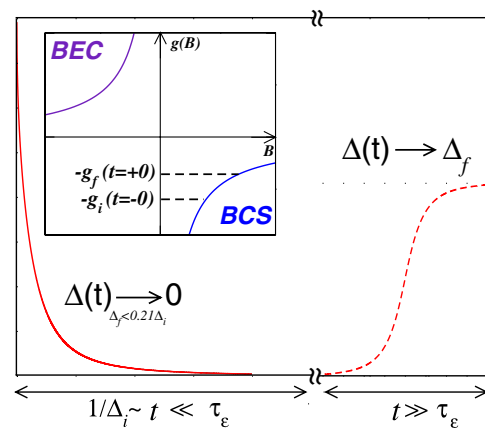

FIG. 1 (color online). Time evolution of the BCS order parameter $\Delta(t)$ following an abrupt change of the coupling constant. The coupling is changed by varying the magnetic field on the BCS side of the Feshbach resonance (inset). The order parameter exponentially decays to zero (solid line) at times $t \sim 1/\Delta_i$ and the system goes into a *gapless* steady state. If the system is continuously cooled, the order parameter recovers to its equilibrium value Δ_f (dashed) at times longer than the quasiparticle relaxation time τ_ϵ.

$$\dot{\mathbf{s}}_j = \mathbf{b}_j \times \mathbf{s}_j, \qquad \mathbf{b}_j = (-2\Delta_x, -2\Delta_y, 2\epsilon_j). \quad (2)$$

Components of spins are related to Bogoliubov amplitudes u_j and v_j

$$2s_j^z = |v_j|^2 - |u_j|^2, \qquad s_j^- = \bar{u}_j v_j, \quad (3)$$

At $t = 0$ the system is in the ground state with gap Δ_i. The ground state is obtained by aligning each spin $\mathbf{s}_j$ antiparallel to its "magnetic" field $\mathbf{b}_j$ [12] in order to minimize the total energy (1) for coupling constant $g = g_i$

$$s_j^x(0) = \frac{\Delta_i}{2\sqrt{\epsilon_j^2 + \Delta_i^2}}, \qquad s_j^z(0) = -\frac{\epsilon_j}{2\sqrt{\epsilon_j^2 + \Delta_i^2}}, \quad (4)$$

and $s_j^y(t = 0) = 0$.

At $t = 0$ the coupling is changed, $g_i \rightarrow g_f$, and the initial spin configuration (4) is no longer an equilibrium for $t > 0$. To determine the time evolution of the system, one has to solve equations of motion (2) with initial conditions (4).

We start with a linear analysis. Solving Eq. (2) (with $g = g_f$) linearized around the spin configuration (4), we obtain up to terms of order $\delta\Delta/\Delta_f$

$$\Delta(t) = \Delta_f - 8\delta\Delta \int_0^\infty d\epsilon \frac{\cos\omega(\epsilon)t}{\omega(\epsilon)[\pi^2 + h^2(\epsilon)]}, \quad (5)$$

where $\delta\Delta = \Delta_f - \Delta_i$, $\omega(\epsilon) = 2\sqrt{\epsilon^2 + \Delta_f^2}$, and $h(\epsilon) = \sinh^{-1}(\epsilon/\Delta_f)$. In Eq. (5), besides the continuum limit, we took the weak coupling limit $E_F/\Delta_i \rightarrow \infty$.

The long time behavior of the order parameter in the linear approximation is obtained from Eq. (5) by the stationary phase method (see also Ref. [13]),

$$\Delta(t) = \Delta_f - \frac{2\delta\Delta}{\pi^{3/2}\sqrt{\Delta_f t}} \cos\left(2\Delta_f t + \frac{\pi}{4}\right), \quad (6)$$

At times $t \gg \tau_\Delta$ the gap approaches a constant value $\Delta_\infty = \Delta_f$ to order $\delta\Delta/\Delta_f$.

Even though the gap is constant, the state of the system is nonstationary [8]. According to Eq. (2), at large times each spin $\mathbf{s}_j \equiv \mathbf{s}(\epsilon_j)$ precesses in its own constant field $\mathbf{b}_j = (-2\Delta_\infty, 0, 2\epsilon_j)$. For example, for the x component of spins we derive from the linearized equations of motion,

$$s_x(\epsilon) = \frac{\Delta_f}{2\sqrt{\epsilon^2 + \Delta_f^2}} - \frac{\delta\Delta\epsilon}{(\epsilon^2 + \Delta_f^2)\sqrt{\pi^2 + h^2(\epsilon)}}$$
$$\times \cos[\omega(\epsilon)t + \phi(\epsilon)], \quad (7)$$

However, the gap $\Delta(t) = g\sum_j s_j^x(t)$ contains oscillations with many different frequencies. At large times they go out of phase and cancel out in the continuum limit.

In the nonlinear case it can be shown that the gap decays to a constant Δ_∞ by a similar mechanism [9]. To determine Δ_∞, we use the exact solution for the dynamics of the BCS model [6]. Consider the following vector function of an

auxiliary parameter u: $\mathbf{L}(u) = -\hat{\mathbf{z}}/g + \sum_j \mathbf{s}_j/(u - \epsilon_j)$, where $\hat{\mathbf{z}}$ is a unit vector along the z axis. Using Eq. (2), we obtain $d\mathbf{L}^2(u)/dt = 0$, i.e., $\mathbf{L}^2(u)$ is conserved for any u.

To determine the steady state gap Δ_∞, we evaluate $\mathbf{L}^2(u)$ at $t = 0$ for the initial spin configuration (4) and at $t \gg \tau_\Delta$, when each spin $\mathbf{s}(\epsilon)$ precesses in a constant field. Matching the two expressions, we derive for $\Delta_f \leq \Delta_i$

$$\Delta_f = \Delta_i \exp(-\kappa \tan\kappa/2), \qquad \Delta_\infty = \Delta_i \cos\kappa, \quad (8)$$

where $0 \leq \kappa \leq \pi/2$. This equation determines Δ_∞ in terms of Δ_i and Δ_f, see Fig. 2.

We make several observations. The steady state gap reaches its maximum $\Delta_\infty = \Delta_f$ at $\Delta_f/\Delta_i = 1$. Otherwise, $\Delta_\infty < \Delta_f$. Expanding around the maximum, we find $\Delta_\infty = \Delta_f - (\delta\Delta)^2/6\Delta_f$ up to terms of higher order in $\delta\Delta/\Delta_f$. The linear term vanishes in agreement with Eq. (6).

Most interestingly, we see from Eq. (8) that the order parameter vanishes in the steady state, $\Delta_\infty = 0$, when $\kappa = \pi/2$, i.e., for $(\Delta_f/\Delta_i)_c = e^{-\pi/2} \approx 0.21$. If Δ_f/Δ_i is below this critical ratio, Eq. (8) has no solutions—the system goes into a gapless steady state, $\Delta_\infty = 0$, for $\Delta_f \leq 0.21\Delta_i$. We also note from Eq. (8) and Fig. 2 that Δ_∞ has a cusp at $\Delta_f/\Delta_i = e^{-\pi/2}$; i.e., the transition to the gapless state is second order.

To gain further insight into properties of the gapless state, we need to know the spin configuration in this state. At large times, spin $\mathbf{s}(\epsilon)$ rotates in a constant field $\mathbf{b}(\epsilon) = -2\Delta_\infty\hat{\mathbf{x}} + 2\epsilon\hat{\mathbf{z}}$, where $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$ are unit vectors along the x and z axes, respectively. In the gapless case, $\mathbf{b}(\epsilon) = 2\epsilon\hat{\mathbf{z}}$. The spin configuration can be characterized by the angle $\theta(\epsilon)$ between $\mathbf{s}(\epsilon)$ and $-\mathbf{b}(\epsilon)$. This angle can be determined by matching the conserved quantity $\mathbf{L}^2(u)$ at $t = 0$ and $t \gg \tau_\Delta$,
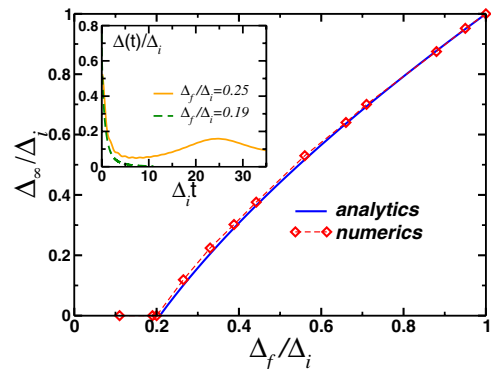

FIG. 2 (color online). After a sudden change of the BCS coupling constant, the order parameter $\Delta(t)$ (inset) saturates to a constant value Δ_∞. The plot shows the steady state gap Δ_∞ in units of Δ_i as a function of the ratio Δ_f/Δ_i. The exact result given by Eq. (8) is compared to numerical solution of Eq. (2). Note that $\Delta_\infty = 0$ for $\Delta_f \leq 0.2\Delta_i$.

$$\sin^2\theta(\epsilon) = \frac{G(\epsilon)}{2\pi^2} - \sqrt{\frac{G^2(\epsilon)}{4\pi^4} - \frac{4\beta^2\Delta_i^2}{\pi^2(\epsilon^2 + \Delta_i^2)}}, \quad (9)$$

where $\beta = \ln(\Delta_i/\Delta_f)$ and

$$G(\epsilon) = \pi^2 + 4[\sinh^{-1}(\epsilon/\Delta_i)]^2$$
$$+ \frac{8\beta\epsilon}{\sqrt{\epsilon^2 + \Delta_i^2}}\sinh^{-1}(\epsilon/\Delta_i) + 4\beta^2. \quad (10)$$

Note from Eq. (2) that in the steady state the component of spin $\mathbf{s}(\epsilon)$ along the field, $s_\parallel(\epsilon) = -\cos\theta(\epsilon)/2$, is constant, while the one perpendicular to the field, $|\mathbf{s}_\perp| = \sin\theta(\epsilon)/2$ rotates with a frequency $\omega(\epsilon) = 2\sqrt{\epsilon^2 + \Delta_\infty^2}$. The approach to the steady state can be studied by linearizing Eq. (2) around this state. In particular, we obtain the asymptotic behavior of $\Delta(t)$

$$\frac{\Delta(t)}{\Delta_\infty} = 1 + a\frac{\cos(2\Delta_\infty t + \pi/4)}{\sqrt{\Delta_\infty t}}, \quad \frac{\Delta_f}{\Delta_i} > e^{-\pi/2}, \quad (11)$$

$$\frac{\Delta(t)}{\Delta_i} = A(t)e^{-2\alpha\Delta_i t} + B(t)e^{-2\Delta_i t}, \quad \frac{\Delta_f}{\Delta_i} < e^{-\pi/2}. \quad (12)$$

Here $\alpha = -\cos p$ and $\pi/2 \leq p \leq \pi$ is the solution of $p = \ln(\Delta_i/\Delta_f)\cot(p/2)$. The parameter α has a property $\alpha \to 0$ when $\Delta_f/\Delta_i \to e^{-\pi/2}$ and $\alpha \to 1$ when $\Delta_f/\Delta_i \to 0$. The coefficient a in Eq. (11) is time independent, $A(t)$ and $B(t)$ in Eq. (12) are decaying power laws, $A(t), B(t) \propto 1/t^\nu$ with $1/2 \leq \nu \leq 2$.

We conclude that the decay law of $\Delta(t)$ changes from power law to exponential as we cross the critical point. Above the critical point spins rotate with frequencies $\omega(\epsilon) = 2\sqrt{\epsilon^2 + \Delta_\infty^2}$. The inverse square root decay and oscillations with frequency $2\Delta_\infty$ in Eq. (11) are due to the square root singularity in the spectral density $d\epsilon/d\omega \propto \omega/\sqrt{\omega^2 - 4\Delta_\infty^2}$, cf. Eqs. (5) and (6). Below the critical point Δ_∞ vanishes, $\omega(\epsilon) = 2\epsilon$, and the square root anomaly disappears.

What happens to the spin configuration as Δ_f/Δ_i is varied across the critical point? Consider a spin at energy $\epsilon_+ \ll \Delta_i$ just above the Fermi energy. Using Eq. (9), we obtain $\sin\theta(\epsilon_+) \approx \sin\theta(0) = 2\ln(\Delta_i/\Delta_f)/\pi$ for $\Delta_f > e^{-\pi/2}\Delta_i$ and $\sin\theta(0) = 1$ otherwise. The field $\mathbf{b}(\epsilon_+) = -2\Delta_\infty\hat{\mathbf{x}} + 2\epsilon_+\hat{\mathbf{z}}$ is along the x axis above the critical point ($\Delta_\infty \gg \epsilon_+$) and along the z axis below ($\Delta_\infty = 0$). At $\Delta_i = \Delta_f$ we have $\sin\theta(0) = 0$, i.e., the spin is parallel to the x axis. As Δ_f/Δ_i decreases, the x component of the spin also decreases until it vanishes at the critical point $\Delta_f = e^{-\pi/2}\Delta_i$. Just above it the spin lies in the yz plane and rotates around the x axis. Below the critical point it rotates in the xy plane around the z axis (Fig. 3).

In the gapless steady state each spin rotates around the z axis. Its z component is time independent, $s_z(\epsilon) = -\cos\theta(\epsilon)/2$. It is related to the average occupation num-
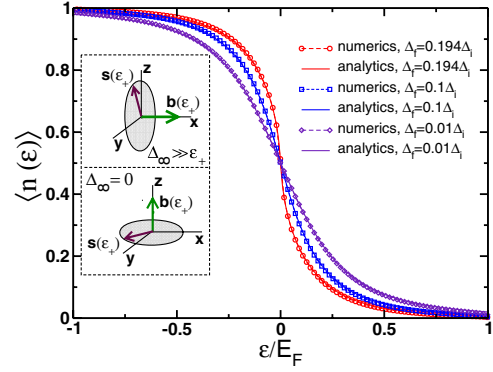

FIG. 3 (color online). Plot of the average occupation number $\langle n(\epsilon)\rangle$ as a function of energy in the gapless state for various values of $\Delta_f/\Delta_i < e^{-\pi/2}$. The exact result given by (9) is compared to the numerical solution of Eq. (2). Note that the distribution function around the Fermi level is smeared over a width $\delta\epsilon \propto \Delta_i$. The inset shows a spin $\mathbf{s}(\epsilon_+)$ at energy $0 < \epsilon_+ \ll \Delta_i$ just above and below the critical point.

ber per fermion species $\langle\hat{n}(\epsilon)\rangle$ at energy ϵ as $\langle\hat{n}(\epsilon)\rangle = s_z(\epsilon) + 1/2$ [12], i.e., $\langle\hat{n}(\epsilon)\rangle = (1 - \cos\theta(\epsilon))/2$. The distribution function $\langle\hat{n}(\epsilon)\rangle$ is smeared near the Fermi energy over a width $\delta\epsilon \propto \Delta_i$ (Fig. 3). Note that in this respect the gapless state is similar to the BCS ground state—the smearing over a width $\delta\epsilon \propto \Delta_i$ due to interactions is also present in the ground state distribution (4). In the normal state $s_z(\epsilon) = -\text{sgn}\epsilon/2$ and $\langle\hat{n}(\epsilon)\rangle = \Theta(-\epsilon)$.

Given relation (3), one can reconstruct the time-dependent condensate wave function and evaluate normal and anomalous correlation functions, e.g.,

$$\langle\hat{c}_{\epsilon\sigma}(t)\hat{c}^\dagger_{\epsilon\sigma}(t')\rangle = e^{i\epsilon(t'-t)}\cos^2\frac{\theta(\epsilon)}{2}, \quad (13)$$

$$\langle\hat{c}^\dagger_{\epsilon\uparrow}(t)\hat{c}^\dagger_{\epsilon\downarrow}(t')\rangle = e^{i\epsilon(t'+t)}\cos\frac{\theta(\epsilon)}{2}\sin\frac{\theta(\epsilon)}{2}, \quad (14)$$

where $\hat{c}_{\epsilon\sigma}$ ($\hat{c}^\dagger_{\epsilon\sigma}$) annihilates (creates) a fermion of one of the two species $\sigma = \uparrow, \downarrow$ on energy level ϵ. Note that even though Δ_∞ vanishes, anomalous averages do not.

Next, we determine the superfluid density n_s in the gapless state. Here we outline the answer, details will be reported elsewhere [14]. Consider a degenerate Fermi gas in an axially symmetric trap slowly rotating around the symmetry axis. The density n_s of the superfluid component can be defined as the rigidity of the superfluid with respect to an infinitesimal twist in the boundary conditions [15], $\hat{\psi}_\sigma(\mathbf{r}) \to e^{-i\alpha\phi}\hat{\psi}_\sigma(\mathbf{r})$, where ϕ is the azimuthal angle with respect to the symmetry axis, $\hat{\psi}_\sigma(\mathbf{r}) = \sum_j\hat{c}_{j\sigma}\varphi_j(\mathbf{r})$, and $\varphi_j(\mathbf{r})$ are the single-particle wave functions. The twist generates a term $-\alpha\hat{J}$ in the Hamiltonian, where $\hat{J}$ is the current operator. The resulting supercurrent $J_s = n_s\alpha/m$ can be expressed in terms of correlation functions (13) and (14) using the standard linear response theory. We find $n_s = n/2$, where n is the particle density in the normal state. A similar calculation for nonzero Δ_∞ yields $n_s = n$—the

superfluid density in the gapped steady state is the same as in the BCS ground state. Since n_s is the second derivative of the free energy with respect to α, the jump from n to $n/2$ at the critical point agrees with the second order character of the transition. The reduction in n_s in the state with zero Δ_∞ is consistent with vanishing of the gap in this state [16].

We see that the superfluid density is finite in the gapless steady state. This reflects the existence of pair correlations (14) between fermions, a property that in equilibrium has been directly linked to nonzero n_s [17]. This situation is similar to the phenomenon of gapless superconductivity in metals [18], where the superconducting state is also characterized by nonvanishing anomalous Green's functions and zero spectral gap. A gapless state in superconductors is usually a consequence of strong perturbations—as the perturbation is increased, the superconducting metal first goes into a gapless state with finite n_s before it becomes normal.

A crude qualitative understanding of the dynamical transition to the gapless steady state can be derived from the following thermodynamic argument. After the change of coupling the initial state (4) has energy $\mathcal{E}(\Delta_f, \Delta_i) > 0$ relative to the ground state with gap Δ_f. In thermal equilibrium the system would have the same energy at a certain temperature $T_0(\Delta_f, \Delta_i)$. Let us keep Δ_i fixed and vary Δ_f. At $\Delta_f = \Delta_i$, $T_0 = 0$, while the critical temperature $T_c \approx 0.57\Delta_f > 0$. As Δ_f increases both T_0 and T_c grow, but T_c grows faster and T_0 never catches up with it. On the other hand, when Δ_f decreases, T_c also decreases, while T_0 grows. Evaluating $T_0(\Delta_f, \Delta_i)$, to the first order in the coupling constant we find that $T_0 \gtrsim T_c$ for $\Delta_f/\Delta_i \lesssim 0.52 + 0.17\lambda$ and $T_0 < T_c$ otherwise. Here $\lambda = g/d$ is the dimensionless BCS coupling constant and $d = \langle \epsilon_{j+1} - \epsilon_j \rangle$ is the mean level spacing. We see that decreasing the coupling beyond a certain value provides enough energy for the transition to the normal state to occur in thermal equilibrium.

Let us discuss experimental manifestations of the dynamical transition to the gapless steady state. After the initial sweep ($g_i \rightarrow g_f$) on the BCS side of the resonance such that $\Delta_f/\Delta_i < e^{-\pi/2}$, the gapless state is reached on a time scale $\tau_\Delta = 1/\Delta_i$. The pairing gap measured in Ref. [4] is the energy cost ΔE of breaking a Cooper pair. This corresponds [12] to removing the spin closest to the Fermi level in (1). The minimum energy cost is $\Delta E = \min|\mathbf{b}(\epsilon) \cdot \mathbf{s}(\epsilon)| = \Delta_\infty \cos\theta(0)$. Using Eq. (9), we obtain $\Delta E = \Delta_\infty \sqrt{1 - 4\ln^2(\Delta_i/\Delta_f)/\pi^2}$. In the steady state, $\Delta E = 0$, i.e., no gap in the rf absorbtion spectrum will be observed. Molecular condensate fraction [1,2] *vanishes* in the gapless state, since $N_{k=0}/N \propto \Delta_\infty^2$ [19].

In conclusion, we determined the dynamics of the paired state of cold atomic fermions following an abrupt lowering of the pairing strength, $g_i \rightarrow g_f$. On a short $1/\Delta_i$ time

scale, where Δ_i is the equilibrium gap for the initial coupling g_i, the system goes to a nonstationary steady state with a constant gap Δ_∞ and a distribution function given by Eqs. (8) and (9). When the coupling is reduced so that Δ_f/Δ_i is smaller than the critical value of $e^{-\pi/2}$, the steady state gap vanishes. The decay law of the time-dependent order parameter $\Delta(t)$ changes from power law to exponential as Δ_f/Δ_i goes through the critical point Eqs. (11) and (12). The gapless state combines features of normal and superfluid states. In particular, the gap and the condensate fraction vanish, while the superfluid density is nonzero.

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